

A Practical Guide to Forecasting
Financial Market Volatility

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Foreword

If one invests in a financial asset today the return received at some pre-specified point in the future should be considered as a random variable. Such a variable can only be fully characterized by a distribution function or, more easily, by a density function. The main, single and most important feature of the density is the expected or mean value, representing the location of the density. Around the mean is the uncertainty or the volatility. If the realized returns are plotted against time, the jagged oscillating appearance illustrates the volatility. This movement contains both welcome elements, when surprisingly large returns occur, and also certainly unwelcome ones, the returns far below the mean. The well-known fact that a poor return can arise from an investment illustrates the fact that investing can be risky and is why volatility is sometimes equated with risk.

Volatility is itself a stock variable, having to be measured over a period of time, rather than a flow variable, measurable at any instant of time. Similarly, a stock price is a flow variable but a return is a stock variable. Observed volatility has to be observed over stated periods of time, such as hourly, daily, or weekly, say.

Having observed a time series of volatilities it is obviously interesting to ask about the properties of the series: is it forecastable from its own past, do other series improve these forecasts, can the series be modeled conveniently and are there useful multivariate generalizations of the results? Financial econometricians have been very inventive and industrious considering such questions and there is now a substantial and often sophisticated literature in this area.

The present book by Professor Ser-Huang Poon surveys this literature carefully and provides a very useful summary of the results available.

By so doing, she allows any interested worker to quickly catch up with the field and also to discover the areas that are still available for further exploration.

Clive W.J. Granger
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Preface

Volatility forecasting is crucial for option pricing, risk management and portfolio management. Nowadays, volatility has become the subject of trading. There are now exchange-traded contracts written on volatility. Financial market volatility also has a wider impact on financial regulation, monetary policy and macroeconomy. This book is about financial market volatility forecasting. The aim is to put in one place models, tools and findings from a large volume of published and working papers from many experts. The material presented in this book is extended from two review papers ('Forecasting Financial Market Volatility: A Review' in the *Journal of Economic Literature*, 2003, 41, 2, pp. 478–539, and 'Practical Issues in Forecasting Volatility' in the *Financial Analysts Journal*, 2005, 61, 1, pp. 45–56) jointly published with Clive Granger.

Since the main focus of this book is on volatility forecasting performance, only volatility models that have been tested for their forecasting performance are selected for further analysis and discussion. Hence, this book is oriented towards practical implementations. Volatility models are not pure theoretical constructs. The practical importance of volatility modelling and forecasting in many finance applications means that the success or failure of volatility models will depend on the characteristics of empirical data that they try to capture and predict. Given the prominent role of option price as a source of volatility forecast, I have also devoted much effort and the space of two chapters to cover Black–Scholes and stochastic volatility option pricing models.

This book is intended for first- and second-year finance PhD students and practitioners who want to implement volatility forecasting models but struggle to comprehend the huge volume of volatility research. Readers who are interested in more technical aspects of volatility modelling

could refer to, for example, Gouriéroux (1997) on ARCH models, Shephard (2003) on stochastic volatility and Fouque, Papanicolaou and Sircar (2000) on stochastic volatility option pricing. Books that cover specific aspects or variants of volatility models include Franses and van Dijk (2000) on nonlinear models, and Beran (1994) and Robinson (2003) on long memory models. Specialist books that cover financial time series modelling in a more general context include Alexander (2001), Tsay (2002) and Taylor (2005). There are also a number of edited series that contain articles on volatility modelling and forecasting, e.g. Rossi (1996), Knight and Satchell (2002) and Jarrow (1998).

I am very grateful to Clive for his teaching and guidance in the last few years. Without his encouragement and support, our volatility survey works and this book would not have got started. I would like to thank all my co-authors on volatility research, in particular Bevan Blair, Namwon Hyung, Eric Jondeau, Martin Martens, Michael Rockinger, Jon Tawn, Stephen Taylor and Konstantinos Vonatsos. Much of the writing here reflects experience gained from joint work with them.