CALCULUS

EARLY TRANSCENDENTALS

EIGHTH EDITION

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McMASTER UNIVERSITY

AND

UNIVERSITY OF TORONTO

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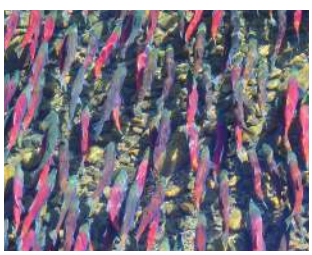

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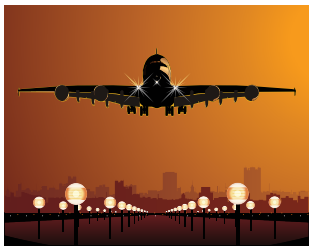

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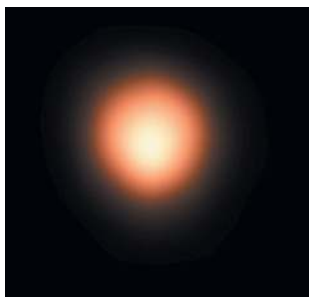

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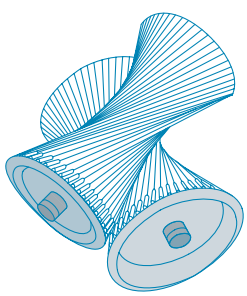

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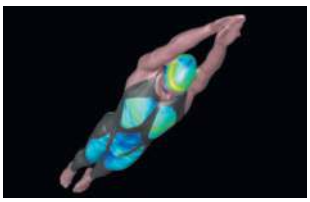

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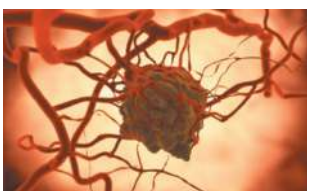

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