

S.R. Otto and J.P. Denier

---

# **An Introduction to Programming and Numerical Methods in MATLAB**

With 111 Figures

 Springer

# Contents

<b>1. Simple Calculations with MATLAB</b> .....	1
1.1 Introduction and a Word of Warning .....	1
1.2 Scalar Quantities and Variables .....	2
1.2.1 Rules for Naming of Variables .....	4
1.2.2 Precedence: The Order in Which Calculations Are Per- formed .....	5
1.2.3 Mathematical Functions .....	8
1.3 Format: The Way in Which Numbers Appear .....	12
1.4 Vectors in MATLAB .....	13
1.4.1 Initialising Vector Objects .....	13
1.4.2 Manipulating Vectors and Dot Arithmetic .....	14
1.5 Setting Up Mathematical Functions .....	17
1.6 Some MATLAB Specific Commands .....	20
1.6.1 Looking at Variables and Their Sizes .....	22
1.7 Accessing Elements of Arrays .....	23
1.8 Tasks .....	24
<b>2. Writing Scripts and Functions</b> .....	27
2.1 Creating Scripts and Functions .....	27
2.1.1 Functions .....	30
2.1.2 Brief Aside .....	35
2.2 Plotting Simple Functions .....	36
2.2.1 Evaluating Polynomials and Plotting Curves .....	41
2.2.2 More on Plotting .....	44
2.3 Functions of Functions .....	49
2.4 Errors .....	51

---

2.4.1	Numerical Errors . . . . .	51
2.4.2	User Error . . . . .	54
2.5	Tasks . . . . .	57
<b>3.</b>	<b>Loops and Conditional Statements . . . . .</b>	<b>63</b>
3.1	Introduction . . . . .	63
3.2	Loops Structures . . . . .	63
3.3	Summing Series . . . . .	68
3.3.1	Sums of Series of the Form $\sum_{j=1}^N j^p, p \in \mathbb{N}$ . . . . .	73
3.3.2	Summing Infinite Series . . . . .	76
3.3.3	Summing Series Using MATLAB Specific Commands . . . . .	79
3.3.4	Loops Within Loops (Nested) . . . . .	82
3.4	Conditional Statements . . . . .	83
3.4.1	Constructing Logical Statements . . . . .	85
3.4.2	The MATLAB Command <code>switch</code> . . . . .	88
3.5	Conditional loops . . . . .	90
3.5.1	The <code>break</code> Command . . . . .	92
3.6	MATLAB Specific Commands . . . . .	92
3.7	Error Checking . . . . .	94
3.8	Tasks . . . . .	97
<b>4.</b>	<b>Root Finding . . . . .</b>	<b>103</b>
4.1	Introduction . . . . .	103
4.2	Initial Estimates . . . . .	104
4.3	Fixed Point Iteration . . . . .	109
4.4	Bisection . . . . .	113
4.5	Newton–Raphson and Secant Methods . . . . .	117
4.5.1	Derivation of the Newton–Raphson Method . . . . .	117
4.6	Repeated Roots of Functions . . . . .	123
4.7	Zeros of Higher-Dimensional Functions(*) . . . . .	125
4.8	MATLAB Routines for Finding Zeros . . . . .	128
4.8.1	Roots of a Polynomial . . . . .	128
4.8.2	The Command <code>fzero</code> . . . . .	128
4.9	Tasks . . . . .	130
<b>5.</b>	<b>Interpolation and Extrapolation . . . . .</b>	<b>133</b>
5.1	Introduction . . . . .	133
5.2	Saving and Reading Data . . . . .	134
5.3	Which Points to Use? . . . . .	139
5.4	Newton Forward Differences and Lagrange Polynomials . . . . .	141
5.4.1	Linear Interpolation/Extrapolation . . . . .	147

---

5.5	Calculating Interpolated and Extrapolated Values . . . . .	148
5.6	Splines . . . . .	150
5.7	Curves of Best Fit . . . . .	152
5.8	Interpolation of Non-Smooth Data . . . . .	155
5.8.1	Insufficient Data Points . . . . .	158
5.9	Minimisation of Functions and Parameter Retrieval . . . . .	161
5.9.1	Parameter Retrieval . . . . .	163
5.9.2	Using <code>fmins</code> for Parameter Retrieval . . . . .	164
5.10	Tasks . . . . .	166
<b>6.</b>	<b>Matrices</b> . . . . .	<b>169</b>
6.1	Introduction . . . . .	169
6.1.1	Initialising Matrices Within MATLAB . . . . .	169
6.1.2	Matrix Operations . . . . .	174
6.1.3	Operations on Elements of Matrices . . . . .	180
6.1.4	More on Special Matrices . . . . .	182
6.1.5	Matrices Containing Strings . . . . .	185
6.2	Properties of Matrices and Systems of Equations . . . . .	186
6.2.1	Determinants of Matrices . . . . .	190
6.3	Elementary Row Operations . . . . .	191
6.3.1	Solving Many Equations at Once . . . . .	198
6.4	Matrix Decomposition . . . . .	199
6.5	Eigenvalues and Eigenvectors . . . . .	204
6.6	Specific MATLAB Commands . . . . .	208
6.7	Characteristic Polynomials . . . . .	212
6.8	Exponentials of Matrices . . . . .	214
6.9	Tasks . . . . .	217
<b>7.</b>	<b>Numerical Integration</b> . . . . .	<b>225</b>
7.1	Introduction . . . . .	225
7.2	Integration Using Straight Lines . . . . .	226
7.2.1	Errors in the Trapezium Method . . . . .	229
7.3	Integration Using Quadratics . . . . .	230
7.4	Integration Using Cubic Polynomials . . . . .	235
7.5	Integrating Using MATLAB Commands . . . . .	237
7.6	Specific Examples of Integrals . . . . .	238
7.6.1	Infinite Integrals and Removable Singularities . . . . .	238
7.6.2	Indefinite Integrals . . . . .	240
7.7	Tasks . . . . .	242

---

<b>8. Solving Differential Equations</b> .....	247
8.1 Introduction .....	247
8.2 Euler's Method and Crank–Nicolson .....	247
8.2.1 Analytical Comparisons .....	253
8.3 Banded Matrices .....	259
8.4 Runge–Kutta Methods .....	263
8.5 Higher-Order Systems .....	266
8.5.1 Second-Order Systems .....	266
8.5.2 Bessel's equation .....	270
8.5.3 Airy's Equation .....	273
8.5.4 Shooting Methods .....	274
8.6 Boundary-Value Problems .....	278
8.7 Population Dynamics .....	282
8.8 Eigenvalues of Differential Systems .....	285
8.9 Tasks .....	286
<b>9. Simulations and Random Numbers</b> .....	291
9.1 Introduction .....	291
9.2 Statistical quantities .....	291
9.2.1 Averages .....	291
9.2.2 Other Statistical Measures .....	293
9.3 Random Numbers and Distributions .....	295
9.3.1 Normal Distribution .....	298
9.3.2 Calculating Probabilities .....	299
9.3.3 Permutations .....	300
9.4 Maps and White Noise .....	300
9.4.1 Modelling Discrete Systems .....	307
9.4.2 Periodicity and Chaos .....	309
9.4.3 Random Motion .....	316
9.5 Tasks .....	319
<b>A. A Mathematical Introduction to Matrices</b> .....	323
A.1 Special Matrices .....	330
A.2 Inverses of Matrices .....	331
<b>B. Glossary of Useful Terms</b> .....	335
B.1 Arithmetic and Logical Operators .....	335
B.2 Symbols .....	343
B.3 Plotting Commands .....	347
B.4 General MATLAB Commands .....	365

---

<b>C. Solutions to Tasks</b> .....	389
C.1 Solutions for Tasks from Chapter 1 .....	389
C.2 Solutions for Tasks from Chapter 2 .....	394
C.3 Solutions for Tasks from Chapter 3 .....	400
C.4 Solutions for Tasks from Chapter 4 .....	408
C.5 Solutions for Tasks from Chapter 5 .....	414
C.6 Solutions for Tasks from Chapter 6 .....	418
C.7 Solutions for Tasks from Chapter 7 .....	429
C.8 Solutions for Tasks from Chapter 8 .....	436
C.9 Solutions for Tasks from Chapter 9 .....	454
<b>Index</b> .....	459