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Binomial Models in Finance

With 3 Figures and 25 Tables

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Preface

This book describes the modelling of prices of financial assets in a simple discrete time, discrete state, binomial framework. By avoiding the mathematical technicalities of continuous time finance we hope we have made the material accessible to a wide audience. Some of the developments and formulae appear here for the first time in book form.

We hope our book will appeal to various audiences. These include MBA students, upper level undergraduate students, beginning doctoral students, quantitative analysts at a basic level and senior executives who seek material on new developments in finance at an accessible level.

The basic building block in our book is the one-step binomial model where a known price today can take one of two possible values at a future time, which might, for example, be tomorrow, or next month, or next year. In this simple situation “risk neutral pricing” can be defined and the model can be applied to price forward contracts, exchange rate contracts and interest rate derivatives. In a few places we discuss multinomial models to explain the notions of incomplete markets and how pricing can be viewed in such a context, where unique prices are no longer available.

The simple one-period framework can then be extended to multi-period models. The Cox-Ross-Rubinstein approximation to the Black Scholes option pricing formula is an immediate consequence. American, barrier and exotic options can all be discussed and priced using binomial models. More precise modelling issues such as implied volatility trees and implied binomial trees are treated, as well as interest rate models like those due to Ho and Lee; and Black, Derman and Toy.

The book closes with a novel discussion of real options. In that chapter we present some new ideas for pricing options on non-tradeable assets where the standard methods from financial options no longer apply. These methods provide an integration of financial and actuarial pricing techniques.

VIII Preface

Practical applications of the ideas and problems can be implemented using a simple spreadsheet program such as Excel. Many practical suggestions for implementing and calibrating the models discussed appear here for the first time in book form.

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