Applied Linear Statistical Models

Fifth Edition

Michael H. Kutner
Emory University

Christopher J. Nachtsheim
University of Minnesota

John Neter
University of Georgia

William Li
University of Minnesota

Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St Louis
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto

Contents

PART ONE
SIMPLE LINEAR REGRESSION 1

Chapter 1
Linear Regression with One Predictor Variable 2

1.1 Relations between Variables 2
Functional Relation between Two Variables 2
Statistical Relation between Two Variables 3
1.2 Regression Models and Their Uses 5
Historical Origins 5
Basic Concepts 5
Construction of Regression Models 7
Uses of Regression Analysis 8
Regression and Causality 8
Use of Computers 9
1.3 Simple Linear Regression Model with Distribution of Error Terms Unspecified 9
Formal Statement of Model 9
Important Features of Model 9
Meaning of Regression Parameters 11
Alternative Versions of Regression Model 12
1.4 Data for Regression Analysis 12
Observational Data 12
Experimental Data 13
Completely Randomized Design 13
1.5 Overview of Steps in Regression Analysis 13
1.6 Estimation of Regression Function 15
Method of Least Squares 15
Point Estimation of Mean Response 21
Residuals 22
Properties of Fitted Regression Line 23
1.7 Estimation of Error Terms Variance σ^2 24
Point Estimator of σ^2 24
1.8 Normal Error Regression Model 26
Model 26
Estimation of Parameters by Method of Maximum Likelihood 27
Cited References 33
Problems 33
Exercises 37
Projects 38

Chapter 2
Inferences in Regression and Correlation Analysis 40

2.1 Inferences Concerning β_1 40
Sampling Distribution of b_1 41
Sampling Distribution of $(b_1 - \beta_1)/s\{b_1\}$ 44
Confidence Interval for β_1 45
Tests Concerning β_1 47
2.2 Inferences Concerning β_0 48
Sampling Distribution of b_0 48
Sampling Distribution of $(b_0 - \beta_0)/s\{b_0\}$ 49
Confidence Interval for β_0 49
2.3 Some Considerations on Making Inferences Concerning β_0 and β_1 50
Effects of Departures from Normality 50
Interpretation of Confidence Coefficient and Risks of Errors 50
Spacing of the X Levels 50
Power of Tests 50
2.4 Interval Estimation of $E\{Y_h\}$ 52
Sampling Distribution of $\hat{Y}_h$ 52
Sampling Distribution of $(\hat{Y}_h - E\{Y_h\})/s\{\hat{Y}_h\}$ 54
Confidence Interval for $E\{Y_h\}$ 54
2.5 Prediction of New Observation 55
Prediction Interval for $Y_{h(\text{new})}$ when Parameters Known 56
Prediction Interval for $Y_{h(\text{new})}$ when Parameters Unknown 57
Prediction of Mean of m New Observations for Given X_h 60
2.6 Confidence Band for Regression Line 61
2.7 Analysis of Variance Approach to Regression Analysis 63
Partitioning of Total Sum of Squares 63
Breakdown of Degrees of Freedom 66

Mean Squares 66
Analysis of Variance Table 67
Expected Mean Squares 68
F Test of $\beta_1 = 0$ *versus* $\beta_1 \neq 0$ 69
2.8 General Linear Test Approach 72
Full Model 72
Reduced Model 72
Test Statistic 73
Summary 73
2.9 Descriptive Measures of Linear Association between X and Y 74
Coefficient of Determination 74
Limitations of R^2 75
Coefficient of Correlation 76
2.10 Considerations in Applying Regression Analysis 77
2.11 Normal Correlation Models 78
Distinction between Regression and Correlation Model 78
Bivariate Normal Distribution 78
Conditional Inferences 80
Inferences on Correlation Coefficients 83
Spearman Rank Correlation Coefficient 87
Cited References 89
Problems 89
Exercises 97
Projects 98

Chapter 3
Diagnostics and Remedial Measures 100

3.1 Diagnostics for Predictor Variable 100
3.2 Residuals 102
Properties of Residuals 102
Semistudentized Residuals 103
Departures from Model to Be Studied by Residuals 103
3.3 Diagnostics for Residuals 103
Nonlinearity of Regression Function 104
Nonconstancy of Error Variance 107
Presence of Outliers 108
Nonindependence of Error Terms 108
Nonnormality of Error Terms 110
Omission of Important Predictor Variables 112
Some Final Comments 114
3.4 Overview of Tests Involving Residuals 114
Tests for Randomness 114
Tests for Constancy of Variance 115
Tests for Outliers 115
Tests for Normality 115
3.5 Correlation Test for Normality 115
3.6 Tests for Constancy of Error Variance 116
Brown-Forsythe Test 116
Breusch-Pagan Test 118
3.7 *F* Test for Lack of Fit 119
Assumptions 119
Notation 121
Full Model 121
Reduced Model 123
Test Statistic 123
ANOVA Table 124
3.8 Overview of Remedial Measures 127
Nonlinearity of Regression Function 128
Nonconstancy of Error Variance 128
Nonindependence of Error Terms 128
Nonnormality of Error Terms 128
Omission of Important Predictor Variables 129
Outlying Observations 129
3.9 Transformations 129
Transformations for Nonlinear Relation Only 129
Transformations for Nonnormality and Unequal Error Variances 132
Box-Cox Transformations 134
3.10 Exploration of Shape of Regression Function 137
Lowess Method 138
Use of Smoothed Curves to Confirm Fitted Regression Function 139
3.11 Case Example—Plutonium Measurement 141
Cited References 146
Problems 146
Exercises 151
Projects 152
Case Studies 153

Chapter 4
Simultaneous Inferences and Other Topics in Regression Analysis 154

4.1 Joint Estimation of β_0 and β_1 154
Need for Joint Estimation 154
Bonferroni Joint Confidence Intervals 155
4.2 Simultaneous Estimation of Mean Responses 157
Working-Hotelling Procedure 158
Bonferroni Procedure 159
4.3 Simultaneous Prediction Intervals for New Observations 160
4.4 Regression through Origin 161
Model 161
Inferences 161
Important Cautions for Using Regression through Origin 164
4.5 Effects of Measurement Errors 165
Measurement Errors in Y 165
Measurement Errors in X 165
Berkson Model 167
4.6 Inverse Predictions 168
4.7 Choice of X Levels 170
Cited References 172
Problems 172
Exercises 175
Projects 175

Chapter 5
Matrix Approach to Simple Linear Regression Analysis 176

5.1 Matrices 176
Definition of Matrix 176
Square Matrix 178
Vector 178
Transpose 178
Equality of Matrices 179
5.2 Matrix Addition and Subtraction 180
5.3 Matrix Multiplication 182
Multiplication of a Matrix by a Scalar 182
Multiplication of a Matrix by a Matrix 182
5.4 Special Types of Matrices 185
Symmetric Matrix 185
Diagonal Matrix 185
Vector and Matrix with All Elements Unity 187
Zero Vector 187
5.5 Linear Dependence and Rank of Matrix 188
Linear Dependence 188
Rank of Matrix 188
5.6 Inverse of a Matrix 189
Finding the Inverse 190
Uses of Inverse Matrix 192
5.7 Some Basic Results for Matrices 193
5.8 Random Vectors and Matrices 193
Expectation of Random Vector or Matrix
Variance-Covariance Matrix of Random Vector 194
Some Basic Results 196
Multivariate Normal Distribution 196
5.9 Simple Linear Regression Model in Matrix Terms 197
5.10 Least Squares Estimation of Regression Parameters 199
Normal Equations 199
Estimated Regression Coefficients 200
5.11 Fitted Values and Residuals 202
Fitted Values 202
Residuals 203
5.12 Analysis of Variance Results 204
Sums of Squares 204
Sums of Squares as Quadratic Forms 205
5.13 Inferences in Regression Analysis 206
Regression Coefficients 207
Mean Response 208
Prediction of New Observation 209
Cited Reference 209
Problems 209
Exercises 212

PART TWO
MULTIPLE LINEAR REGRESSION 213

Chapter 6
Multiple Regression I 214

6.1 Multiple Regression Models 214

Need for Several Predictor Variables 214
First-Order Model with Two Predictor Variables 215
First-Order Model with More than Two Predictor Variables 217
General Linear Regression Model 217
6.2 General Linear Regression Model in Matrix Terms 222
6.3 Estimation of Regression Coefficients 223
6.4 Fitted Values and Residuals 224
6.5 Analysis of Variance Results 225
Sums of Squares and Mean Squares 225
F Test for Regression Relation 226
Coefficient of Multiple Determination 226
Coefficient of Multiple Correlation 227
6.6 Inferences about Regression Parameters 227
Interval Estimation of β_k 228
Tests for β_k 228
Joint Inferences 228
6.7 Estimation of Mean Response and Prediction of New Observation 229
Interval Estimation of $E\{Y_h\}$ 229
Confidence Region for Regression Surface 229
Simultaneous Confidence Intervals for Several Mean Responses 230
Prediction of New Observation $Y_{h(\text{new})}$ 230
Prediction of Mean of m New Observations at X_h 230
Predictions of g New Observations 231
Caution about Hidden Extrapolations 231
6.8 Diagnostics and Remedial Measures 232
Scatter Plot Matrix 232
Three-Dimensional Scatter Plots 233
Residual Plots 233
Correlation Test for Normality 234
Brown-Forsythe Test for Constancy of Error Variance 234
Breusch-Pagan Test for Constancy of Error Variance 234
F Test for Lack of Fit 235
Remedial Measures 236
6.9 An Example—Multiple Regression with Two Predictor Variables 236
Setting 236
Basic Calculations 237
Estimated Regression Function 240
Fitted Values and Residuals 241
Analysis of Appropriateness of Model 241
Analysis of Variance 243
Estimation of Regression Parameters 245
Estimation of Mean Response 245
Prediction Limits for New Observations 247
Cited Reference 248
Problems 248
Exercises 253
Projects 254

Chapter 7
Multiple Regression II 256

7.1 Extra Sums of Squares 256
Basic Ideas 256
Definitions 259
Decomposition of SSR into Extra Sums of Squares 260
ANOVA Table Containing Decomposition of SSR 261
7.2 Uses of Extra Sums of Squares in Tests for Regression Coefficients 263
Test whether a Single $\beta_k = 0$ 263
Test whether Several $\beta_k = 0$ 264
7.3 Summary of Tests Concerning Regression Coefficients 266
Test whether All $\beta_k = 0$ 266
Test whether a Single $\beta_k = 0$ 267
Test whether Some $\beta_k = 0$ 267
Other Tests 268
7.4 Coefficients of Partial Determination 268
Two Predictor Variables 269
General Case 269
Coefficients of Partial Correlation 270
7.5 Standardized Multiple Regression Model 271
Roundoff Errors in Normal Equations Calculations 271
Lack of Comparability in Regression Coefficients 272
Correlation Transformation 272
Standardized Regression Model 273
X'X Matrix for Transformed Variables 274

Estimated Standardized Regression Coefficients 275
7.6 Multicollinearity and Its Effects 278
Uncorrelated Predictor Variables 279
Nature of Problem when Predictor Variables Are Perfectly Correlated 281
Effects of Multicollinearity 283
Need for More Powerful Diagnostics for Multicollinearity 289
Cited Reference 289
Problems 289
Exercise 292
Projects 293

Chapter 8
Regression Models for Quantitative and Qualitative Predictors 294

8.1 Polynomial Regression Models 294
Uses of Polynomial Models 294
One Predictor Variable—Second Order 295
One Predictor Variable—Third Order 296
One Predictor Variable—Higher Orders 296
Two Predictor Variables—Second Order 297
Three Predictor Variables—Second Order 298
Implementation of Polynomial Regression Models 298
Case Example 300
Some Further Comments on Polynomial Regression 305
8.2 Interaction Regression Models 306
Interaction Effects 306
Interpretation of Interaction Regression Models with Linear Effects 306
Interpretation of Interaction Regression Models with Curvilinear Effects 309
Implementation of Interaction Regression Models 311
8.3 Qualitative Predictors 313
Qualitative Predictor with Two Classes 314
Interpretation of Regression Coefficients 315
Qualitative Predictor with More than Two Classes 318
Time Series Applications 319
8.4 Some Considerations in Using Indicator Variables 321
Indicator Variables versus Allocated Codes 321
Indicator Variables versus Quantitative Variables 322
Other Codings for Indicator Variables 323
8.5 Modeling Interactions between Quantitative and Qualitative Predictors 324
Meaning of Regression Coefficients 324
8.6 More Complex Models 327
More than One Qualitative Predictor Variable 328
Qualitative Predictor Variables Only 329
8.7 Comparison of Two or More Regression Functions 329
Soap Production Lines Example 330
Instrument Calibration Study Example 334
Cited Reference 335
Problems 335
Exercises 340
Projects 341
Case Study 342

Chapter 9
Building the Regression Model I: Model Selection and Validation 343

9.1 Overview of Model-Building Process 343
Data Collection 343
Data Preparation 346
Preliminary Model Investigation 346
Reduction of Explanatory Variables 347
Model Refinement and Selection 349
Model Validation 350
9.2 Surgical Unit Example 350
9.3 Criteria for Model Selection 353
R_p^2 or SSE_p Criterion 354
$R_{a,p}^2$ or MSE_p Criterion 355
Mallows' C_p Criterion 357
AIC_p and SBC_p Criteria 359
$PRESS_p$ Criterion 360
9.4 Automatic Search Procedures for Model Selection 361
"Best" Subsets Algorithm 361
Stepwise Regression Methods 364

Forward Stepwise Regression 364
Other Stepwise Procedures 367
9.5 Some Final Comments on Automatic Model Selection Procedures 368
9.6 Model Validation 369
Collection of New Data to Check Model 370
Comparison with Theory, Empirical Evidence, or Simulation Results 371
Data Splitting 372
Cited References 375
Problems 376
Exercise 380
Projects 381
Case Studies 382

Chapter 10
Building the Regression Model II: Diagnostics 384

10.1 Model Adequacy for a Predictor Variable—Added-Variable Plots 384
10.2 Identifying Outlying *Y* Observations—Studentized Deleted Residuals 390
Outlying Cases 390
Residuals and Semistudentized Residuals 392
Hat Matrix 392
Studentized Residuals 394
Deleted Residuals 395
Studentized Deleted Residuals 396
10.3 Identifying Outlying *X* Observations—Hat Matrix Leverage Values 398
Use of Hat Matrix for Identifying Outlying X Observations 398
Use of Hat Matrix to Identify Hidden Extrapolation 400
10.4 Identifying Influential Cases—*DFFITS*, Cook's Distance, and *DFBETAS* Measures 400
Influence on Single Fitted Value—DFFITS 401
Influence on All Fitted Values—Cook's Distance 402
Influence on the Regression Coefficients—DFBETAS 404
Influence on Inferences 405
Some Final Comments 406
10.5 Multicollinearity Diagnostics—Variance Inflation Factor 406
Informal Diagnostics 407
Variance Inflation Factor 408
10.6 Surgical Unit Example—Continued 410
Cited References 414
Problems 414
Exercises 419
Projects 419
Case Studies 420

Chapter 11
Building the Regression Model III: Remedial Measures 421

11.1 Unequal Error Variances Remedial Measures—Weighted Least Squares 421
Error Variances Known 422
Error Variances Known up to Proportionality Constant 424
Error Variances Unknown 424
11.2 Multicollinearity Remedial Measures—Ridge Regression 431
Some Remedial Measures 431
Ridge Regression 432
11.3 Remedial Measures for Influential Cases—Robust Regression 437
Robust Regression 438
IRLS Robust Regression 439
11.4 Nonparametric Regression: Lowess Method and Regression Trees 449
Lowess Method 449
Regression Trees 453
11.5 Remedial Measures for Evaluating Precision in Nonstandard Situations—Bootstrapping 458
General Procedure 459
Bootstrap Sampling 459
Bootstrap Confidence Intervals 460
11.6 Case Example—MNDOT Traffic Estimation 464
The AADT Database 464
Model Development 465
Weighted Least Squares Estimation 468

Cited References 471
Problems 472
Exercises 476
Projects 476
Case Studies 480

Chapter 12
Autocorrelation in Time Series Data 481

12.1 Problems of Autocorrelation 481
12.2 First-Order Autoregressive Error Model 484
Simple Linear Regression 484
Multiple Regression 484
Properties of Error Terms 485
12.3 Durbin-Watson Test for Autocorrelation 487
12.4 Remedial Measures for Autocorrelation 490
Addition of Predictor Variables 490
Use of Transformed Variables 490
Cochrane-Orcutt Procedure 492
Hildreth-Lu Procedure 495
First Differences Procedure 496
Comparison of Three Methods 498
12.5 Forecasting with Autocorrelated Error Terms 499
Cited References 502
Problems 502
Exercises 507
Projects 508
Case Studies 508

PART THREE
NONLINEAR REGRESSION 509

Chapter 13
Introduction to Nonlinear Regression and Neural Networks 510

13.1 Linear and Nonlinear Regression Models 510
Linear Regression Models 510
Nonlinear Regression Models 511
Estimation of Regression Parameters 514
13.2 Least Squares Estimation in Nonlinear Regression 515
Solution of Normal Equations 517
Direct Numerical Search—Gauss-Newton Method 518
Other Direct Search Procedures 525
13.3 Model Building and Diagnostics 526
13.4 Inferences about Nonlinear Regression Parameters 527
Estimate of Error Term Variance 527
Large-Sample Theory 528
When Is Large-Sample Theory Applicable? 528
Interval Estimation of a Single γ_k 531
Simultaneous Interval Estimation of Several γ_k 532
Test Concerning a Single γ_k 532
Test Concerning Several γ_k 533
13.5 Learning Curve Example 533
13.6 Introduction to Neural Network Modeling 537
Neural Network Model 537
Network Representation 540
Neural Network as Generalization of Linear Regression 541
Parameter Estimation: Penalized Least Squares 542
Example: Ischemic Heart Disease 543
Model Interpretation and Prediction 546
Some Final Comments on Neural Network Modeling 547
Cited References 547
Problems 548
Exercises 552
Projects 552
Case Studies 554

Chapter 14
Logistic Regression, Poisson Regression, and Generalized Linear Models 555

14.1 Regression Models with Binary Response Variable 555
Meaning of Response Function when Outcome Variable Is Binary 556

Special Problems when Response Variable Is Binary 557
14.2 Sigmoidal Response Functions for Binary Responses 559
Probit Mean Response Function 559
Logistic Mean Response Function 560
Complementary Log-Log Response Function 562
14.3 Simple Logistic Regression 563
Simple Logistic Regression Model 563
Likelihood Function 564
Maximum Likelihood Estimation 564
Interpretation of b_1 567
Use of Probit and Complementary Log-Log Response Functions 568
Repeat Observations—Binomial Outcomes 568
14.4 Multiple Logistic Regression 570
Multiple Logistic Regression Model 570
Fitting of Model 571
Polynomial Logistic Regression 575
14.5 Inferences about Regression Parameters 577
Test Concerning a Single β_k: Wald Test 578
Interval Estimation of a Single β_k 579
Test whether Several $\beta_k = 0$: Likelihood Ratio Test 580
14.6 Automatic Model Selection Methods 582
Model Selection Criteria 582
Best Subsets Procedures 583
Stepwise Model Selection 583
14.7 Tests for Goodness of Fit 586
Pearson Chi-Square Goodness of Fit Test 586
Deviance Goodness of Fit Test 588
Hosmer-Lemeshow Goodness of Fit Test 589
14.8 Logistic Regression Diagnostics 591
Logistic Regression Residuals 591
Diagnostic Residual Plots 594
Detection of Influential Observations 598
14.9 Inferences about Mean Response 602
Point Estimator 602
Interval Estimation 602
Simultaneous Confidence Intervals for Several Mean Responses 603
14.10 Prediction of a New Observation 604
Choice of Prediction Rule 604
Validation of Prediction Error Rate 607
14.11 Polytomous Logistic Regression for Nominal Response 608
Pregnancy Duration Data with Polytomous Response 609
$J - 1$ Baseline-Category Logits for Nominal Response 610
Maximum Likelihood Estimation 612
14.12 Polytomous Logistic Regression for Ordinal Response 614
14.13 Poisson Regression 618
Poisson Distribution 618
Poisson Regression Model 619
Maximum Likelihood Estimation 620
Model Development 620
Inferences 621
14.14 Generalized Linear Models 623
Cited References 624
Problems 625
Exercises 634
Projects 635
Case Studies 640

PART FOUR
DESIGN AND ANALYSIS OF SINGLE-FACTOR STUDIES 641

Chapter 15
Introduction to the Design of Experimental and Observational Studies 642

15.1 Experimental Studies, Observational Studies, and Causation 643
Experimental Studies 643
Observational Studies 644
Mixed Experimental and Observational Studies 646
15.2 Experimental Studies: Basic Concepts 647

Factors 647
Crossed and Nested Factors 648
Treatments 649
Choice of Treatments 649
Experimental Units 652
Sample Size and Replication 652
Randomization 653
Constrained Randomization: Blocking 655
Measurements 658
15.3 An Overview of Standard Experimental Designs 658
Completely Randomized Design 659
Factorial Experiments 660
Randomized Complete Block Designs 661
Nested Designs 662
Repeated Measures Designs 663
Incomplete Block Designs 664
Two-Level Factorial and Fractional Factorial Experiments 665
Response Surface Experiments 666
15.4 Design of Observational Studies 666
Cross-Sectional Studies 666
Prospective Studies 667
Retrospective Studies 667
Matching 668
15.5 Case Study: Paired-Comparison Experiment 669
15.6 Concluding Remarks 672
Cited References 672
Problems 672
Exercise 676

Chapter 16
Single-Factor Studies 677

16.1 Single-Factor Experimental and Observational Studies 677
16.2 Relation between Regression and Analysis of Variance 679
Illustrations 679
Choice between Two Types of Models 680
16.3 Single-Factor ANOVA Model 681
Basic Ideas 681
Cell Means Model 681
Important Features of Model 682
The ANOVA Model Is a Linear Model 683
Interpretation of Factor Level Means 68
Distinction between ANOVA Models I and II 685
16.4 Fitting of ANOVA Model 685
Notation 686
Least Squares and Maximum Likelihood Estimators 687
Residuals 689
16.5 Analysis of Variance 690
Partitioning of SSTO *690*
Breakdown of Degrees of Freedom 693
Mean Squares 693
Analysis of Variance Table 694
Expected Mean Squares 694
16.6 *F* Test for Equality of Factor Level Means 698
Test Statistic 698
Distribution of F* *699*
Construction of Decision Rule 699
16.7 Alternative Formulation of Model 701
Factor Effects Model 701
Definition of $\mu_.$ *702*
Test for Equality of Factor Level Means 704
16.8 Regression Approach to Single-Factor Analysis of Variance 704
Factor Effects Model with Unweighted Mean 705
Factor Effects Model with Weighted Mean 709
Cell Means Model 710
16.9 Randomization Tests 712
16.10 Planning of Sample Sizes with Power Approach 716
Power of F *Test 716*
Use of Table B.12 for Single-Factor Studies 718
Some Further Observations on Use of Table B.12 720
16.11 Planning of Sample Sizes to Find "Best" Treatment 721
Cited Reference 722

Problems 722
Exercises 730
Projects 730
Case Studies 732

Chapter 17
Analysis of Factor Level Means 733

17.1 Introduction 733
17.2 Plots of Estimated Factor Level Means 735
Line Plot 735
Bar Graph and Main Effects Plot 736
17.3 Estimation and Testing of Factor Level Means 737
Inferences for Single Factor Level Mean 737
Inferences for Difference between Two Factor Level Means 739
Inferences for Contrast of Factor Level Means 741
Inferences for Linear Combination of Factor Level Means 743
17.4 Need for Simultaneous Inference Procedures 744
17.5 Tukey Multiple Comparison Procedure 746
Studentized Range Distribution 746
Simultaneous Estimation 747
Simultaneous Testing 747
Example 1—Equal Sample Sizes 748
Example 2—Unequal Sample Sizes 750
17.6 Scheffé Multiple Comparison Procedure 753
Simultaneous Estimation 753
Simultaneous Testing 754
Comparison of Scheffé and Tukey Procedures 755
17.7 Bonferroni Multiple Comparison Procedure 756
Simultaneous Estimation 756
Simultaneous Testing 756
Comparison of Bonferroni Procedure with Scheffé and Tukey Procedures 757
Analysis of Means 758
17.8 Planning of Sample Sizes with Estimation Approach 759
Example 1—Equal Sample Sizes 759
Example 2—Unequal Sample Sizes 761
17.9 Analysis of Factor Effects when Factor Is Quantitative 762
Cited References 766
Problems 767
Exercises 773
Projects 774
Case Studies 774

Chapter 18
ANOVA Diagnostics and Remedial Measures 775

18.1 Residual Analysis 775
Residuals 776
Residual Plots 776
Diagnosis of Departures from ANOVA Model 778
18.2 Tests for Constancy of Error Variance 781
Hartley Test 782
Brown-Forsythe Test 784
18.3 Overview of Remedial Measures 786
18.4 Weighted Least Squares 786
18.5 Transformations of Response Variable 789
Simple Guides to Finding a Transformation 789
Box-Cox Procedure 791
18.6 Effects of Departures from Model 793
Nonnormality 793
Unequal Error Variances 794
Nonindependence of Error Terms 794
18.7 Nonparametric Rank F Test 795
Test Procedure 795
Multiple Pairwise Testing Procedure 797
18.8 Case Example—Heart Transplant 798
Cited References 801
Problems 801
Exercises 807
Projects 807
Case Studies 809

PART FIVE
MULTI-FACTOR STUDIES 811

Chapter 19
Two-Factor Studies with Equal Sample Sizes 812

19.1 Two-Factor Observational and Experimental Studies 812
- *Examples of Two-Factor Experiments and Observational Studies 812*
- *The One-Factor-at-a-Time (OFAAT) Approach to Experimentation 815*
- *Advantages of Crossed, Multi-Factor Designs 816*

19.2 Meaning of ANOVA Model Elements 817
- *Illustration 817*
- *Treatment Means 817*
- *Factor Level Means 818*
- *Main Effects 818*
- *Additive Factor Effects 819*
- *Interacting Factor Effects 822*
- *Important and Unimportant Interactions 824*
- *Transformable and Nontransformable Interactions 826*
- *Interpretation of Interactions 827*

19.3 Model I (Fixed Factor Levels) for Two-Factor Studies 829
- *Cell Means Model 830*
- *Factor Effects Model 831*

19.4 Analysis of Variance 833
- *Illustration 833*
- *Notation 834*
- *Fitting of ANOVA Model 834*
- *Partitioning of Total Sum of Squares 836*
- *Partitioning of Degrees of Freedom 839*
- *Mean Squares 839*
- *Expected Mean Squares 840*
- *Analysis of Variance Table 840*

19.5 Evaluation of Appropriateness of ANOVA Model 842

19.6 *F* Tests 843
- *Test for Interactions 844*
- *Test for Factor* A *Main Effects 844*
- *Test for Factor* B *Main Effects 845*
- *Kimball Inequality 846*

19.7 Strategy for Analysis 847

19.8 Analysis of Factor Effects when Factors Do Not Interact 848
- *Estimation of Factor Level Mean 848*
- *Estimation of Contrast of Factor Level Means 849*
- *Estimation of Linear Combination of Factor Level Means 850*
- *Multiple Pairwise Comparisons of Factor Level Means 850*
- *Multiple Contrasts of Factor Level Means 852*
- *Estimates Based on Treatment Means 853*
- *Example 1—Pairwise Comparisons of Factor Level Means 853*
- *Example 2—Estimation of Treatment Means 855*

19.9 Analysis of Factor Effects when Interactions Are Important 856
- *Multiple Pairwise Comparisons of Treatment Means 856*
- *Multiple Contrasts of Treatment Means 857*
- *Example 1—Pairwise Comparisons of Treatment Means 857*
- *Example 2—Contrasts of Treatment Means 860*

19.10 Pooling Sums of Squares in Two-Factor Analysis of Variance 861

19.11 Planning of Sample Sizes for Two-Factor Studies 862
- *Power Approach 862*
- *Estimation Approach 863*
- *Finding the "Best" Treatment 864*

Problems 864
Exercises 876
Projects 876
Case Studies 879

Chapter 20
Two-Factor Studies—One Case per Treatment 880

20.1 No-Interaction Model 880
Model 881
Analysis of Variance 881
Inference Procedures 881
Estimation of Treatment Mean 884
20.2 Tukey Test for Additivity 886
Development of Test Statistic 886
Remedial Actions if Interaction Effects Are Present 888
Cited Reference 889
Problems 889
Exercises 891
Case Study 891

Chapter 21
Randomized Complete Block Designs 892

21.1 Elements of Randomized Complete Block Designs 892
Description of Designs 892
Criteria for Blocking 893
Advantages and Disadvantages 894
How to Randomize 895
Illustration 895
21.2 Model for Randomized Complete Block Designs 897
21.3 Analysis of Variance and Tests 898
Fitting of Randomized Complete Block Model 898
Analysis of Variance 898
21.4 Evaluation of Appropriateness of Randomized Complete Block Model 901
Diagnostic Plots 901
Tukey Test for Additivity 903
21.5 Analysis of Treatment Effects 904
21.6 Use of More than One Blocking Variable 905
21.7 Use of More than One Replicate in Each Block 906
21.8 Factorial Treatments 908
21.9 Planning Randomized Complete Block Experiments 909
Power Approach 909
Estimation Approach 910
Efficiency of Blocking Variable 911
Problems 912
Exercises 916

Chapter 22
Analysis of Covariance 917

22.1 Basic Ideas 917
How Covariance Analysis Reduces Error Variability 917
Concomitant Variables 919
22.2 Single-Factor Covariance Model 920
Notation 921
Development of Covariance Model 921
Properties of Covariance Model 922
Generalizations of Covariance Model 923
Regression Formula of Covariance Model 924
Appropriateness of Covariance Model 925
Inferences of Interest 925
22.3 Example of Single-Factor Covariance Analysis 926
Development of Model 926
Test for Treatment Effects 928
Estimation of Treatment Effects 930
Test for Parallel Slopes 932
22.4 Two-Factor Covariance Analysis 933
Covariance Model for Two-Factor Studies 933
Regression Approach 934
Covariance Analysis for Randomized Complete Block Designs 937
22.5 Additional Considerations for the Use of Covariance Analysis 939
Covariance Analysis as Alternative to Blocking 939
Use of Differences 939
Correction for Bias 940

Interest in Nature of Treatment Effects 940
Problems 941
Exercise 947
Projects 947
Case Studies 950

Chapter 23
Two-Factor Studies with Unequal Sample Sizes 951

23.1 Unequal Sample Sizes 951
Notation 952
23.2 Use of Regression Approach for Testing Factor Effects when Sample Sizes Are Unequal 953
Regression Approach to Two-Factor Analysis of Variance 953
23.3 Inferences about Factor Effects when Sample Sizes Are Unequal 959
Example 1—Pairwise Comparisons of Factor Level Means 962
Example 2—Single-Degree-of-Freedom Test 964
23.4 Empty Cells in Two-Factor Studies 964
Partial Analysis of Factor Effects 965
Analysis if Model with No Interactions Can Be Employed 966
Missing Observations in Randomized Complete Block Designs 967
23.5 ANOVA Inferences when Treatment Means Are of Unequal Importance 970
Estimation of Treatment Means and Factor Effects 971
Test for Interactions 972
Tests for Factor Main Effects by Use of Equivalent Regression Models 972
Tests for Factor Main Effects by Use of Matrix Formulation 975
Tests for Factor Effects when Weights Are Proportional to Sample Sizes 977
23.6 Statistical Computing Packages 980
Problems 981
Exercises 988
Projects 988
Case Studies 990

Chapter 24
Multi-Factor Studies 992

24.1 ANOVA Model for Three-Factor Studies 992
Notation 992
Illustration 993
Main Effects 993
Two-Factor Interactions 995
Three-Factor Interactions 996
Cell Means Model 996
Factor Effects Model 997
24.2 Interpretation of Interactions in Three-Factor Studies 998
Learning Time Example 1: Interpretation of Three-Factor Interactions 998
Learning Time Example 2: Interpretation of Multiple Two-Factor Interactions 999
Learning Time Example 3: Interpretation of a Single Two-Factor Interaction 1000
24.3 Fitting of ANOVA Model 1003
Notation 1003
Fitting of ANOVA Model 1003
Evaluation of Appropriateness of ANOVA Model 1005
24.4 Analysis of Variance 1008
Partitioning of Total Sum of Squares 1008
Degrees of Freedom and Mean Squares 1009
Tests for Factor Effects 1009
24.5 Analysis of Factor Effects 1013
Strategy for Analysis 1013
Analysis of Factor Effects when Factors Do Not Interact 1014
Analysis of Factor Effects with Multiple Two-Factor Interactions or Three-Factor Interaction 1016
Analysis of Factor Effects with Single Two-Factor Interaction 1016
Example—Estimation of Contrasts of Treatment Means 1018
24.6 Unequal Sample Sizes in Multi-Factor Studies 1019
Tests for Factor Effects 1019
Inferences for Contrasts of Factor Level Means 1020

24.7 Planning of Sample Sizes 1021
Power of F *Test for Multi-Factor Studies 1021*
Use of Table B.12 for Multi-Factor Studies 1021
Cited Reference 1022
Problems 1022
Exercises 1027
Projects 1027
Case Studies 1028

Chapter 25
Random and Mixed Effects Models 1030

25.1 Single-Factor Studies—ANOVA Model II 1031
Random Cell Means Model 1031
Questions of Interest 1034
Test whether $\sigma_\mu^2 = 0$ *1035*
Estimation of $\mu_\cdot$ *1038*
Estimation of $\sigma_\mu^2/(\sigma_\mu^2 + \sigma^2)$ *1040*
Estimation of σ^2 *1041*
Point Estimation of σ_μ^2 *1042*
Interval Estimation of σ_μ^2 *1042*
Random Factor Effects Model 1047
25.2 Two-Factor Studies—ANOVA Models II and III 1047
ANOVA Model II—Random Factor Effects 1047
ANOVA Model III—Mixed Factor Effects 1049
25.3 Two-Factor Studies—ANOVA Tests for Models II and III 1052
Expected Mean Squares 1052
Construction of Test Statistics 1053
25.4 Two-Factor Studies—Estimation of Factor Effects for Models II and III 1055
Estimation of Variance Components 1055
Estimation of Fixed Effects in Mixed Model 1056
25.5 Randomized Complete Block Design: Random Block Effects 1060
Additive Model 1061
Interaction Model 1064
25.6 Three-Factor Studies—ANOVA Models II and III 1066
ANOVA Model II—Random Factor Effects 1066
ANOVA Model III—Mixed Factor Effects 1066
Appropriate Test Statistics 1067
Estimation of Effects 1069
25.7 ANOVA Models II and III with Unequal Sample Sizes 1070
Maximum Likelihood Approach 1072
Cited References 1077
Problems 1077
Exercises 1085
Projects 1085

PART SIX
SPECIALIZED STUDY DESIGNS 1087

Chapter 26
Nested Designs, Subsampling, and Partially Nested Designs 1088

26.1 Distinction between Nested and Crossed Factors 1088
26.2 Two-Factor Nested Designs 1091
Development of Model Elements 1091
Nested Design Model 1092
Random Factor Effects 1093
26.3 Analysis of Variance for Two-Factor Nested Designs 1093
Fitting of Model 1093
Sums of Squares 1094
Degrees of Freedom 1095
Tests for Factor Effects 1097
Random Factor Effects 1099
26.4 Evaluation of Appropriateness of Nested Design Model 1099
26.5 Analysis of Factor Effects in Two-Factor Nested Designs 1100
Estimation of Factor Level Means $\mu_{i\cdot}$ *1100*
Estimation of Treatment Means μ_{ij} *1102*
Estimation of Overall Mean $\mu_{\cdot\cdot}$ *1103*
Estimation of Variance Components 1103

26.6 Unbalanced Nested Two-Factor Designs 1104
26.7 Subsampling in Single-Factor Study with Completely Randomized Design 1106
Model 1107
Analysis of Variance and Tests of Effects 1108
Estimation of Treatment Effects 1110
Estimation of Variances 1111
26.8 Pure Subsampling in Three Stages 1113
Model 1113
Analysis of Variance 1113
Estimation of $\mu_{..}$ 1113
26.9 Three-Factor Partially Nested Designs 1114
Development of Model 1114
Analysis of Variance 1115
Cited Reference 1119
Problems 1119
Exercises 1125
Projects 1125

Chapter 27
Repeated Measures and Related Designs 1127

27.1 Elements of Repeated Measures Designs 1127
Description of Designs 1127
Advantages and Disadvantages 1128
How to Randomize 1128
27.2 Single-Factor Experiments with Repeated Measures on All Treatments 1129
Model 1129
Analysis of Variance and Tests 1130
Evaluation of Appropriateness of Repeated Measures Model 1134
Analysis of Treatment Effects 1137
Ranked Data 1138
Multiple Pairwise Testing Procedure 1138
27.3 Two-Factor Experiments with Repeated Measures on One Factor 1140
Description of Design 1140
Model 1141
Analysis of Variance and Tests 1142
Evaluation of Appropriateness of Repeated Measures Model 1144
Analysis of Factor Effects: Without Interaction 1145
Analysis of Factor Effects: With Interaction 1148
Blocking of Subjects in Repeated Measures Designs 1153
27.4 Two-Factor Experiments with Repeated Measures on Both Factors 1153
Model 1154
Analysis of Variance and Tests 1155
Evaluation of Appropriateness of Repeated Measures Model 1157
Analysis of Factor Effects 1157
27.5 Regression Approach to Repeated Measures Designs 1161
27.6 Split-Plot Designs 1162
Cited References 1164
Problems 1164
Exercise 1171
Projects 1171

Chapter 28
Balanced Incomplete Block, Latin Square, and Related Designs 1173

28.1 Balanced Incomplete Block Designs 1173
Advantages and Disadvantages of BIBDs 1175
28.2 Analysis of Balanced Incomplete Block Designs 1177
BIBD Model 1177
Regression Approach to Analysis of Balanced Incomplete Block Designs 1177
Analysis of Treatment Effects 1180
Planning of Sample Sizes with Estimation Approach 1182
28.3 Latin Square Designs 1183
Basic Ideas 1183
Description of Latin Square Designs 1184
Advantages and Disadvantages of Latin Square Designs 1185

Randomization of Latin Square Design 1185
28.4 Latin Square Model 1187
28.5 Analysis of Latin Square Experiments 1188
Notation 1188
Fitting of Model 1188
Analysis of Variance 1188
Test for Treatment Effects 1190
Analysis of Treatment Effects 1190
Residual Analysis 1191
Factorial Treatments 1192
Random Blocking Variable Effects 1193
Missing Observations 1193
28.6 Planning Latin Square Experiments 1193
Power of F Test 1193
Necessary Number of Replications 1193
Efficiency of Blocking Variables 1193
28.7 Additional Replications with Latin Square Designs 1195
Replications within Cells 1195
Additional Latin Squares 1196
28.8 Replications in Repeated Measures Studies 1198
Latin Square Crossover Designs 1198
Use of Independent Latin Squares 1200
Carryover Effects 1201
Cited References 1202
Problems 1202

Chapter 29
Exploratory Experiments: Two-Level Factorial and Fractional Factorial Designs 1209

29.1 Two-Level Full Factorial Experiments 1210
Design of Two-Level Studies 1210
Notation 1210
Estimation of Factor Effects 1212
Inferences about Factor Effects 1214
29.2 Analysis of Unreplicated Two-Level Studies 1216
Pooling of Interactions 1218
Pareto Plot 1219
Dot Plot 1220
Normal Probability Plot 1221
Center Point Replications 1222
29.3 Two-Level Fractional Factorial Designs 1223
Confounding 1224
Defining Relation 1227
Half-Fraction Designs 1228
Quarter-Fraction and Smaller-Fraction Designs 1229
Resolution 1231
Selecting a Fraction of Highest Resolution 1232
29.4 Screening Experiments 1239
2^{k-f}_{III} *Fractional Factorial Designs 1239*
Plackett-Burman Designs 1240
29.5 Incomplete Block Designs for Two-Level Factorial Experiments 1240
Assignment of Treatments to Blocks 1241
Use of Center Point Replications 1243
29.6 Robust Product and Process Design 1244
Location and Dispersion Modeling 1246
Incorporating Noise Factors 1250
Case Study—Clutch Slave Cylinder Experiment 1252
Cited References 1256
Problems 1256
Exercises 1266

Chapter 30
Response Surface Methodology 1267

30.1 Response Surface Experiments 1267
30.2 Central Composite Response Surface Designs 1268
Structure of Central Composite Designs 1268
Commonly Used Central Composite Designs 1270
Rotatable Central Composite Designs 1271
Other Criteria for Choosing a Central Composite Design 1273
Blocking Central Composite Designs 1275

Additional General-Purpose Response Surface Designs 1276
30.3 Optimal Response Surface Designs 1276
Purpose of Optimal Designs 1276
Optimal Design Approach 1278
Design Criteria for Optimal Design Selection 1279
Construction of Optimal Response Surface Designs 1282
Some Final Cautions 1283
30.4 Analysis of Response Surface Experiments 1284
Model Interpretation and Visualization 1284
Response Surface Optimum Conditions 1286
30.5 Sequential Search for Optimum Conditions—Method of Steepest Ascent 1290
Cited References 1292
Problems 1292
Projects 1295

Appendix A
Some Basic Results in Probab and Statistics 1297

Appendix B
Tables 1315

Appendix C
Data Sets 1348

Appendix D
Rules for Developing ANOVA Tables for Balanced Designs

Appendix E
Selected Bibliography 1374

Index 1385