

# Basic Econometrics

**Fifth Edition**

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After teaching for more than 25 years at the City University of New York and 17 years in the Department of Social Sciences, U.S. Military Academy at West Point, New York, Dr. Gujarati is currently Professor Emeritus of economics at the Academy. Dr. Gujarati received his M.Com. degree from the University of Bombay in 1960, his M.B.A. degree from the University of Chicago in 1963, and his Ph.D. degree from the University of Chicago in 1965. Dr. Gujarati has published extensively in recognized national and international journals, such as the *Review of Economics and Statistics*, the *Economic Journal*, the *Journal of Financial and Quantitative Analysis*, and the *Journal of Business*. Dr. Gujarati was a member of the Board of Editors of the *Journal of Quantitative Economics*, the official journal of the Indian Econometric Society. Dr. Gujarati is also the author of *Pensions and the New York City Fiscal Crisis* (the American Enterprise Institute, 1978), *Government and Business* (McGraw-Hill, 1984), and *Essentials of Econometrics* (McGraw-Hill, 3d ed., 2006). Dr. Gujarati's books on econometrics have been translated into several languages.

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