CALCULUS

EARLY TRANSCENDENTALS

SEVENTH EDITION

JAMES STEWART

McMASTER UNIVERSITY

AND

UNIVERSITY OF TORONTO

BROOKS/COLE
CENGAGE Learning

Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Contents

Preface xi

To the Student xxiii

Diagnostic Tests xxiv

A PREVIEW OF CALCULUS 1

1 Functions and Models 9

1.1 Four Ways to Represent a Function 10

1.2 Mathematical Models: A Catalog of Essential Functions 23

1.3 New Functions from Old Functions 36

1.4 Graphing Calculators and Computers 44

1.5 Exponential Functions 51

1.6 Inverse Functions and Logarithms 58

Review 72

Principles of Problem Solving 75

2 Limits and Derivatives 81

2.1 The Tangent and Velocity Problems 82

2.2 The Limit of a Function 87

2.3 Calculating Limits Using the Limit Laws 99

2.4 The Precise Definition of a Limit 108

2.5 Continuity 118

2.6 Limits at Infinity; Horizontal Asymptotes 130

2.7 Derivatives and Rates of Change 143

Writing Project ▪ Early Methods for Finding Tangents 153

2.8 The Derivative as a Function 154

Review 165

Problems Plus 170

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

3 Differentiation Rules 173

3.1 Derivatives of Polynomials and Exponential Functions 174

Applied Project ▪ Building a Better Roller Coaster 184

3.2 The Product and Quotient Rules 184

3.3 Derivatives of Trigonometric Functions 191

3.4 The Chain Rule 198

Applied Project ▪ Where Should a Pilot Start Descent? 208

3.5 Implicit Differentiation 209

Laboratory Project ▪ Families of Implicit Curves 217

3.6 Derivatives of Logarithmic Functions 218

3.7 Rates of Change in the Natural and Social Sciences 224

3.8 Exponential Growth and Decay 237

3.9 Related Rates 244

3.10 Linear Approximations and Differentials 250

Laboratory Project ▪ Taylor Polynomials 256

3.11 Hyperbolic Functions 257

Review 264

Problems Plus 268

4 Applications of Differentiation 273

4.1 Maximum and Minimum Values 274

Applied Project ▪ The Calculus of Rainbows 282

4.2 The Mean Value Theorem 284

4.3 How Derivatives Affect the Shape of a Graph 290

4.4 Indeterminate Forms and l'Hospital's Rule 301

Writing Project ▪ The Origins of l'Hospital's Rule 310

4.5 Summary of Curve Sketching 310

4.6 Graphing with Calculus *and* Calculators 318

4.7 Optimization Problems 325

Applied Project ▪ The Shape of a Can 337

4.8 Newton's Method 338

4.9 Antiderivatives 344

Review 351

Problems Plus 355

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

5 Integrals 359

5.1 Areas and Distances 360

5.2 The Definite Integral 371

Discovery Project ▪ Area Functions 385

5.3 The Fundamental Theorem of Calculus 386

5.4 Indefinite Integrals and the Net Change Theorem 397

Writing Project ▪ Newton, Leibniz, and the Invention of Calculus 406

5.5 The Substitution Rule 407

Review 415

Problems Plus 419

6 Applications of Integration 421

6.1 Areas Between Curves 422

Applied Project ▪ The Gini Index 429

6.2 Volumes 430

6.3 Volumes by Cylindrical Shells 441

6.4 Work 446

6.5 Average Value of a Function 451

Applied Project ▪ Calculus and Baseball 455

Applied Project ▪ Where to Sit at the Movies 456

Review 457

Problems Plus 459

7 Techniques of Integration 463

7.1 Integration by Parts 464

7.2 Trigonometric Integrals 471

7.3 Trigonometric Substitution 478

7.4 Integration of Rational Functions by Partial Fractions 484

7.5 Strategy for Integration 494

7.6 Integration Using Tables and Computer Algebra Systems 500

Discovery Project ▪ Patterns in Integrals 505

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

7.7 Approximate Integration 506
7.8 Improper Integrals 519
Review 529

Problems Plus 533

8 Further Applications of Integration 537

8.1 Arc Length 538
Discovery Project ▪ Arc Length Contest 545
8.2 Area of a Surface of Revolution 545
Discovery Project ▪ Rotating on a Slant 551
8.3 Applications to Physics and Engineering 552
Discovery Project ▪ Complementary Coffee Cups 562
8.4 Applications to Economics and Biology 563
8.5 Probability 568
Review 575

Problems Plus 577

9 Differential Equations 579

9.1 Modeling with Differential Equations 580
9.2 Direction Fields and Euler's Method 585
9.3 Separable Equations 594
Applied Project ▪ How Fast Does a Tank Drain? 603
Applied Project ▪ Which Is Faster, Going Up or Coming Down? 604
9.4 Models for Population Growth 605
9.5 Linear Equations 616
9.6 Predator-Prey Systems 622
Review 629

Problems Plus 633

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

10 Parametric Equations and Polar Coordinates 635

10.1 Curves Defined by Parametric Equations 636

Laboratory Project ▪ Running Circles around Circles 644

10.2 Calculus with Parametric Curves 645

Laboratory Project ▪ Bézier Curves 653

10.3 Polar Coordinates 654

Laboratory Project ▪ Families of Polar Curves 664

10.4 Areas and Lengths in Polar Coordinates 665

10.5 Conic Sections 670

10.6 Conic Sections in Polar Coordinates 678

Review 685

Problems Plus 688

11 Infinite Sequences and Series 689

11.1 Sequences 690

Laboratory Project ▪ Logistic Sequences 703

11.2 Series 703

11.3 The Integral Test and Estimates of Sums 714

11.4 The Comparison Tests 722

11.5 Alternating Series 727

11.6 Absolute Convergence and the Ratio and Root Tests 732

11.7 Strategy for Testing Series 739

11.8 Power Series 741

11.9 Representations of Functions as Power Series 746

11.10 Taylor and Maclaurin Series 753

Laboratory Project ▪ An Elusive Limit 767

Writing Project ▪ How Newton Discovered the Binomial Series 767

11.11 Applications of Taylor Polynomials 768

Applied Project ▪ Radiation from the Stars 777

Review 778

Problems Plus 781

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

12 Vectors and the Geometry of Space 785

12.1 Three-Dimensional Coordinate Systems 786
12.2 Vectors 791
12.3 The Dot Product 800
12.4 The Cross Product 808
Discovery Project ▪ The Geometry of a Tetrahedron 816
12.5 Equations of Lines and Planes 816
Laboratory Project ▪ Putting 3D in Perspective 826
12.6 Cylinders and Quadric Surfaces 827
Review 834

Problems Plus 837

13 Vector Functions 839

13.1 Vector Functions and Space Curves 840
13.2 Derivatives and Integrals of Vector Functions 847
13.3 Arc Length and Curvature 853
13.4 Motion in Space: Velocity and Acceleration 862
Applied Project ▪ Kepler's Laws 872
Review 873

Problems Plus 876

14 Partial Derivatives 877

14.1 Functions of Several Variables 878
14.2 Limits and Continuity 892
14.3 Partial Derivatives 900
14.4 Tangent Planes and Linear Approximations 915
14.5 The Chain Rule 924
14.6 Directional Derivatives and the Gradient Vector 933
14.7 Maximum and Minimum Values 946
Applied Project ▪ Designing a Dumpster 956
Discovery Project ▪ Quadratic Approximations and Critical Points 956

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

14.8 Lagrange Multipliers 957

Applied Project ▪ Rocket Science 964

Applied Project ▪ Hydro-Turbine Optimization 966

Review 967

Problems Plus 971

15 Multiple Integrals 973

15.1 Double Integrals over Rectangles 974

15.2 Iterated Integrals 982

15.3 Double Integrals over General Regions 988

15.4 Double Integrals in Polar Coordinates 997

15.5 Applications of Double Integrals 1003

15.6 Surface Area 1013

15.7 Triple Integrals 1017

Discovery Project ▪ Volumes of Hyperspheres 1027

15.8 Triple Integrals in Cylindrical Coordinates 1027

Discovery Project ▪ The Intersection of Three Cylinders 1032

15.9 Triple Integrals in Spherical Coordinates 1033

Applied Project ▪ Roller Derby 1039

15.10 Change of Variables in Multiple Integrals 1040

Review 1049

Problems Plus 1053

16 Vector Calculus 1055

16.1 Vector Fields 1056

16.2 Line Integrals 1063

16.3 The Fundamental Theorem for Line Integrals 1075

16.4 Green's Theorem 1084

16.5 Curl and Divergence 1091

16.6 Parametric Surfaces and Their Areas 1099

16.7 Surface Integrals 1110

16.8 Stokes' Theorem 1122

Writing Project ▪ Three Men and Two Theorems 1128

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

16.9 The Divergence Theorem 1128
16.10 Summary 1135
Review 1136

Problems Plus 1139

17 **Second-Order Differential Equations 1141**

17.1 Second-Order Linear Equations 1142
17.2 Nonhomogeneous Linear Equations 1148
17.3 Applications of Second-Order Differential Equations 1156
17.4 Series Solutions 1164
Review 1169

Appendixes A1

A Numbers, Inequalities, and Absolute Values A2
B Coordinate Geometry and Lines A10
C Graphs of Second-Degree Equations A16
D Trigonometry A24
E Sigma Notation A34
F Proofs of Theorems A39
G The Logarithm Defined as an Integral A50
H Complex Numbers A57
I Answers to Odd-Numbered Exercises A65

Index A135

Copyright 2010 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s).
Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.