CALCULUS OF VARIATIONS

I. M. GELFAND

S. V. FOMIN

Moscow State University

Revised English Edition
Translated and Edited by

Richard A. Silverman

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AUTHORS' PREFACE

The present course is based on lectures given by I. M. Gelfand in the Mechanics and Mathematics Department of Moscow State University. However, the book goes considerably beyond the material actually presented in the lectures. Our aim is to give a treatment of the elements of the calculus of variations in a form which is both easily understandable and sufficiently modern. Considerable attention is devoted to physical applications of variational methods, e.g., canonical equations, variational principles of mechanics and conservation laws.

The reader who merely wishes to become familiar with the most basic concepts and methods of the calculus of variations need only study the first chapter. The first three chapters, taken together, form a more comprehensive course on the elements of the calculus of variations,-but one which is still quite elementary (involving only necessary conditions for extrema). The first six chapters contain, more or less, the material given in the usual university course in the calculus of variations (with applications to the mechanics of systems with a finite number of degrees of freedom), including the theory of fields (presented in a somewhat novel way) and sufficient conditions for weak and strong extrema. Chapter 7 is devoted to the application of variational methods to the study of systems with infinitely many degrees of freedom. Chapter 8 contains a brief treatment of direct methods in the calculus of variations.

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I. M. G.
S. V. F.

TRANSLATOR'S PREFACE

This book is a modern introduction to the calculus of variations and certain of its ramifications, and I trust that its fresh and lively point of view will serve to make it a welcome addition to the English-language literature on the subject. The present edition is rather different from the Russian original. With the authors' consent, I have given free rein to the tendency of any mathematically educated translator to assume the functions of annotator and stylist. In so doing, I have had two special assets: 1) A substantial list of revisions and corrections from Professor S. V. Fomin himself, and 2) A variety of helpful suggestions from Professor J. T. Schwartz of New York University, who read the entire translation in typescript.

The problems appearing at the end of each of the eight chapters and two appendices were made specifically for the English edition, and many of them comment further on the corresponding parts of the text. A variety of Russian sources have played an important role in the synthesis of this material. In particular, I have consulted the textbooks on the calculus of variations by N. I. Akhiezer, by L. E. Elsgolts, and by M. A. Lavrentev and L. A. Lyusternik, as well as Volume 2 of the well-known problem collection by N. M. Gyunter and R. O. Kuzmin, and Chapter 3 of G. E. Shilov's "Mathematical Analysis, A Special Course."

At the end of the book I have added a Bibliography containing suggestions for collateral and supplementary reading. This list is not intended as an exhaustive catalog of the literature, and is in fact confined to books available in English.

R. A. S.

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