



# Contemporary Linear Algebra

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## ABOUT THE AUTHORS

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# GUIDE FOR THE INSTRUCTOR

## Number of Lectures

The Syllabus Guide below provides for a 29-lecture core and a 35-lecture core. The 29-lecture core is for schools with time constraints, as with abbreviated summer courses. Both core programs can be supplemented by starred topics as time permits. The omission of starred topics does not affect the readability or continuity of the core topics.

## Pace

The core program is based on covering one section per lecture, but whether you can do this in every instance will depend on your teaching style and the capabilities of your particular students. For longer sections we recommend that you just highlight the main points in class and leave the details for the students to read. Since the reviews of this text have praised the clarity of the exposition, you should find this workable. If, in certain cases, you want to devote more than one lecture to

a core topic, you can do so by adjusting the number of starred topics that you cover.

By the end of Lecture 15 the following concepts will have been covered in a basic form: linear combination, spanning, subspace, dimension, eigenvalues, and eigenvectors. Thus, even with a relatively slow pace you will have no trouble touching on all of the main ideas in the course.

## Organization

It is our feeling that the most effective way to teach abstract vector spaces is to place that material at the end (Chapter 9), at which point it occurs as a “natural generalization” of the earlier material, and the student has developed the “linear algebra maturity” to understand its purpose. However, we recognize that not everybody shares that philosophy, so we have designed that chapter so it can be moved forward, if desired.

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