

INTRODUCTION TO OPERATIONS RESEARCH

Tenth Edition

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ABOUT THE AUTHORS

Frederick S. Hillier was born and raised in Aberdeen, Washington, where he was an award winner in statewide high school contests in essay writing, mathematics, debate, and music. As an undergraduate at Stanford University, he ranked first in his engineering class of over 300 students. He also won the McKinsey Prize for technical writing, won the Outstanding Sophomore Debater award, played in the Stanford Woodwind Quintet and Stanford Symphony Orchestra, and won the Hamilton Award for combining excellence in engineering with notable achievements in the humanities and social sciences. Upon his graduation with a BS degree in industrial engineering, he was awarded three national fellowships (National Science Foundation, Tau Beta Pi, and Danforth) for graduate study at Stanford with specialization in operations research. During his three years of graduate study, he took numerous additional courses in mathematics, statistics, and economics beyond what was required for his MS and PhD degrees while also teaching two courses (including “Introduction to Operations Research”). Upon receiving his PhD degree, he joined the faculty of Stanford University and began work on the 1st edition of this textbook two years later. He subsequently earned tenure at the age of 28 and the rank of full professor at 32. He also received visiting appointments at Cornell University, Carnegie-Mellon University, the Technical University of Denmark, the University of Canterbury (New Zealand), and the University of Cambridge (England). After 35 years on the Stanford faculty, he took early retirement from his faculty responsibilities in order to focus full time on textbook writing, and now is Professor Emeritus of Operations Research at Stanford.

Dr. Hillier’s research has extended into a variety of areas, including integer programming, queueing theory and its application, statistical quality control, and the application of operations research to the design of production systems and to capital budgeting. He has published widely, and his seminal papers have been selected for republication in books of selected readings at least 10 times. He was the first-prize winner of a research contest on “Capital Budgeting of Interrelated Projects” sponsored by The Institute of Management Sciences (TIMS) and the U.S. Office of Naval Research. He and Dr. Lieberman also received the honorable mention award for the 1995 Lanchester Prize (best English-language publication of any kind in the field of operations research), which was awarded by the Institute of Operations Research and the Management Sciences (INFORMS) for the 6th edition of this book. In addition, he was the recipient of the prestigious 2004 INFORMS Expository Writing Award for the 8th edition of this book.

Dr. Hillier has held many leadership positions with the professional societies in his field. For example, he has served as treasurer of the Operations Research Society of America (ORSA), vice president for meetings of TIMS, co-general chairman of the 1989 TIMS International Meeting in Osaka, Japan, chair of the TIMS Publications Committee, chair of the ORSA Search Committee for Editor of *Operations Research*, chair of the ORSA Resources Planning Committee, chair of the ORSA/TIMS Combined Meetings Committee, and chair of the John von Neumann Theory Prize Selection Committee for INFORMS. He also is a Fellow of INFORMS. In addition, he recently completed a 20-year tenure as the series editor for Springer’s International Series in Operations Research and Management Science, a particularly prominent book series with over 200 published books that he founded in 1993.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *The Evaluation of Risky Interrelated Investments* (North-Holland, 1969), *Queueing Tables and Graphs* (Elsevier North-Holland, 1981, co-authored by O. S. Yu, with D. M. Avis, L. D. Fossett, F. D. Lo, and M. I. Reiman), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (5th ed., McGraw-Hill/Irwin, 2014, co-authored by his son Mark Hillier).

The late **Gerald J. Lieberman** sadly passed away in 1999. He had been Professor Emeritus of Operations Research and Statistics at Stanford University, where he was the founding chair of the Department of Operations Research. He was both an engineer (having received an undergraduate degree in mechanical engineering from Cooper Union) and an operations research statistician (with an AM from Columbia University in mathematical statistics, and a PhD from Stanford University in statistics).

Dr. Lieberman was one of Stanford's most eminent leaders in recent decades. After chairing the Department of Operations Research, he served as associate dean of the School of Humanities and Sciences, vice provost and dean of research, vice provost and dean of graduate studies, chair of the faculty senate, member of the University Advisory Board, and chair of the Centennial Celebration Committee. He also served as provost or acting provost under three different Stanford presidents.

Throughout these years of university leadership, he also remained active professionally. His research was in the stochastic areas of operations research, often at the interface of applied probability and statistics. He published extensively in the areas of reliability and quality control, and in the modeling of complex systems, including their optimal design, when resources are limited.

Highly respected as a senior statesman of the field of operations research, Dr. Lieberman served in numerous leadership roles, including as the elected president of The Institute of Management Sciences. His professional honors included being elected to the National Academy of Engineering, receiving the Shewhart Medal of the American Society for Quality Control, receiving the Cuthbertson Award for exceptional service to Stanford University, and serving as a fellow at the Center for Advanced Study in the Behavioral Sciences. In addition, the Institute of Operations Research and the Management Sciences (INFORMS) awarded him and Dr. Hillier the honorable mention award for the 1995 Lanchester Prize for the 6th edition of this book. In 1996, INFORMS also awarded him the prestigious Kimball Medal for his exceptional contributions to the field of operations research and management science.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *Handbook of Industrial Statistics* (Prentice-Hall, 1955, co-authored by A. H. Bowker), *Tables of the Non-Central t-Distribution* (Stanford University Press, 1957, co-authored by G. J. Resnikoff), *Tables of the Hypergeometric Probability Distribution* (Stanford University Press, 1961, co-authored by D. Owen), *Engineering Statistics*, (2nd ed., Prentice-Hall, 1972, co-authored by A. H. Bowker), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (McGraw-Hill/Irwin, 2000, co-authored by F. S. Hillier and M. S. Hillier).

ABOUT THE CASE WRITERS

Karl Schmedders is professor of quantitative business administration at the University of Zurich in Switzerland and a visiting associate professor at the Kellogg Graduate School of Management (Northwestern University). His research interests include management science, financial economics, and computational economics and finance. In 2003, a paper by Dr. Schmedders received a nomination for the Smith-Breeden Prize for the best paper in *Journal of Finance*. He received his doctorate in operations research from Stanford University, where he taught both undergraduate and graduate classes in operations research, including a case studies course in operations research. He received several teaching awards at Stanford, including the university's prestigious Walter J. Gores Teaching Award. After post-doctoral research at the Hoover Institution, a think tank on the Stanford campus, he became assistant professor of managerial economics and decision sciences at the Kellogg School. He was promoted to associate professor in 2001 and received tenure in 2005. In 2008, he joined the University of Zurich, where he currently teaches courses in management science, spreadsheet modeling, and computational economics and finance. At Kellogg he received several teaching awards, including the L. G. Lavengood Professor of the Year Award. More recently he won the best professor award of the Kellogg School's European EMBA program (2008, 2009, and 2011) and its Miami EMBA program (2011).

Molly Stephens is a partner in the Los Angeles office of Quinn, Emanuel, Urquhart & Sullivan, LLP. She graduated from Stanford University with a BS degree in industrial engineering and an MS degree in operations research. Ms. Stephens taught public speaking in Stanford's School of Engineering and served as a teaching assistant for a case studies course in operations research. As a teaching assistant, she analyzed operations research problems encountered in the real world and the transformation of these problems into classroom case studies. Her research was rewarded when she won an undergraduate research grant from Stanford to continue her work and was invited to speak at an INFORMS conference to present her conclusions regarding successful classroom case studies. Following graduation, Ms. Stephens worked at Andersen Consulting as a systems integrator, experiencing real cases from the inside, before resuming her graduate studies to earn a JD degree (with honors) from the University of Texas Law School at Austin. She is a partner in the largest law firm in the United States devoted solely to business litigation, where her practice focuses on complex financial and securities litigation.

DEDICATION

To the memory of our parents

and

To the memory of my beloved mentor,
Gerald J. Lieberman, who was one of the true
giants of our field

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