



ESSENTIAL CALCULUS

EARLY TRANSCENDENTALS

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PREFACE

This book is a response to those instructors who feel that calculus textbooks are too big. In writing the book I asked myself: What is essential for a three-semester calculus course for scientists and engineers?

The book is about two-thirds the size of my other calculus books (*Calculus*, Fifth Edition and *Calculus, Early Transcendentals*, Fifth Edition) and yet it contains almost all of the same topics. I have achieved relative brevity mainly by condensing the exposition and by putting some of the features on the website www.stewartcalculus.com. Here, in more detail are some of the ways I have reduced the bulk:

- I have organized topics in an efficient way and rewritten some sections with briefer exposition.
- The design saves space. In particular, chapter opening spreads and photographs have been eliminated.
- The number of examples is slightly reduced. Additional examples are provided online.
- The number of exercises is somewhat reduced, though most instructors will find that there are plenty. In addition, instructors have access to the archived problems on the website.
- Although I think projects can be a very valuable experience for students, I have removed them from the book and placed them on the website.
- A discussion of the principles of problem solving and a collection of challenging problems for each chapter have been moved to the web.

Despite the reduced size of the book, there is still a modern flavor: Conceptual understanding and technology are not neglected, though they are not as prominent as in my other books.

CONTENT

This book treats the exponential, logarithmic, and inverse trigonometric functions early, in Chapter 3. Those who wish to cover such functions later, with the logarithm defined as an integral, should look at my book titled simply *Essential Calculus*.

CHAPTER 1 ■ FUNCTIONS AND LIMITS After a brief review of the basic functions, limits and continuity are introduced, including limits of trigonometric functions, limits involving infinity, and precise definitions.

CHAPTER 2 ■ DERIVATIVES The material on derivatives is covered in two sections in order to give students time to get used to the idea of a derivative as a function. The

formulas for the derivatives of the sine and cosine functions are derived in the section on basic differentiation formulas. Exercises explore the meanings of derivatives in various contexts.

CHAPTER 3 ■ INVERSE FUNCTIONS: EXPONENTIAL, LOGARITHMIC, AND INVERSE TRIGONOMETRIC FUNCTIONS Exponential functions are defined first and the number e is defined as a limit. Logarithms are then defined as inverse functions. Applications to exponential growth and decay follow. Inverse trigonometric functions and hyperbolic functions are also covered here. L'Hospital's Rule is included in this chapter because limits of transcendental functions so often require it.

CHAPTER 4 ■ APPLICATIONS OF DIFFERENTIATION The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. The section on curve sketching includes a brief treatment of graphing with technology. The section on optimization problems contains a brief discussion of applications to business and economics.

CHAPTER 5 ■ INTEGRALS The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix C.) A quite general definition of the definite integral (with unequal subintervals) is given initially before regular partitions are employed. Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.

CHAPTER 6 ■ TECHNIQUES OF INTEGRATION All the standard methods are covered, as well as computer algebra systems, numerical methods, and improper integrals.

CHAPTER 7 ■ APPLICATIONS OF INTEGRATION General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral. The chapter concludes with an introduction to differential equations, including separable equations and direction fields.

CHAPTER 8 ■ SERIES The convergence tests have intuitive justifications as well as formal proofs. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those based on Taylor's Formula (with Lagrange's form of the remainder term) and those from graphing devices.

CHAPTER 9 ■ PARAMETRIC EQUATIONS AND POLAR COORDINATES This chapter introduces parametric and polar curves and applies the methods of calculus to them. A brief treatment of conic sections in polar coordinates prepares the way for Kepler's Laws in Chapter 10.

CHAPTER 10 ■ VECTORS AND THE GEOMETRY OF SPACE In addition to the material on vectors, dot and cross products, lines, planes, and surfaces, this chapter covers vector-valued functions, length and curvature of space curves, and velocity and acceleration along space curves, culminating in Kepler's laws.

CHAPTER 11 ■ PARTIAL DERIVATIVES In view of the fact that many students have difficulty forming mental pictures of the concepts of this chapter, I've placed a special emphasis on graphics to elucidate such ideas as graphs, contour maps, directional derivatives, gradients, and Lagrange multipliers.

CHAPTER 12 ■ MULTIPLE INTEGRALS Cylindrical and spherical coordinates are introduced in the context of evaluating triple integrals.

CHAPTER 13 ■ VECTOR CALCULUS The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

WEBSITE

The website www.stewartcalculus.com includes the following.

- Review of Algebra, Analytic Geometry, and Conic Sections
- Additional Examples
- Projects
- Archived Problems (drill exercises that have appeared in previous editions of my other books), together with their solutions
- Challenge Problems
- Complex Numbers
- Graphing Calculators and Computers
- Lies My Calculator and Computer Told Me
- Additional Topics (complete with exercise sets): Principles of Problem Solving, Strategy for Integration, Strategy for Testing Series, Fourier Series, Area of a Surface of Revolution, Linear Differential Equations, Second-Order Linear Differential Equations, Nonhomogeneous Linear Equations, Applications of Second-Order Differential Equations, Using Series to Solve Differential Equations, Complex Numbers, Rotation of Axes
- Links, for particular topics, to outside web resources
- History of Mathematics, with links to the better historical websites

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The idea for this book came from my editor, Bob Pirtle, who had been hearing of the desire for a much shorter calculus text from numerous instructors. I thank him for encouraging me to pursue this idea and for his advice and assistance whenever I needed it.

JAMES STEWART