

---

# AN INTRODUCTION TO OPTIMIZATION

---

Fourth Edition

**Edwin K. P. Chong**

Colorado State University

**Stanislaw H. Żak**

Purdue University

 **WILEY**

A JOHN WILEY & SONS, INC., PUBLICATION

# CONTENTS

---

Preface xiii

## PART I MATHEMATICAL REVIEW

<b>1</b>	<b>Methods of Proof and Some Notation</b>	<b>3</b>
1.1	Methods of Proof	3
1.2	Notation	5
	Exercises	6
<b>2</b>	<b>Vector Spaces and Matrices</b>	<b>7</b>
2.1	Vector and Matrix	7
2.2	Rank of a Matrix	13
2.3	Linear Equations	17
2.4	Inner Products and Norms	19
	Exercises	22
<b>3</b>	<b>Transformations</b>	<b>25</b>
3.1	Linear Transformations	25

vii

3.2	Eigenvalues and Eigenvectors	26
3.3	Orthogonal Projections	29
3.4	Quadratic Forms	31
3.5	Matrix Norms	35
	Exercises	40
<b>4</b>	<b>Concepts from Geometry</b>	<b>45</b>
4.1	Line Segments	45
4.2	Hyperplanes and Linear Varieties	46
4.3	Convex Sets	48
4.4	Neighborhoods	50
4.5	Polytopes and Polyhedra	52
	Exercises	53
<b>5</b>	<b>Elements of Calculus</b>	<b>55</b>
5.1	Sequences and Limits	55
5.2	Differentiability	62
5.3	The Derivative Matrix	63
5.4	Differentiation Rules	67
5.5	Level Sets and Gradients	68
5.6	Taylor Series	72
	Exercises	77
<b>PART II UNCONSTRAINED OPTIMIZATION</b>		
<b>6</b>	<b>Basics of Set-Constrained and Unconstrained Optimization</b>	<b>81</b>
6.1	Introduction	81
6.2	Conditions for Local Minimizers	83
	Exercises	93
<b>7</b>	<b>One-Dimensional Search Methods</b>	<b>103</b>
7.1	Introduction	103
7.2	Golden Section Search	104
7.3	Fibonacci Method	108
7.4	Bisection Method	116
7.5	Newton's Method	116
7.6	Secant Method	120
7.7	Bracketing	123

7.8	Line Search in Multidimensional Optimization	124
	Exercises	126
<b>8</b>	<b>Gradient Methods</b>	<b>131</b>
8.1	Introduction	131
8.2	The Method of Steepest Descent	133
8.3	Analysis of Gradient Methods	141
	Exercises	153
<b>9</b>	<b>Newton's Method</b>	<b>161</b>
9.1	Introduction	161
9.2	Analysis of Newton's Method	164
9.3	Levenberg-Marquardt Modification	168
9.4	Newton's Method for Nonlinear Least Squares	168
	Exercises	171
<b>10</b>	<b>Conjugate Direction Methods</b>	<b>175</b>
10.1	Introduction	175
10.2	The Conjugate Direction Algorithm	177
10.3	The Conjugate Gradient Algorithm	182
10.4	The Conjugate Gradient Algorithm for Nonquadratic Problems	186
	Exercises	189
<b>11</b>	<b>Quasi-Newton Methods</b>	<b>193</b>
11.1	Introduction	193
11.2	Approximating the Inverse Hessian	194
11.3	The Rank One Correction Formula	197
11.4	The DFP Algorithm	202
11.5	The BFGS Algorithm	207
	Exercises	211
<b>12</b>	<b>Solving Linear Equations</b>	<b>217</b>
12.1	Least-Squares Analysis	217
12.2	The Recursive Least-Squares Algorithm	227
12.3	Solution to a Linear Equation with Minimum Norm	231
12.4	Kaczmarz's Algorithm	232
12.5	Solving Linear Equations in General	236

	Exercises	244
<b>13</b>	<b>Unconstrained Optimization and Neural Networks</b>	<b>253</b>
13.1	Introduction	253
13.2	Single-Neuron Training	256
13.3	The Backpropagation Algorithm	258
	Exercises	270
<b>14</b>	<b>Global Search Algorithms</b>	<b>273</b>
14.1	Introduction	273
14.2	The Nelder-Mead Simplex Algorithm	274
14.3	Simulated Annealing	278
14.4	Particle Swarm Optimization	282
14.5	Genetic Algorithms	285
	Exercises	298
<b>PART III LINEAR PROGRAMMING</b>		
<b>15</b>	<b>Introduction to Linear Programming</b>	<b>305</b>
15.1	Brief History of Linear Programming	305
15.2	Simple Examples of Linear Programs	307
15.3	Two-Dimensional Linear Programs	314
15.4	Convex Polyhedra and Linear Programming	316
15.5	Standard Form Linear Programs	318
15.6	Basic Solutions	324
15.7	Properties of Basic Solutions	327
15.8	Geometric View of Linear Programs	330
	Exercises	335
<b>16</b>	<b>Simplex Method</b>	<b>339</b>
16.1	Solving Linear Equations Using Row Operations	339
16.2	The Canonical Augmented Matrix	346
16.3	Updating the Augmented Matrix	349
16.4	The Simplex Algorithm	350
16.5	Matrix Form of the Simplex Method	357
16.6	Two-Phase Simplex Method	361
16.7	Revised Simplex Method	364
	Exercises	369

<b>17</b>	<b>Duality</b>	<b>379</b>
	17.1 Dual Linear Programs	379
	17.2 Properties of Dual Problems	387
	Exercises	394
<b>18</b>	<b>Nonsimplex Methods</b>	<b>403</b>
	18.1 Introduction	403
	18.2 Khachiyan's Method	405
	18.3 Affine Scaling Method	408
	18.4 Karmarkar's Method	413
	Exercises	426
<b>19</b>	<b>Integer Linear Programming</b>	<b>429</b>
	19.1 Introduction	429
	19.2 Unimodular Matrices	430
	19.3 The Gomory Cutting-Plane Method	437
	Exercises	447
<b>PART IV NONLINEAR CONSTRAINED OPTIMIZATION</b>		
<b>20</b>	<b>Problems with Equality Constraints</b>	<b>453</b>
	20.1 Introduction	453
	20.2 Problem Formulation	455
	20.3 Tangent and Normal Spaces	456
	20.4 Lagrange Condition	463
	20.5 Second-Order Conditions	472
	20.6 Minimizing Quadratics Subject to Linear Constraints	476
	Exercises	481
<b>21</b>	<b>Problems with Inequality Constraints</b>	<b>487</b>
	21.1 Karush-Kuhn-Tucker Condition	487
	21.2 Second-Order Conditions	496
	Exercises	501
<b>22</b>	<b>Convex Optimization Problems</b>	<b>509</b>
	22.1 Introduction	509
	22.2 Convex Functions	512
	22.3 Convex Optimization Problems	521

22.4	Semidefinite Programming	527
	Exercises	540
<b>23</b>	<b>Algorithms for Constrained Optimization</b>	<b>549</b>
23.1	Introduction	549
23.2	Projections	549
23.3	Projected Gradient Methods with Linear Constraints	553
23.4	Lagrangian Algorithms	557
23.5	Penalty Methods	564
	Exercises	571
<b>24</b>	<b>Multiobjective Optimization</b>	<b>577</b>
24.1	Introduction	577
24.2	Pareto Solutions	578
24.3	Computing the Pareto Front	581
24.4	From Multiobjective to Single-Objective Optimization	585
24.5	Uncertain Linear Programming Problems	588
	Exercises	596
	<b>References</b>	<b>599</b>
	<b>Index</b>	<b>609</b>