Essays in International Economics and Macroeconomics

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Dedication

To my parents, Queiroz and Vera, and my wife, Daniela.

Abstract

The three chapters of this dissertation investigate major puzzles in international economics and macroeconomics. Chapter 1 proposes a new measure of knowledge production within corporations and analyzes how the production and flow of knowledge within multinational corporations can account for the cross-country correlation in corporate sector GDP fluctuations. Chapter 2 studies how fluctuations in the price of primary commodities can account for fluctuations in bilateral real exchange rates between the United States and United Kingdom, Germany, and Japan. Finally, Chapter 3 studies the role of firm entry in accounting for the slow recovery in employment following the World Economic Crisis in 2008–2009.[1]

Chapter 1 proposes a new measure of knowledge production by U.S. multinational corporations in the United States and European Union, and assesses its quantitative implications for international business cycles. I use a two-country general equilibrium model of knowledge flows within multinationals and compute the parameter values related to knowledge production such that, in steady-state, the model matches the observed factor share differentials between the operations of U.S. multinationals in the United States and European Union. This is an alternative measure in comparison to previous studies that focus on firm expenditures in R&D, marketing, among others. The difficulties involved in measuring the expenditures on knowledge investment from the data is that it is not observable and part of it is produced and used within a firm, so this literature relies heavily on extrapolations and on the use of proxies to estimate the production of knowledge that takes place within firms. So in Chapter 1 I use a theory that allows me to quantify the production of knowledge indirectly. The main assumptions are: i) U.S. multinationals produce knowledge in the United States; ii) this knowledge is used by its subsidiaries in the European Union; and iii) investment in knowledge is either unobserved or expensed in corporate accounts. The results show that investment in knowledge is 1.4 times larger than investment in tangible capital. I then move on to study its quantitative implications for international business cycles. In particular, Chapter 1 focuses on its implications for the correlation in GDP fluctuations in the corporate sector across countries. The model with knowledge flows generates cross-country correlations in corporate sector GDP fluctuations that are much closer to the data in comparison to the standard international real business cycle model. The main reason is

[1] Chapter 2 is co-authored with Constantino Hevia and Juan Pablo Nicolini. Chapter 3 is co-authored with Gajendran Raveendranathan.

that the presence of an input in production that is available simultaneously (nonrival) in both countries reduces the incentives for corporations to move other inputs across countries when facing a country-specific shock.

Chapter 2 shows that a substantial fraction of the volatility of real exchange rates between developed economies like the United States, Germany, Japan, and the United Kingdom in 1960–2014 can be accounted for by shocks that affect primary commodity prices, like oil, aluminum, maize, or copper. The idea explored in Chapter 2 is the following: fluctuations in the prices of commodities affect manufacturing costs, and therefore manufacturing prices, which in turn induce changes in final good costs. These cost fluctuations translate into price fluctuations at the country level. If changes in commodity prices have differential effects on the domestic cost of any two countries, primary commodity price changes will affect the real exchange rate between those two countries. Chapter 2 presents the production side of a totally standard model of an open economy that makes explicit the production of commodities and the use of commodities in the production of manufactured goods. It derives an equilibrium condition relating the bilateral real exchange rate between two countries to primary commodity prices that, in the case of Cobb-Douglas production functions, is linear in the logarithms of the variables. This relation is then used for the empirical analysis. A summary of the results is that with just 3 or 4 primary commodity prices, one can account for between 34% and 91% of the volatility of the RER between the US and those three countries, depending on the country and the period considered. These results are remarkable, given the so called *exchange rate disconnect puzzle*: the fact that real exchange rates across developed economies are very volatile, very persistent, and very hard to relate to fundamentals, as documented by previous studies. Therefore, the analysis in Chapter 2 implies that existing models used to analyze real exchange rates between large economies that mostly focus on trade between differentiated final goods could benefit, in terms of matching the evidence, by also considering trade in primary commodities.

Chapter 3 shows that lack of firm entry has been an important factor causing the slow recovery of employment in the United States after the 2008-2009 Financial Crisis. A counterfactual exercise shows that lack of firm entry accounts for 22% of the difference between the actual level of employment per labor force participant in March 2012 and its pre-recession level, in March 2007. It then moves forward to study what types of shocks could explain this pattern. In a standard model of firm dynamics featuring aggregate uncertainty and firm heterogeneity, it shows that a negative aggregate productivity shock does not generate a drop in firm entry, while a negative demand shock does. The latter causes a significant drop in firm entry that is similar to the one observed during the Financial Crisis. However, the demand shock alone does not generate a slow recovery. Finally, Chapter 3 also provides empirical evidence that contradicts common explanations for the slow recovery, such as financial constraints, offshoring, increased uncertainty at the firm level, and increased self-employment.

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Chapter 1

Knowledge Flows, Dark Matter, and International Business Cycles

1.1 Introduction

The production of knowledge by profit-maximizing firms is at the core of the main theories of economic growth and firm dynamics.[1] Firms invest in knowledge and grow, while spillover effects allow the rest of the economy to benefit from it. Knowledge production also has implications for firms' decisions to access foreign markets through foreign direct investment and trade.[2] Yet, the process through which knowledge is produced is largely unknown. The main reasons are that knowledge is an intangible good, and that part of it is produced and used within firms, so that there are no observable transactions involved in the knowledge production process. In this paper, I abstract from endogenous growth and spillover effects and focus my analysis on knowledge production. I focus on a particular group of firms, multinational corporations, and use data on the operations of U.S. multinationals in the United States and European Union to compute the parameter values related to knowledge production.[3] The results show that the investment in knowledge is 1.4 times larger than the investment in physical capital. Furthermore, I show that the calibrated model with knowledge production has quantitative implications for the cross-country correlation in corporate sector GDP fluctuations. This is another contribution of the paper, since knowledge flows are usually restricted to the analysis of long-run growth patterns.[4]

The production of knowledge and its flow within multinational corporations make up one of the main theories for explaining the existence of multinational corporations in the first place (see, e.g., Markusen (1984) and Helpman (1984)). Knowledge is produced within corporations, and that knowledge can be used in different locations at no additional cost. This creates an incentive for these corporations to expand their operations across different locations, including different countries. This theory is particularly relevant to a study of foreign direct investment between the United States and European Union, which is the focus of this paper. Alternative theories explore the differentials in factor costs across countries, such as labor and tangible capital. While they apply well to the U.S. foreign direct investment in China and Mexico, for example, they do not perform as well in the case of the European Union, since its factor costs are not much different from those in the United States.

The international flow of knowledge through multinational corporations is also one of

[1] In this paper, knowledge is defined as a nonrival and excludable good. Examples of growth theories with knowledge production are Aghion and Howitt (1992), Grossman and Helpman (1991), and Romer (1986, 1990). See Luttmer (2011) for a theory of firm dynamics with knowledge production.

[2] See Melitz (2003) for an example with trade and Helpman, Melitz and Yeaple (2004) for an example with foreign direct investment.

[3] Multinational corporations are corporations with establishments in multiple countries.

[4] Exceptions are Ramondo and Rappoport (2010) and Menno (2015).

the leading explanations for the fact that the United States has been a net recipient of investment income despite its increasing external debt position—the United States has been a net borrower since the early 1980s. Commonly known as dark matter (Hausmann and Sturzenegger (2005)), this literature shows that the positive U.S. net investment income is explained by the high rates of return on U.S. foreign direct investment, and that unobservable flows of intangible goods and capital could account for it (e.g., McGrattan and Prescott (2010)).[5]

In order to be able to compute the parameters related to knowledge production, I assume that multinationals produce knowledge in a single location, the country where its headquarters is located. This assumption is supported by the fact that 86% of the research and development (R&D) expenditures of U.S. multinationals take place in the United States, and that these multinationals export knowledge-intensive services such as royalties and license fees from the United States to their foreign subsidiaries, instead of the other way around.

Under the assumption that U.S. multinationals produce knowledge in the United States and that it is used by its subsidiaries abroad, the theory implies differentials in observed labor shares between the activities of U.S. multinationals in the United States and European Union. In particular, the measured labor share of U.S. multinationals in the United States will be higher than its true counterpart if investment in knowledge is either unobserved or expensed in corporate accounts. In addition, the fact that investment in knowledge is expensed does not affect the observed ratio of capital expenditures to compensation of employees. I use data on the activity of U.S. multinationals in the United States and European Union from the Bureau of Economic Analysis (BEA) and show that this is exactly the pattern observed. The labor share of U.S. multinational operations in the European Union, 46.4%, is significantly lower than the labor share of their operations in the United States, 57.4%, whereas the ratios of capital expenditures to compensation of employees—29.7% in the European Union and 32.8% in the United States—are roughly the same.

In order to estimate the share of knowledge in production and its depreciation rate, I build on the technology capital model of McGrattan and Prescott (2010) and compute the parameter values such that the model matches the observed factor shares of U.S. multinationals in the United States and European Union in steady state. The estimates for the knowledge share and its annual depreciation rate are 30% and 11%, respectively. This share is significantly higher than previous studies suggest. These studies rely on empirical estimates of expenditures in R&D, advertisement, organizational capital, and so on. However,

[5]Profit-shifting strategies such as transfer pricing could also generate the same pattern. See Bernard, Jensen and Schott (2006) for a study on transfer pricing.

such estimates cannot capture all the investment in knowledge that takes place within a firm. For example, the time that a worker spends in routine tasks versus the time that he spends in creating knowledge cannot be observed. In this sense, I am using a broader definition than technology capital, which I simply call knowledge. On the other hand, the estimate for the annual depreciation rate of knowledge, 11%, is close to the value that is usually used in the literature, 15%. While my prior is that a significant fraction of investment in knowledge was not being accounted for by previous studies, there is no reason to think that the depreciation rate of the part of knowledge that is actually observed should be different from the depreciation rate of the part that is not observed.

Next, using the estimated parameter values, I show that the flow of knowledge through multinational corporations have quantitative implications for the international transmission of shocks. More specifically, a negative productivity shock in the headquarters country disrupts its production of knowledge. The lower amount of knowledge causes a reduction in the production of its subsidiaries, and this implies a reduction of economic activity in the host country. At the same time, the negative productivity shock reduces the incentives of multinationals whose subsidiaries are located in that country to produce more knowledge. On top of that, the fact that the knowledge stock of a corporation is present in both regions (its nonrival within the corporation) reduces its incentive to move other factors of productions across countries when the economy is hit by a country-specific shock. I start by documenting the following empirical facts that connect multinational activity to the correlation in GDP fluctuations between the corporate sectors in the United States and European Union: i) the correlation in GDP fluctuations between the United States and major European Union countries during 1995–2007 is explained by the correlation between the corporate sectors of these countries; ii) operations of U.S. multinationals in the United States and European Union are highly correlated during the period 1995–2007; iii) the correlation between corporate sectors increases as a function of the share of U.S. multinationals in the corporate sector GDP. Next, I simulate the model to assess its business cycle properties. The model nests the standard international real business cycle model by Backus, Kehoe, and Kydland (1994) as a particular case. One of the main puzzles in international macroeconomics is the failure of the standard international real business cycle model to account for the observed cross-country correlation in GDP fluctuations. Using the parameter values that I estimate, I show that incorporating knowledge flows within multinational corporations reduces the discrepancy between the correlation implied by the model and the correlation observed in the data by 48%.

1.1.1 Related Literature

This paper is related to the literature that studies intangible flows within firms and the issues related to its measurement (e.g., Corrado, Hulten, and Sichel (2005)). The closest paper to my analysis is McGrattan and Prescott (2010). In their paper, in order to pin down the values for the parameters related to both knowledge and plant-specific intangible capital, the authors use estimates of expenditures on R&D, advertisement, and so on, and calibrate the remaining parameters using data on the value of corporations. Using these estimates, the authors account for the differentials in tangible capital rates of return between U.S. foreign direct investment and foreign direct investment in the United States. In this paper, I take a different route. I compute the parameter values related to knowledge such that, in steady state, the model matches the moments in the data (labor share differentials and ratio of capital expenditures to compensation of employees), and then I use the results to study international business cycles. Ramondo, Rappoport and Ruhl (2016) use firm-level data on U.S. multinational corporations to document that the median subsidiary ships nothing to the rest of the corporation. Atalay, Hortaçsu and Syverson (2014) find similar evidence for multi-establishment corporations in the United States. These findings can be interpreted as supporting the theory of knowledge flows within corporations.

This paper is also related to the literature on international real business cycles (e.g., Backus, Kehoe and Kydland (1992) and Heathcote and Perri (2002)). In particular, this paper is related to the literature that analyzes the connection between foreign direct investment and international business cycles, which includes the empirical evidence in Kleinert, Martin and Toubal (2015), and Cravino and Levchenko (2016), and the quantitative studies based on general equilibrium models of foreign direct investment, such as Cravino and Levchenko (2016), Ramondo and Rappoport (2010), and Menno (2015). In particular, Menno (2015) was the first to incorporate the technology capital model of McGrattan and Prescott (2010) into an international real business cycle framework. While the author focuses on investment synchronization and uses the parameter values of McGrattan and Prescott (2010), my analysis focuses on corporate sector GDP correlation and uses the parameter values based on the calibration strategy described above.

Finally, this paper is related to the literature on dark matter (e.g., Hausmann and Sturzenegger (2005)), which studies the U.S. external debt position and its net investment income. As mentioned before, leading explanations include the exports of intangibles from the United States to the rest of the world that are not observed in the data, which is a feature of the model that I use in this paper. While there are no global imbalances in my analysis, the empirical facts that I present, together with the large share of knowledge that I estimate, provide further support for this type of explanation.

This chapter is organized as follows: Section 1.2 presents data on U.S. multinationals regarding intrafirm trade in knowledge intensive services and factor shares in production; Section 1.3 presents a two-country general equilibrium model of knowledge flows through multinational corporations that replicates the pattern observed in the data; in Section 1.4, I estimate the parameters related to knowledge production; Section 1.5 presents data on the correlation between corporate sector GDP fluctuations in the United States and European Union, and between GDP fluctuations of U.S. multinational operations in the United States and European Union; Section 1.6 assesses the business cycle properties of the model with knowledge flows; Section 1.7 concludes.

1.2 Data on U.S. multinationals

In this section, I document the following facts: i) 86% of R&D expenditures of U.S. multinationals takes place in the U.S.; ii) U.S. multinationals export knowledge-intensive services such as royalties and license fees from the United States to their foreign subsidiaries; iii) the labor share of U.S. multinational operations in the European Union is much lower than the labor share of U.S. multinational operations in the United States, whereas the ratios of physical capital expenditures to compensation of employees are roughly the same. Tables and figures are included at the end of the paper.

1.2.1 R&D expenditures and intrafirm trade in royalties and license fees

Table I shows statistics on the operations of U.S. multinationals. First, it shows that their R&D expenditures are concentrated in the United States, representing 85.9% of the total. Second, it shows that most of the exports of royalties and license fees from the United States to the European Union represent exports from U.S. multinationals to their foreign subsidiaries. In this case, intrafirm trade in royalties and license fees accounts for 66.4% of the total exports. Finally, Table I shows that U.S. multinationals account for a significant share of the U.S. corporate sector GDP, 31.4%. Figure 1.1, on the other hand, shows that U.S. multinationals export royalties and license fees from the United States to their affiliates in the European Union, and not the other way around.

Table I: Data on U.S. multinationals

	Share (%)
Share of U.S. multinationals R&D expenditures that takes place in the United States	85.9
Share of exports of royalties and license fees to the European Union that is intrafirm	66.4
Share of U.S. multinationals in U.S. corporate sector GDP	31.4

Source: BEA.
Note: The table shows that most of R&D expenditures of U.S. multinationals take place in the United States, and that most of exports of royalties and license fees are intrafirm. The share of U.S. multinationals R&D expenditures in the United States is the average share in 2004–2007. The share of intrafirm exports of royalties and license fees to the European Union is the average share in 2006–2007. The share of U.S. multinationals in U.S. corporate sector GDP is the average in share in 1995–2007. The changes in time intervals reflect data availability.

Together, these statistics support the model assumptions that I make in Section 1.3. In particular, I focus on intrafirm flow of knowledge, which is supported by the fact that most of the trade in knowledge intensive services such as royalties and license fees between the United States and European Union represent intrafirm trade. In addition, in the model I assume that the production of knowledge within corporations takes place in a single location, the country where the headquarters is located, which is supported by the fact that U.S. multinationals are net exporters of royalties and license fees (see Figure 1.1).

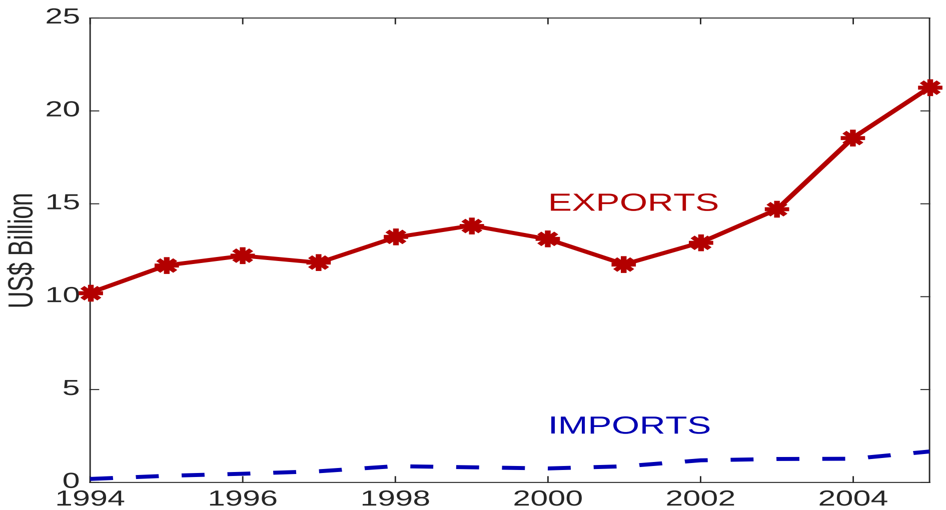
1.2.2 Labor shares and capital expenditures of U.S. multinationals in the United States and European Union

In this section, I compare the labor shares and the ratio of capital expenditures to compensation of employees between U.S. multinational operations in the United States and European Union. Table II summarizes the results. The difference between the shares is significant. While the labor share of U.S. multinationals in the United States is 57.4%, the labor share of their operations in the European Union is much lower, 46.4%.[6] This is surprising, since the labor shares in the United States and European Union corporate sectors are not much different from each other (61.8% and 58.6%, respectively). On the other hand, the ratios between capital expenditures and compensation of employees are similar to each

[6]Labor share is defined as the ratio of compensation of employees to gross product (GDP), without adjustments for mixed income. See Mataloni and Goldberg (1994) for a description of the BEA measure of U.S. multinationals value-added.

other: 32.8% in the United States versus 29.7% in the European Union.

Figure 1.1: U.S. multinationals intrafirm trade of royalties and license fees with the European Union (1994–2005)



Sources: BEA.

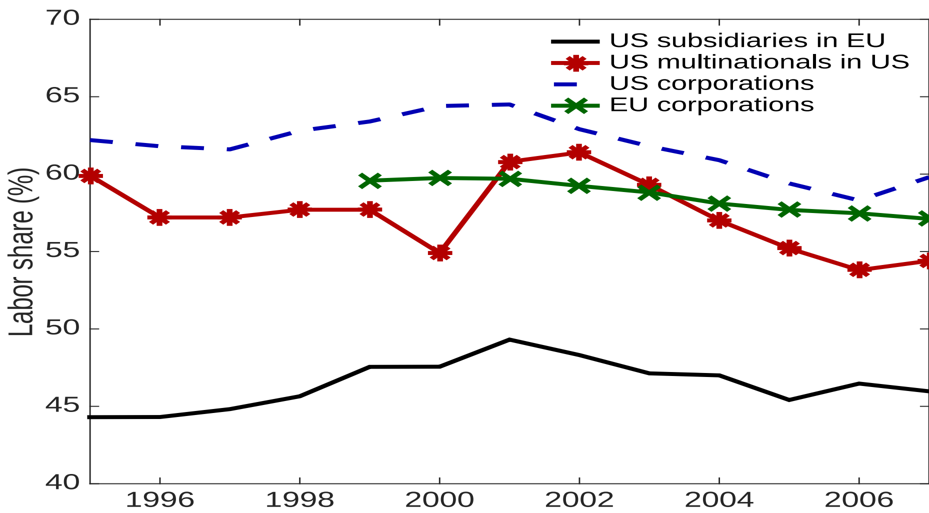
Note: The figure shows that U.S. multinationals export royalties and license fees to their subsidiaries in the European Union.

Figures 1.2a and 1.2b show the series. Figure 1.2a shows that the difference in labor shares is persistent across time, while Figure 1.2b shows that the ratios of capital expenditures to the sum of capital expenditures and compensation of employees are roughly the same across time.

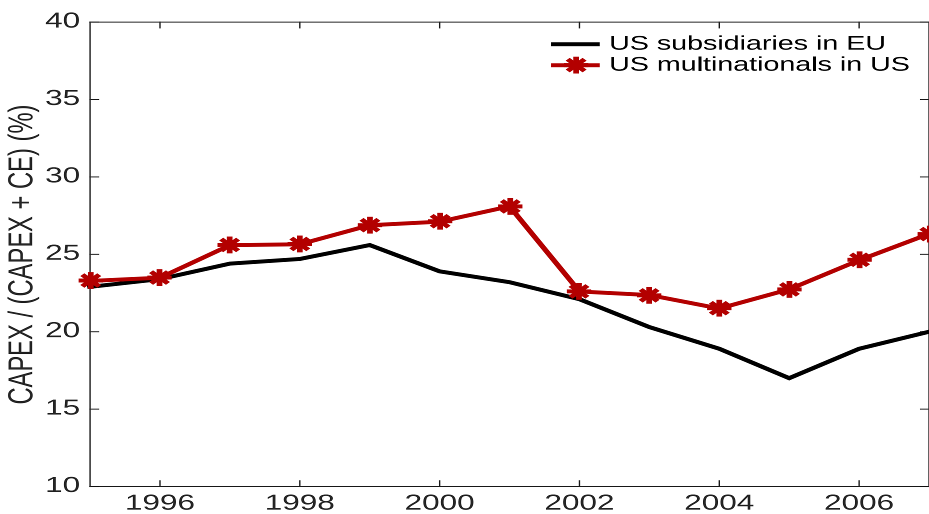
In the next section, I show that this pattern emerges in a model in which multinationals produce knowledge in their headquarters (home) country and that knowledge is used by their subsidiaries abroad. The investment in knowledge is either unobserved or expensed in corporate accounts, which implies higher (measured) labor shares at home versus abroad.

Figure 1.2: Factor shares of U.S. multinational operations

(a) Labor share



(b) Capital expenditures



Sources: OECD and BEA.

Note: Figure 4a shows the labor share (compensation of employees over GDP) for four different groups: i) U.S. subsidiaries operating in the European Union; ii) U.S. multinationals operating in the United States; iii) U.S. corporations; iv) EU corporations. Although the labor shares in the U.S. and EU are similar to each other, the labor share of subsidiaries of U.S. multinationals in the European Union is much lower, and this difference is persistent across time. Figure 4b shows the ratio of capital expenditures (CAPEX) to the sum of capital expenditures and compensation of employees (CE), for both U.S. multinationals in the United States and European Union. It shows that they are roughly the same, specially in 1995–2004.

Table II: Factor shares: (%) average in 1997–2015

	Labor share (%)	Capital expenditures to compensation of employees ratio (%)
U.S. subsidiaries in EU	**46.4**	29.7
U.S. multinationals in U.S.	**57.4**	32.8
U.S. corporate sector	61.8	-
EU corporate sector (1999:2007)	58.6	-

Source: BEA for the United States, and OECD for the European Union.
Note: The table shows that the labor share of U.S. subsidiaries in the European Union is much lower than in the United States, while the ratios of capital expenditures to compensation of employees are similar. Labor share is defined as the ratio of compensation of employees to GDP. EU corporate sector data from OECD include 28 countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Sweden, and the United Kingdom. Data on U.S. subsidiaries in the European Union cover 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and United Kingdom. The fact that they cover two different groups should not be a problem because the EU-15 group accounts for more than 92% of GDP in the European Union.

1.3 Model of international knowledge flows through multinational corporations

In this section, I present a two-country general equilibrium model with knowledge flows within multinational corporations. The model builds on McGrattan and Prescott (2010) by allowing for imperfect substitutability between intermediate goods produced by multinational corporations from different countries. Time is discrete, and the world consists of two symmetric countries ($i = 1, 2$), each characterized by its population size (normalized to one) and a finite measure of locations assumed to be proportional to its population size (the proportionality factor is also normalized to one).[7] To simplify the exposition, I begin by describing the production structure of the economy, and later I describe the households.

[7]The concept of "location" allows the introduction of a firm-specific nonrival input (knowledge) into a standard model in which agents take prices as given. Corporations are able to use the nonrival input in different locations simultaneously, but the finite measure of locations in each country prevents them from expanding without bound. See McGrattan and Prescott (2009, 2010) for a full description of a model with locations.

Final good production: Final good producers use intermediate goods to produce a nontradable final good that is used for consumption and (tangible) capital investment. I use the final good in country 1 as the numeraire. The nontradable final good in each country is produced according to the following technology:

$$G_1(a_{1,t}, b_{1,t}) = \left(\omega a_{1,t}^{\frac{\sigma-1}{\sigma}} + (1-\omega) b_{1,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \quad (1.1)$$

$$G_2(a_{2,t}, b_{2,t}) = \left((1-\omega) a_{2,t}^{\frac{\sigma-1}{\sigma}} + \omega b_{2,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \quad (1.2)$$

where σ, $\sigma \geq 0$, is the elasticity of substitution between the tradable intermediate goods produced by firms from country 1, a, and by firms from country 2, b. The production technology is an Armington aggregator and ω, $0 < \omega < 1$, determines the degree of home bias.[8] In each period t, the final good producers in country 1 solve the maximization problem:

$$\max_{a_{1,t}, b_{1,t} \geq 0} G_1(a_{1,t}, b_{1,t}) - q_{a,t} a_{1,t} - q_{b,t} b_{1,t}. \quad (1.3)$$

The problem for the final good producer in country 2 is analogous, but note that the price of its final good, $p_{2,t}$, might be different than one.

The structure so far is very close to the standard international real business cycle model (e.g., Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002)) without aggregate uncertainty.[9] Besides abstracting from aggregate uncertainty, the difference is in the production structure of the intermediate goods a and b, which I describe next.

Instead of restricting intermediate goods a and b to be produced in a single country and then exported, I allow for their production to take place in both countries. The intermediate good producers will then be multinational corporations.

Intermediate good production: Multinational corporations from country 1 produce good a according to the following technology:

$$a_{1,t} + a_{2,t} = y_{11,t} + y_{12,t}, \quad (1.4)$$

$$y_{11,t} = z_{1,t} F(k^y_{11,t}, m_{1,t}, n^y_{11,t}), \quad (1.5)$$

$$y_{12,t} = z_{2,t} F(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}), \quad (1.6)$$

[8] In this paper, home bias is not related to the location where the good is produced, but to the nationality of the corporation that produces the good.

[9] In Section 1.6, I introduce aggregate uncertainty and analyze the model's business cycle properties.

where $F(k,m,n) = (k^{\alpha}n^{1-\alpha})^{1-\phi}m^{\phi}$, $0 < \alpha < 1, 0 < \phi < 1$. Multinational corporations from country 1 produce the intermediate good a using capital k, firm-specific knowledge m, and labor, n. The amount of good a produced in country 1 is y_{11}, and y_{12} is the amount of good a produced by subsidiaries in country 2. Note that m is nonrival, it is used both in (1.5) and (1.6).[10] There is a cost of adapting the firm-specific knowledge to a different country, and its denoted by θ, $0 < \theta < 1$. As long as $m > 0$ and $\theta > 0$, the corporation will always choose to operate in both countries. Knowledge is produced according to the following technology:

$$g^m_{1,t} = z_{1,t}F(k^m_{11,t}, m_{1,t}, n^m_{11,t}), \tag{1.7}$$

$$m_{1,t+1} = (1-\delta^m)m_{1,t} + g^m_{1,t}, \tag{1.8}$$

where F is the same production function as in (1.5) and (1.6), δ^m is the depreciation rate of knowledge, and g^m is the investment in knowledge.[11] Productivity in countries 1 and 2 are denoted by $z_{1,t}$ and $z_{2,t}$, respectively, and they are assumed to be constant over time and normalized to one, so that $z_{1,t} = z_{2,t} = 1$ for all t.[12] Note that corporations from country 1 produce knowledge only in country 1. The multinational corporation from country 1 then comprises its headquarters in country 1, that produces knowledge, and its subsidiaries in both countries 1 and 2, that produce the intermediate good a.

Given the initial capital and knowledge stocks, multinationals from country 1 choose labor, capital investment, and knowledge investment, in order to maximize the discounted value of dividend payments:

$$\max \sum_{t=0}^{\infty} \Lambda_{0,t} D_{1,t}, \tag{1.9}$$

where $\Lambda_{t_0,t}$ is the pricing kernel.[13] The dividends payment $D_{1,t}$ is given by

$$\begin{aligned} D_{1,t} = \; & q_{a,t}(y_{11,t} + y_{12,t}) + p^m_{1,t}g^{m,s}_{1,t} - (w_{1,t}(n^y_{11,t} + n^m_{11,t}) + w_{2,t}n^y_{12,t}) \\ & -(x^y_{11,t} + p_{2,t}x^y_{12,t} + x^m_{11,t} + p^m_{1,t}g^{m,d}_{1,t}), \end{aligned} \tag{1.10}$$

where p_2 is the price of the final good in country 2, p^m_1 is the price of the investment in knowledge from multinationals of country 1, and w_1 and w_2 are the wage rates in country 1 and 2, respectively. The capital investment for knowledge production in country 1 is denoted

[10] It is also used in knowledge production.

[11] Note that I am allowing for the relative price of knowledge with respect to the price of the intermediate good to differ from one.

[12] In Section 1.6, I allow for productivities to follow an exogenous stochastic process.

[13] In equilibrium, $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t},n_{1,t})}{U_c(c_{1,0},n_{1,0})} = \beta^t \frac{U_c(c_{2,t},n_{2,t})}{U_c(c_{2,0},n_{2,0})}$.

by x^m_{11}, whereas the capital investment for intermediate-good production in countries 1 and 2 are denoted by x^y_{11} and x^y_{12}, respectively. Remember that the capital investment good is nontradable and the corporation must invest in capital in country 2 in order to produce there. I also make a distinction between the amount of knowledge investment supplied by the firm, $p^m_1 g^{m,s}_1$, and the amount of knowledge investment demanded by the firm, $p^m_1 g^{m,d}_1$, so I am assuming there is a competitive market for it and its price will be determined accordingly. In equilibrium, the supply and demand of knowledge investment good will be the same and will cancel each other out.

The production structure and maximization problem for multinationals from country 2 are analogous.

Capital accumulation: Capital evolves according to:

$$k^y_{ij,t+1} = (1-\delta^k)k^y_{ij,t} + x^y_{ij,t}, \tag{1.11}$$

$$k^m_{ii,t+1} = (1-\delta^k)k^m_{ii,t} + x^m_{ii,t}, \tag{1.12}$$

for all $i, j = 1, 2$, $i \neq j$, and δ^k, $0 < \delta^k < 1$, is the depreciation rate of capital.

Households: Households from country i value consumption, $c_{i,t}$, and leisure, $1 - n_{i,t}$. The utility function of the household is given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}), \tag{1.13}$$

where $U(c,n) = \left(c^\mu (1-n)^{1-\gamma}\right)^\gamma / \gamma$, $0 < \mu < 1$, $\gamma < 1$.

The budget constraint of the household in country 1 is given by

$$c_{1,t} + \sum_{j=1,2} p^v_{j,t}(V^j_{1,t+1} - V^j_{1,t}) = w_{1,t}n_{1,t} + \sum_{j=1,2} V^j_{1,t}D_{j,t}, \tag{1.14}$$

where w_1 is the wage rate in country 1. Households trade stocks of intermediate-good producing firms (V^j_i denotes the shares of multinational corporations from country j owned by households in country i).[14] The price of the stocks of intermediate-goods producing firms from country j is denoted by p^v_j. The budget constraint of the household in country 2 is analogous. Households maximize their utility subject to the budget constraints, initial stock holdings, and usual non-negativity constraints.

[14] They also receive profits from the final-good producing firms. It is zero in equilibrium.

Market-clearing conditions: The market clearing conditions are:

$$\text{(final goods)} \qquad c_{i,t} + x^y_{ii,t} + x^y_{ji,t} + x^m_{ii,t} = G_i(a_{i,t}, b_{i,t}), \tag{1.15}$$

$$\text{(labor)} \qquad n_{i,t} = n^y_{ii,t} + n^y_{ji,t} + n^m_{ii,t}, \tag{1.16}$$

$$\text{(intermediate good a)} \qquad a_{1,t} + a_{2,t} = y_{11,t} + y_{12,t}, \tag{1.17}$$

$$\text{(intermediate good b)} \qquad b_{1,t} + b_{2,t} = y_{21,t} + y_{22,t}, \tag{1.18}$$

$$\text{(knowledge investment)} \qquad g^{m,s}_{i,t} = g^{m,d}_{i,t} = z_{i,t}F(k^m_{ii,t}, m_{i,t}, n^m_{ii,t}), \tag{1.19}$$

for all $i, j = 1, 2$, $i \neq j$.

Equilibrium: a competitive equilibrium is a sequence of prices ($p_{2,t}$, $p^m_{1,t}$, $p^m_{2,t}$, $p^v_{1,t}$, $p^v_{2,t}$, $q_{a,t}$, $q_{b,t}$, $w_{1,t}$, $w_{2,t}$), a sequence of labor allocations ($n_{1,t}$, $n_{2,t}$, $n^y_{11,t}$, $n^y_{12,t}$, $n^y_{12,t}$, $n^y_{22,t}$, $n^m_{11,t}$, $n^m_{22,t}$), a sequence of consumption ($c_{1,t}, c_{2,t}$), a sequence of pricing kernel ($\Lambda_{0,t}$), a sequence of intermediate-good allocations ($a_{1,t}$, $a_{2,t}$, $b_{1,t}$, $b_{2,t}$, $y_{11,t}$, $y_{12,t}$, $y_{21,t}$, $y_{22,t}$), a sequence of capital allocations ($k^y_{11,t}$, $k^y_{12,t}$, $k^y_{21,t}$, $k^y_{22,t}$, $k^m_{11,t}$, $k^m_{22,t}$), a sequence of capital investment allocations ($x^y_{11,t}$, $x^y_{12,t}$, $x^y_{21,t}$, $x^y_{22,t}$, $x^m_{11,t}$, $x^m_{22,t}$), a sequence of knowledge allocations ($m_{1,t}$, $m_{2,t}$), a sequence of knowledge investment allocations ($g^{m,d}_{1,t}$, $g^{m,s}_{1,t}$,$g^{m,d}_{2,t}$, $g^{m,s}_{2,t}$), a sequence of dividends ($D_{1,t}$,$D_{2,t}$), and a sequence of stock holdings ($V^1_{1,t}$,$V^2_{1,t}$,$V^1_{2,t}$,$V^2_{2,t}$) such that, given initial capital stocks, knowledge stocks, stock holdings, and given the sequence of prices: i) the allocations solve the households problem; ii) the allocations solve both final-good and intermediate-good firm's problems; iii) market-clearing conditions are satisfied; iv) pricing kernel satisfies $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t},n_{1,t})}{U_c(c_{1,0},n_{1,0})}$.

Optimality conditions: final-good producers face a sequence of static problems. The optimality conditions of their profit-maximization problem imply that the following relation must hold in equilibrium:

$$\frac{G_{1a}(a_{i,t}, b_{i,t})}{G_{ib}(a_{i,t}, b_{i,t})} = -\frac{q_{a,t}}{q_{b,t}}, \tag{1.20}$$

for $i = 1, 2$.

Regarding the maximization problem of the intermediate-good producers of country 1 (multinationals from country 1), the following conditions must hold in equilibrium:

$$w_{1,t} = q_{a,t} z_{1,t} F_n(k^y_{11,t}, m_{1,t}, n^y_{11,t}), \tag{1.21}$$

$$w_{2,t} = q_{a,t} z_{2,t} F_n(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}), \tag{1.22}$$

$$w_{1,t} = p^m_{1,t} z_{1,t} F_n(k^m_{11,t}, m_{1,t}, n^m_{11,t}), \tag{1.23}$$

$$1 = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[q_{a,t+1} z_{1,t+1} F_k(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) + (1-\delta^k) \right], \tag{1.24}$$

$$p_{2,t} = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[q_{a,t+1} z_{2,t+1} F_k(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) + p_{2,t+1}(1-\delta^k) \right], \tag{1.25}$$

$$1 = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \left[p^m_{1,t+1} z_{1,t+1} F_k(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + (1-\delta^k) \right], \tag{1.26}$$

$$p^m_{1,t} = \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} \times \left[\begin{array}{c} q_{a,t+1} z_{1,t+1} F_m(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) \\ + \, q_{a,t+1} \theta z_{2,t+1} F_m(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) \\ + \, p^m_{1,t+1} \left(z_{1,t+1} F_m(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + (1-\delta^m) \right) \end{array} \right], \tag{1.27}$$

together with the transversality conditions.[15] Conditions (1.21)–(1.23) and (1.24)–(1.26) are the standard optimality conditions for labor and capital investment decisions, respectively.

Equation (1.27), on the other hand, is not standard. It is the optimality condition with respect to investment in knowledge, and it shows how the price of the knowledge investment good is determined in equilibrium. In particular, the cost of acquiring a unit of knowledge investment good in period t, $p^m_{1,t}$, must be equal to the benefit of acquiring it. The benefit is given by the extra value of intermediate-good production in the following period (produced by subsidiaries in both countries), the extra value of knowledge production in the following period, and the value of the stock of knowledge after depreciation in the following period, all discounted by $\frac{\Lambda_{0,t+1}}{\Lambda_{0,t}}$ to reflect period t values. The optimality conditions for the multinationals from country 2 are analogous.

Finally, the following optimality conditions are derived from the household maximization problem:

$$\frac{w_{i,t}}{p_{i,t}} = -\frac{U_n(c_{i,t}, n_{i,t})}{U_c(c_{i,t}, n_{i,t})}, \tag{1.28}$$

$$1 = \beta \frac{U_c(c_{i,t+1}, n_{i,t+1})}{U_c(c_{i,t}, n_{i,t})} (D_{j,t+1} + p^v_{j,t+1}), \tag{1.29}$$

[15] The transversality conditions are: $\lim_{t\to\infty} \Lambda_{0,t+1} k^y_{11,t+1} = 0$, $\lim_{t\to\infty} \Lambda_{0,t+1} k^y_{12,t+1} = 0$, $\lim_{t\to\infty} \Lambda_{0,t+1} k^m_{11,t+1} = 0$, and $\lim_{t\to\infty} \Lambda_{0,t+1} p^m_{1,t+1} m_{1,t+1} = 0$.

for $j = 1, 2$, together with the transversality conditions.[16] Equation (1.28) is the standard intratemporal Euler equation, and (1.29) is the standard intertemporal Euler equation with respect to stock holdings of corporations from country j, $j = 1, 2$.

Therefore, the equilibrium is characterized by the optimality conditions (1.20)–(1.29), together with the budget constraints of the households (1.14), the constraints of the multinational corporations (1.4)–(1.8), the transversality conditions of both the household and multinational corporation problems, the capital accumulation equations (1.11)–(1.12), the market clearing conditions (1.15)–(1.19), and $\Lambda_{0,t} = \beta^t \frac{U_c(c_{1,t}, n_{1,t})}{U_c(c_{1,0}, n_{1,0})}$.

In the next section, I analyze the steady state properties of the model outlined above. In this environment, the competitive equilibrium is equivalent to a solution to the planner's problem that maximizes a weighted sum of households utility in country 1 and 2 subject to the resource constraints (equal to the market clearing conditions). In other words, the welfare theorems apply. I analyze the solution to the problem in which the planner gives equal weight to both countries.

1.4 Steady state, measurement, and knowledge production

In steady state, allocations and prices are constant. I compute the steady state equilibrium of the planner's problem in which the planner gives equal weight to households from both countries. The symmetric steady-state allocation is computed in Appendix A.1.

As discussed in the introduction, in this section I compute the parameter values for the share of knowledge in production, ϕ, and the depreciation rate of knowledge, δ^m, such that, in steady state, the model matches the factor shares observed in the data. But first, I make a distinction between the true measures in the model and their data-equivalent counterparts.

Let country 1 represent the United States, and country 2 the European Union. An important observation is that, in the model, I define a multinational corporation as a U.S. multinational corporation if it produces knowledge in the United States.[17] This definition has no direct relation to its ownership structure. On the other hand, BEA defines U.S. multinationals according to its ownership structure.[18] Therefore, when using the data on U.S. multinational corporations, I am making the assumption that both definitions coincide.[19]

[16] The transversality conditions are: $\lim_{t\to\infty} \Lambda_{0,t+1} p^v_{j,t+1} V^j_{1,t+1} = 0$, $j = 1, 2$.

[17] In the case of imperfect substitutability between intermediate goods produced by multinational corporations from different countries, the nationality of each multinational is also directly related to the good it produces.

[18] In particular, BEA uses the 10% ownership threshold to define foreign direct investment.

[19] This will be the case if knowledge is produced by the headquarters of the multinational corporation.

Another observation is that the measurement issues that I discuss below have no effect on the actual allocation of resources. However, I show that I can use the measures in the data to infer the parameters from the model.

Royalties and license fees: According to the model, the true value-added of U.S. multinational subsidiaries in the European Union is:

$$gdp_{12,t} = \underbrace{q_{a,t} y_{12,t}}_{\text{output approach}} = \underbrace{w_{2,t} n^y_{12,t} + r^k_{12,t} k^y_{12,t} + r^m_{12,t} m_{1,t}}_{\text{income approach}},$$

where $r^k_{12,t} = q_{a,t} z_{2,t} F_k(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t})$ is the return on capital invested in U.S. multinational subsidiaries in the European Union, and $r^m_{12,t} = q_{a,t} \theta z_{2,t} F_m(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t})$ is the return on knowledge investment from operations of U.S. multinational subsidiaries in the European Union. However, in the data, a fraction τ of the return on knowledge, $\tau r^m_{12,t} m_{1,t}$, is treated as exports of royalties and license fees—see Section 1.2.1. In this case, the measured value-added of U.S. subsidiaries in the European Union is:

$$\widetilde{gdp}_{12,t} = \underbrace{q_{a,t} y_{12,t} + \tau_1 r^m_{12,t} m_{1,t}}_{\text{output approach}} = \underbrace{w_{2,t} n^y_{12,t} + r^k_{12,t} k^y_{12,t} + (1-\tau_1) r^m_{12,t} m_{1,t}}_{\text{income approach}}.$$

Therefore, the first adjustment that I make is to add the value of net exports of royalties and license fees from U.S. multinationals to their foreign affiliates in the European Union to the value-added of these subsidiaries, and to subtract its value from the value-added of U.S. multinational operations in the United States.[20]

Knowledge investment: Investment in knowledge by private corporations is either unobserved or expensed in corporate accounts. Regarding the data on U.S. multinational operations for the period 1995–2007, value-added (GDP) does not include expenditures for R&D. Data on U.S. corporate sector GDP, on the other hand, includes investment in R&D.[21] However, I am assuming that the investment in R&D that is observed in the data on U.S. corporate sector GDP does not necessarily account for all the investment in knowledge that takes place within U.S. corporations. In order to make the data on U.S. corporate sector GDP comparable to the data on value-added (GDP) of U.S. multinationals, I subtract the investment in intellectual property rights by private businesses from corporate sector

[20] One can also think of $\widetilde{p} = \tau_1 r^m_{12,t}$ as the transfer price of royalties and license fees.

[21] After BEA's 2013 comprehensive revision, expenditures for R&D started to be treated as investment in intellectual property rights and to be included in private fixed investment. See McCulla, Holdren, and Smith (2013) for a description of the revision.

GDP. Therefore, the true and the measured value-added of operations of U.S. multinational corporations in the United States are (I use tilde for measured values):[22,23]

$$
\begin{aligned}
gdp_{11,t} &= q_{a,t} y_{11,t} + p^m_{1,t} g^m_{1,t}, \\
\widetilde{gdp}_{11,t} &= q_{a,t} y_{11,t},
\end{aligned}
$$

where GDP is computed according to the output approach. So measured GDP does not include investment in knowledge, $p^m_{1,t} g^m_{1,t}$, whereas true GDP does.

Labor share: The fact that measured GDP is different from its true counterpart has implications for the observed labor shares, which is defined as the ratio of compensation of employees to GDP. The payments to employees are observable, so true and measured compensation of employees are the same. The fact that measured GDP does not include investment in knowledge implies that measured GDP is higher than its true counterpart if investment in knowledge is positive, so that the measured labor share is higher than the true labor share.

According to the model presented above, the true labor share of domestic operations, LS_{11}, and foreign operations, LS_{12}, of U.S. multinationals are both equal to $(1-\alpha)(1-\phi)$. This comes from the fact that I am assuming a Cobb-Douglas production function, $F(k,m,n) = (k^\alpha n^{1-\alpha})^{1-\phi} m^\phi$, in which $\alpha(1-\phi)$, $(1-\alpha)(1-\phi)$, and ϕ are the actual factor shares of capital, labor, and knowledge, respectively. So the true labor shares are the same in both countries. The assumption that production functions are the same in both countries is supported by the fact that measured factor shares in the United States and European Union corporate sectors are similar to each other. Their data-equivalent counterparts are:

$$
\widetilde{LS}_{11,t} = \frac{w_{1,t}(n^y_{11,t} + n^m_{11,t})}{q_{a,t} y_{11,t}} = (1-\alpha)(1-\phi)\left(1 + \frac{p^m_{1,t} g^m_{1,t}}{q_{a,t} y_{11,t}}\right), \tag{1.30}
$$

$$
\widetilde{LS}_{12,t} = \frac{w_{2,t} n^y_{12,t}}{q_{a,t} y_{12,t}} = (1-\alpha)(1-\phi). \tag{1.31}
$$

Therefore, the measured labor share of U.S. multinationals in the European Union is not distorted ($\widetilde{LS}_{12,t} = LS_{12,t}$), only their measured labor share in the United States is. The reason why the measured labor share is only distorted in the United States is because

[22] I am assuming that measured values already include the adjustment for exports of royalties and license fees discussed above.

[23] Note that I am assuming that U.S. corporate sector GDP is divided into two groups: GDP of foreign subsidiaries of EU multinationals, and GDP of U..S. corporations. So I am treating all U.S. corporations that are not EU multinational subsidiaries as U.S. multinationals.

I am assuming that investment in knowledge only takes place in the headquarters country, the United States. Since investment in knowledge is positive for $\phi > 0$, it implies that the measured labor share in the U.S. will be higher than in the European Union. Therefore, the model with unobserved knowledge investment replicates the pattern observed in the data, and I will use this information to compute both the share of knowledge in the production function, ϕ, and the depreciation rate of knowledge, δ^m.

1.4.1 Steady-state and knowledge production

I use the steady-state equilibrium of the model to pin-down the parameter values related to knowledge production. In particular, I use the following relations:[24]

$$\widetilde{LS}_{11} = (1-\alpha)(1-\phi)\left(1+\frac{\delta^m\phi\beta(1+\theta)}{1-\beta+\beta\delta^m(1-\phi)}\right), \tag{1.32}$$

$$LS_{12} = (1-\alpha)(1-\phi), \tag{1.33}$$

$$\frac{CAPEX_{11}}{CE_{11}} = \frac{\delta^k\beta}{1-\beta(1-\delta^k)}\frac{\alpha}{1-\alpha}, \tag{1.34}$$

$$\frac{CE_{12}}{CE_{11}} = \theta\frac{1-\beta(1-\delta^m)-\delta^m\phi\beta}{1-\beta(1-\delta^m)+\delta^m\phi\beta\theta}, \tag{1.35}$$

where $CAPEX_{11} = x^y_{11} + x^m_{11}$ denotes capital expenditures of U.S. multinationals in the United States, and $CE_{11} = w_1(n^y_{11} + n^m_{11})$ and $CE_{12} = w_2 n^y_{12}$ denote compensation of employees of U.S. multinational operations in the United States and in the European Union, respectively. Note that I have six parameters, $(\alpha, \phi, \delta^k, \delta^m, \beta, \theta)$, and I selected four moments. Parameters β and δ^k are selected according to observed real interest rates and the actual estimates of capital depreciation from BEA. One important thing to note is that the six parameters, $(\alpha, \phi, \delta^k, \delta^m, \beta, \theta)$, will not depend on the remaining parameters, $(\mu, \sigma, \gamma, \omega)$. So I do not need to specify their values in order to compute the share of knowledge, ϕ, and its depreciation rate, δ^m.

Table III shows the results. The estimated share of knowledge, $\phi = 0.29$, is much larger than previous studies suggest. For example, McGrattan and Prescott (2010) use 7.0% for the share of technology capital. On the other hand, its depreciation rate, $\delta^m = 0.17$, is close to the value used by the BEA, 15%.

Investment in knowledge: The estimates presented above imply that the investment in knowledge is actually 1.4 times the investment in capital, whereas the BEA reports that

[24] See Appendix A.1 for derivation.

this ratio is only 0.4.[25] Therefore, the data accounts for less than 30% of the total investment in knowledge by U.S. corporations.

Table III: Matching Moments

Matched moments		Values
Labor share in the U.S. corporate sector		0.58
Labor share of U.S. multinationals in the European Union		0.45
Ratio of capital expenditures to compensation of employees		0.32
Ratio of compensation of employees in subsidiaries over headquarters		0.045
Parameters calibrated to match the moments above		
Share of knowledge	ϕ	**0.30**
Capital/Labor share	α	0.36
Depreciation rate of knowledge	δ^m	**0.11**
Foreign direct investment cost	θ	0.06
Other parameters		
Annual depreciation rate of (tangible) capital	δ^k	0.06
Discount factor	β	0.96

International flow of knowledge: Let $r^m_{12,t}m_{1,t}$ denote the flow of knowledge from the United States to the European Union that takes place within U.S. multinationals. The estimates imply that the observed intrafirm net exports of royalties and license fees of U.S. multinationals from the United States to the European Union represent only 13% of the total flow of knowledge. That is, $\tau = 0.13$.

Dark matter (U.S. FDI position): BEA only reports the FDI flows of tangible capital. The value of the stock of knowledge used by U.S. subsidiaries in the European Union can be computed as the present value of the flow of returns on knowledge, $r^m_{12,t}m_{1,t}$, taking into account its depreciation rate. Its values is on average 6.1% of U.S. GDP during the period 1995-2007, which implies that the statistics on U.S. FDI stock are biased downwards by that amount.

[25] This is the ratio of private domestic investment in intellectual property rights to private domestic nonresidential investment excluding intellectual property rights.

Other implications The high estimates for the share of knowledge have also other important implications. For example, it has implications for international business cycle fluctuations. In Section 1.6, I simulate the stochastic version of the model outlined above and show that it greatly improves the capacity of the standard international real business cycle model to generate GDP correlations closer to the ones observed in the data. Another example is the welfare implications of the Base Erosion and Profit Shifting (BEPS) action plan by the Organisation for Economic Co-operation and Development (OECD), which can (potentially) have a significant impact on the effective tax rate faced by multinational corporations, therefore affecting the international flow of knowledge. This is part of future work.

1.5 GDP fluctuations in corporate sectors and U.S. multinationals

Before presenting the business cycle model with international knowledge flows, I present empirical evidence that suggests a central role to U.S. multinationals in accounting for the corporate sector GDP correlation between the United States and European Union. In this section, I document the following facts: i) the correlation in GDP fluctuations between the United States and European Union in 1995–2007 is explained by the correlation between their corporate sectors; ii) fluctuations in the operations of U.S. multinationals in the United States and European Union are highly correlated in 1995–2007; iii) the corporate sector correlation with the United States increases as a function of the share of U.S. multinationals in corporate sector GDP.

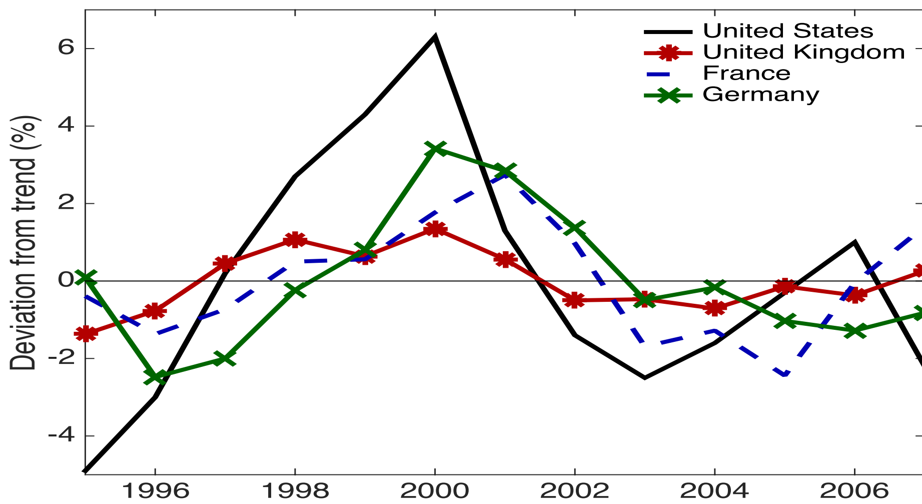
1.5.1 Cross-country correlation in corporate GDP fluctuations

In this section, I document the positive correlation between corporate sector real GDP fluctuations in the United States and major EU countries during the period 1995-2007. Data are from the Organisation for Economic Co-operation and Development (OECD) for EU countries and from the Bureau of Economic Analysis (BEA) for the United States. The corporate sector comprises both financial and nonfinancial corporations.[26] Data for the noncorporate sector are computed by subtracting the corporate sector from the whole economy, and real values are computed using the respective country's GDP deflator. Table V reports the cross-country correlations of GDP fluctuations between the United States and major EU countries, together with the share of the corporate sector in each economy. Fluctuations are computed as the difference between log values and a log-linear trend.

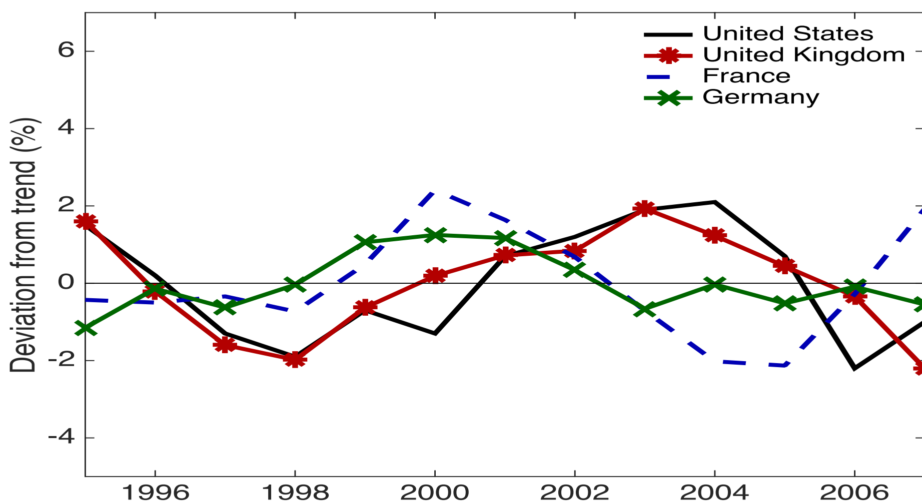
[26] The noncorporate sector comprises general government, households, and nonprofit institutions serving households. I also separated financial and nonfinancial corporations, and the results did not change.

Figure 1.3: Corporate and Noncorporate GDP Fluctuations

(a) Corporate Sector GDP



(b) Noncorporate Sector GDP



Sources: OECD and BEA.

Note: The figure shows the deviation of the log values of real GDP from their log-linear trends. GDP is divided between (a) corporate sectors and (b) noncorporate sectors, and I compute the log-linear trend for each series separately.

The results show a high correlation between corporate sectors' GDP. For example, the corporate sector GDP correlation between Germany and the United States in 1995–2007 is 0.48, whereas the correlation in the noncorporate sector is -0.36. Table V also reports the

GDP-weighted average of each statistic for a group of 18 EU countries.[27] The average GDP correlation in the corporate and noncorporate sectors are 0.55 and -0.06, respectively.

Although the average share of the corporate sector in the economy is close to one-half (56%), the difference in correlations is striking. While the corporate sectors show a high positive correlation, the noncorporate sectors show a weak negative correlation. However, the negative correlation in the noncorporate sector is not robust to small changes in the sample period. In the case of Germany, for example, the correlation increases from -0.36 to -0.09 once the sample period is expanded to 1991–2007 using data from the German Federal Statistical Office (Destatis).

The high correlation in the corporate sector is also surprising because this period (1995–2007) is part of the period known as the Great Moderation (1985–2007), which is characterized by the low volatility of main macroeconomic variables. Table IV shows that the standard deviation of the annual U.S. corporate GDP growth rate in that period was 2.2%, which is 35% lower than the standard deviation during the whole sample (1949–2015).[28] These results suggest that the explanation for the comovement based on large correlated shocks is less plausible, since there were no evident large shocks in 1995–2007 compared to the oil price variations in the 1970s or the financial crisis in 2008–2009.

Figure 1.3 shows the series of fluctuations for the United States, United Kingdom, France, and Germany for both corporate and noncorporate sectors. Figure 1.3a shows that there was a period of fast growth in corporate GDP between 1995 and 2000 for all countries, followed by a sharp decline in growth afterward. This interpretation is based on the fact that deviations move from negative to positive values between 1995 and 2000, and back to negative values in 2005. On the other hand, in the noncorporate sector, Figure 1.3b, there is less variation in deviations and no clear pattern such as the one observed in the corporate sector.

[27] Weights are based on the nominal GDP of each country in 2001 in U.S. dollars. I only kept those countries with data for all years in 1995–2007. For example, Spain and Ireland were excluded because their series start in 1999. The countries are Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Netherlands, Portugal, Slovak Republic, Slovenia, Sweden, and United Kingdom.

[28] Refer to Stock and Watson (2002) for an extensive description of that period.

Table IV: Standard deviation of real GDP growth rates by period (%)

	1949–2015	1962–1984	1985–2007
United States			
Total economy	2.4	2.5	1.2
Corporate sector	3.4	3.1	**2.2**
Noncorporate sector	2.1	2.2	1.6
France			
Total economy	2.2	2.1	1.2
Corporate sector	2.9	2.8	1.8
Noncorporate sector	2.0	2.1	0.9

Source: BEA for the United States, Insee for France.

Therefore, I interpret these results as indicating a close link between the corporate sectors in the United States and in the European Union, a link that I try to account for in this paper by incorporating knowledge flows through multinational corporations. In the next section, I show that the operations of U.S. multinationals in the United States and of their subsidiaries in the European Union are even more correlated, and that the correlation of corporate GDP fluctuations increases as a function of the share of U.S. multinationals in corporate GDP.

1.5.2 Correlation in GDP fluctuations between U.S. multinational operations in the United States and European Union

Table VI reports the correlation between fluctuations in economic activity of U.S. multinational operations in the United States and the operations of their subsidiaries in the European Union. Data are from the BEA and cover the period 1983–2013 (except for the GDP series, which starts in 1994). The BEA uses the threshold of 10% of ownership to define foreign direct investment; that is, a foreign enterprise is classified as a subsidiary of a U.S. corporation if more than 10% of its business is owned by a U.S. corporation. However, the BEA reports separate statistics for those subsidiaries that are majority owned (ownership share above 50%), which include a richer set of data. Unless otherwise stated, the statistics in this paper refer to the majority-owned group.[29]

[29] See Mataloni (1995) for a complete description of the data. Note that I am not following the terminology used by the BEA. In particular, foreign direct investment is associated with entities that are not necessarily corporations, although corporations make up the bulk of it.

Table V: Cross-country correlations in real GDP fluctuations

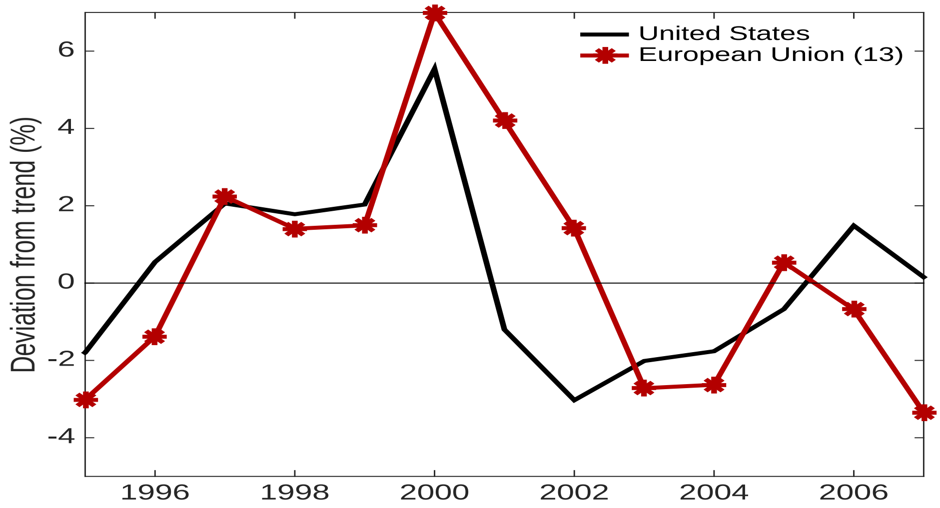
Correlation of GDP fluctuations with the United States	Whole economy	Corporate sector	Noncorporate sector	Share of corporate sector in GDP (%)
Period 1995–2007				
GDP-weighted average of 18 EU countries	0.39	**0.55**	-0.06	56
Germany	0.38	0.48	-0.36	59
United Kingdom	0.15	0.88	0.83	58
France	0.76	0.54	-0.21	54
Italy	0.16	0.24	-0.28	50
Netherlands	0.77	0.76	-0.35	63
Sweden	0.15	0.73	0.39	61
Belgium	0.70	0.50	-0.64	59
Austria	0.67	0.65	-0.33	57
Other periods				
Germany (1991–2007)	0.11	0.30	-0.09	64.7
France (1985–2007)	0.48	0.35	-0.04	50.4
France (1949–2007)	0.24	0.22	0.13	54.4

Source: OECD for European Union countries 1995–2007, BEA for the United States 1950–2007. Insee for France 1950–2007, Destatis for Germany 1992–2007.

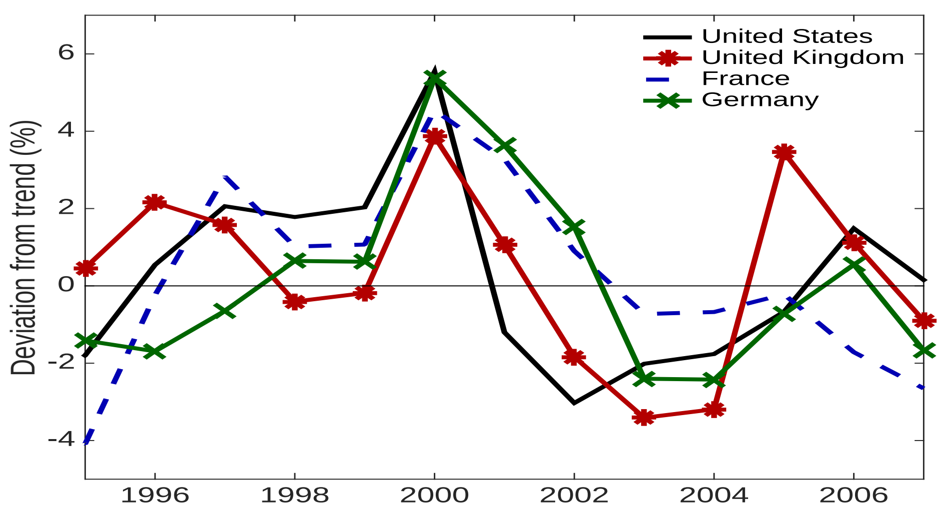
Note: Data frequency is annual. GDP values are in logarithms and the respective GDP deflator is used to compute real values. I assume a log-linear trend, except for Germany (1991–2007) and France (1985–2007 and 1950–2007), where data are HP-filtered with parameter 6.25. The corporate sector comprises both financial and non-inancial corporations (except for Insee data, which contain only nonfinancial corporations). The share of the corporate sector is the average share in the period. For the United States, the average shares are 57.3%, 58.5%, and 58.5% for 1949–2007, 1985–2007, and 1995–2007, respectively. I only included countries with data for all the years in 1995–2007. The 18 EU countries are Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Latvia, Netherlands, Portugal, Slovak Republic, Slovenia, Sweden, and United Kingdom. Weights based on U.S. dollar GDP in 2002 from World Bank Development Indicators.

Figure 1.4: GDP fluctuations of U.S. multinationals in the United States and European Union

(a)



(b)



Sources: BEA.

Note: The figure shows the deviation of the log values of real GDP from their log-linear trends. All series are based on the operations of U.S. multinationals. For example, the series labeled "United States" shows the fluctuations in GDP of their operations in the United States, whereas the series labeled "United Kingdom" shows the fluctuations in GDP of their subsidiaries in the United Kingdom. The European Union (13) includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom.

Table VI shows that the activities of U.S. multinationals in the United States and their subsidiaries in the European Union are highly correlated. For example, the correlation in GDP fluctuations between operations of U.S. multinationals in the United States and their subsidiaries in the United Kingdom in 1995–2007 is 0.67. For the aggregate of 13 EU countries, the correlation is 0.66.[30] Table VI also reports the correlation for other variables related to the economic activity of these multinationals, such as compensation of employees and employment. They follow the same pattern as GDP, that is, the correlation is positive and high.

Table VI: Cross-country correlations of U.S. multinational operations

	Gross domestic product	Compensation of employees	Employment
Period 1995–2007			
Aggregate of 13 EU countries	**0.66**	0.83	0.96
Germany	0.53	0.80	0.50
United Kingdom	0.67	0.76	0.82
France	0.58	0.83	0.87
Period 1983–2007			
Aggregate of 13 EU countries	-	0.59	0.63
Germany	-	0.53	0.22
United Kingdom	-	0.59	0.62
France	-	0.50	0.61

Source: BEA.

Note: Data frequency is annual. The GDP series for multinationals start in 1994. BEA data are in U.S. dollars, and I convert the values to their respective original currency using average market exchange rates. Variables are in logarithms, and the respective GDP deflator is used to compute real values (except for employment). I used the GDP deflator in France in the case of EU-13. I assume a log-linear trend. The 13 EU countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom. The years 1999 and 2004 correspond to benchmark revision years in the BEA data, and we can observe large variations in the number of multinationals observed. I opted for using a linear interpolation for these years, where I assumed the average growth rate in the period without including 1994 and 1999. Otherwise, the correlations would be even larger, reflecting large variations in the extensive margin.

Figure 1.4 shows the series of GDP fluctuations of U.S. multinationals broken down by location (United States, EU-13, United Kingdom, France, and Germany). The series follow a pattern similar to the corporate GDP series; that is, there is a period of fast growth

[30] This group includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal, Sweden, and United Kingdom.

between 1995 and 2000, and a sharp slowdown afterward.

These results are in line with the findings in Cravino and Levchenko (2014), who use the ORBIS database to compute the correlation in sales growth rates between headquarters and subsidiaries of multinational corporations worldwide.[31] They show that sales growth rates of multinational headquarters (operations in the country where the headquarters is located) and their subsidiaries (operations in a foreign country) are positively correlated. However, U.S. corporations do not report unconsolidated financial statements of their operations; that is, their financial statements do not separate U.S. operations from foreign operations. This means that one cannot use the ORBIS database to compare operations of U.S. multinationals in the United States and European Union, so my results complement their analysis.

1.5.3 Corporate sector GDP correlation and share of U.S. multinationals in corporate GDP

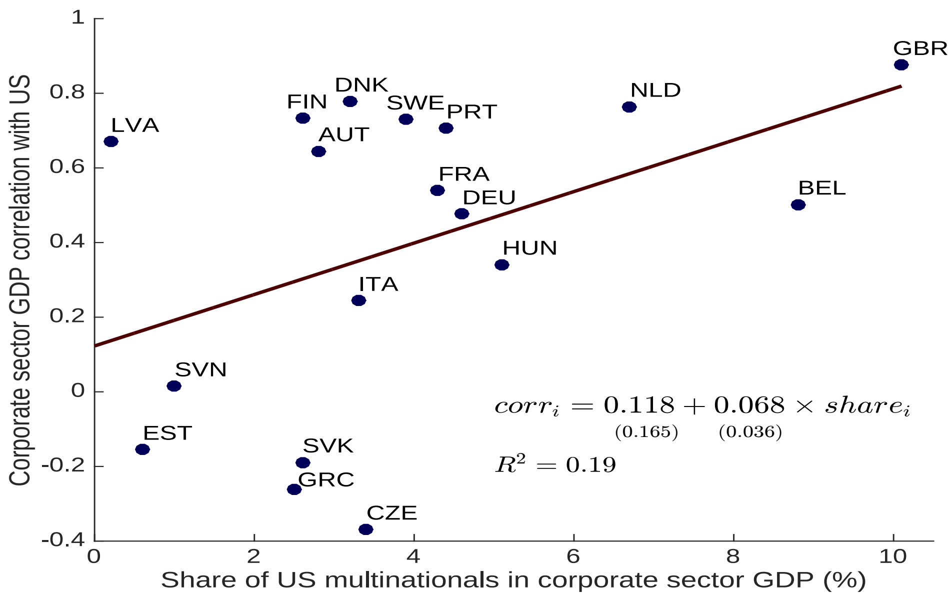
In this section, I document that the corporate sector GDP correlations with the United States described in Section 1.5.1 are closely related to the share of U.S. multinationals in the corporate sector of the respective country. Figure 1.5 shows a scatter plot of the bilateral correlations with the United States and the share of U.S. multinationals in each country's corporate sector GDP. They are positively correlated—correlation equal to 0.43.

In the United Kingdom (GBR), for example, the share of U.S. multinationals in corporate sector GDP is 10.1%, the largest share among the countries in my sample. At the same time, its corporate GDP correlation with the United States is 0.88, also the largest among the countries in the sample. Slovenia (SVN) and Estonia (EST), on the other hand, have a low average share of U.S. multinationals in their GDP (less than 1%), and at the same time, the correlation with the United States is also very low (around zero for Slovenia and negative for Estonia).

Latvia (LVA) seems to be an outlier. It has the lowest share of U.S. multinationals in corporate sector GDP, yet its correlation with the United States is relatively high, at 0.67. The results are robust to the inclusion of either the United Kingdom or Latvia. The correlations after excluding either the United Kingdom or Latvia or both are 0.33, 0.53, and 0.46, respectively.

[31] They also analyze value added and employment growth. See Cravino and Levchenko (2016) for a complete description of the ORBIS database.

Figure 1.5: Correlation with United States versus share of U.S. multinationals in corporate sector GDP



Sources: BEA and OECD.

Note: *corr* = correlation between corporate sector GDP fluctuations with United States in 1995–2007; *share* = average share of U.S. multinationals in the corporate sector in the period 1995–2007. The line corresponds to the fitted values of the ordinary least squares regression $corr_i = \alpha + \beta share_i + e_i$. The countries are: Austria (AUT), Belgium (BEL), Czech Republic (CZE), Denmark (DNK), Estonia (EST), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), Hungary (HUN), Italy (ITA), Latvia (LVA), Netherlands (NLD), Portugal (PRT), Slovak Republic (SVK), Slovenia (SVN), Sweden (SWE), and United Kingdom (GBR).

Again, the empirical findings indicate a close connection between the corporate sectors in these countries and the United States, and that the strength of this connection is associated with the degree of U.S. multinational activities in these countries. Therefore, any theory that attempts to explain the cross-country correlation must be able to account for these patterns, including the activities of multinational corporations.

1.6 Knowledge Flows and International Real Business Cycles

In this section, I assess the business cycle properties of the model outlined in Section 1.3 by allowing the productivity parameters to follow a stochastic process. In particular, I am interested in how much GDP correlation the model can generate when compared to the standard international real business cycle (IRBC) model by Backus, Kehoe, and Kydland (1994). As mentioned before, the model of knowledge flows that I present in this paper nests the standard IRBC model. More specifically, the model in this paper is equivalent to the standard IRBC model when I set the knowledge share parameter, ϕ, and the FDI cost, θ, to be equal to zero. Productivity in each country evolves according to:

$$\ln z_{1,t} = \rho \ln z_{1,t-1} + \epsilon_{1,t}, \tag{1.36}$$

$$\ln z_{2,t} = \rho \ln z_{2,t-1} + \epsilon_{2,t}, \tag{1.37}$$

where $\epsilon_{1,t}$ and $\epsilon_{1,t}$ follow a multivariate normal distribution, with mean equal to zero and standard deviations such that the standard deviation of GDP in the model matches the standard deviation of U.S. corporate sector GDP fluctuations in the data. The persistence parameter, ρ, is set to be equal to 0.90.

I assume that markets are complete and solve the planner's problem with equal weights. Next, I log-linearize the solution around the steady-state and use the method of undetermined coefficients to compute the dynamics of the model.[32]

Calibration: First, I use the parameter values listed in Table III. Next, I still have to set values for the remaining parameters that were not used in Section 1.4. They are: μ, γ, σ, and ω. Table VII shows their values. The value of μ, $\mu = 0.34$, is chosen such that households allocate 33% of their endowment of time to market activities (work) in steady state. For the home bias parameter, ω, I compute its value in order to match the ratio of the total value of U.S. exports to the European Union net of intrafirm exports plus the GDP of subsidiaries of U.S. multinationals in the European Union to the EU corporate sector GDP. Finally, I set $\gamma = -1$ and $\sigma = 1.5$, equal to the values used by Backus, Kehoe and Kydland (1994).

Results: Table VIII shows the GDP correlation implied by the model, together with the cross-country correlation of other variables such as consumption, labor, and investment. I compare the model with knowledge flows to the model without it (standard IRBC model).

[32] See Uhlig (2002).

Table VII: Calibration

Parameters		
Consumption share	μ	0.34
Home bias	ω	0.78
Share of knowledge	ϕ	**0.30**
Labor/capital share	α	0.35
Depreciation rate of knowledge	δ^m	**0.11**
Foreign direct investment cost	θ	0.06
Discount factor	β	0.96
Annual depreciation rate of (tangible) capital	δ^k	0.06
Elasticity of substitution between intermediate goods	σ	1.50
Risk aversion parameter	γ	-1.00
Autocorrelation of exogenous productivity	ρ	0.90
Standard deviation of productivity shocks	σ^ϵ	0.0082
Exogenous correlation of productivity shocks	$corr_\epsilon$	0.30

Table VIII shows that the model with knowledge greatly improves the performance of the model regarding the correlation of GDP fluctuations. As Table VIII shows, the standard IRBC model fails to generate GDP correlations close to the ones observed in the data, even when I allow for positive correlation between the exogenous shocks. On the other hand, the model with knowledge reduces its discrepancy with the data by 50%, i.e., it explains 50% of the distance between the correlation implied by the standard IRBC model and the data.

1.7 Conclusion

In this paper, I quantified the flow of knowledge within U.S. multinational corporations in the United States and European Union. I built on the general equilibrium model of knowledge flows through multinationals by McGrattan and Prescott (2010) and computed both the share of knowledge in the production function and its depreciation rate such that in steady state the model matches the observed factor share differentials between the operations of U.S. multinationals in the United States and European Union. The main assumptions are: i) U.S. multinationals produce knowledge in the United States; ii) this knowledge is used by its subsidiaries in the European Union; and iii) investment in knowledge is expensed in corporate accounts. Under these assumptions, the model predicts that the observed labor share of U.S. multinational operations in the United States must be lower than the labor share of their subsidiaries in the European Union, whereas the ratio of capital expenditures to compensation of employees must be the same, both patterns found in the data. The estimated share of knowledge is 30%, and its annual depreciation rate is 11%.

Table VIII: Results: cross-country correlations

	Data	with knowledge $\phi = 0.3$	without knowledge $\phi = 0.0$ (BKK-94)
Cross-country correlations			
GDP	0.55	0.13	-0.30
Consumption		0.69	0.86
Labor		-0.25	-0.61
Investment		-0.03	-0.37
Std. relative to output			
Net exports	0.18	0.19	1.15
Investment	1.83	2.64	8.84

The high estimates for the share of knowledge have important implications. For example, I show that the model calibrated with these parameter values has quantitative implications for international real business cycles. I provide empirical evidence that connects the operations of U.S. multinationals in the European Union to the correlation between corporate sector GDP fluctuations in the United State and European Union in 1995–2007. Accounting for the corporate sector GDP correlation, the model with knowledge flows reduces the distance between the standard international real business cycle model and data by 48%.

The results also suggest large welfare implications for the Base Erosion and Profit Shifting (BEPS) action plan by the Organisation for Economic Co-operation and Development (OECD) which is likely to have a big impact on the effective tax rate faced by multinational corporations, therefore affecting the international flow of knowledge.

Chapter 2

Real Exchange Rates and Primary Commodity Prices

2.1 Introduction

This chapter shows that shocks that generate fluctuations in a small number of primary commodity prices can explain a substantial fraction of the movements in real exchange rates (RER) among industrialized countries. Specifically, we study the behavior of the RER of Germany, Japan, and the United Kingdom against the United States for the 1960-2014 period. A rough summary of the results is that with just 3 or 4 primary commodity prices, we can account for between 34% and 91% of the volatility of the RER between the US and those three countries, depending on the country and the period considered.

We find these results remarkable, given the so called *exchange rate disconnect puzzle*: The fact that real exchange rates across developed economies are very volatile, very persistent, and very hard to relate to fundamentals, as documented, for example, in Meese and Rogoff (1983), Engel (1999), Obtsfeld and Rogoff (2001), and Betts and Kehoe (2004). This difficulty opened the door for theoretical explorations of models with nominal rigidities as the source of RER movements, like for example Chari, Kehoe and McGrattan (2002). We will ignore nominal rigidities in our analysis and explore how far one can go with shocks that affect relative prices of the main primary commodities.

The disconnect puzzle is not present in small open economies where exports of one or two primary commodities are a sizeable share of their exports, as shown in Chen and Rogoff (2003) and more recently by Hevia and Nicolini (2013). For countries like Australia, Chile, or Norway, changes in the international prices of the commodities each country exports are highly correlated to changes in their real exchange rate, as we document below. As we show, that same idea can go a long way in explaining movements in RER among developed economies.[1]

The idea that we exploit in the paper is very simple: Fluctuations in the prices of commodities affect manufacturing costs, and therefore manufacturing prices, which in turn induce changes in final good costs. These cost fluctuations translate into price fluctuations at the country level. If changes in commodity prices have differential effects on the domestic cost of any two countries, primary commodity price changes will affect the real exchange rate between those two countries.

Relating primary commodity price changes to real exchange rate changes is a promising avenue to explore for several reasons. First, primary commodity prices are very volatile (much more than real exchange rates as we show in Tables I.a and I.b) and very persistent,

[1] In the small open economy literature it is common to assume that commodity prices are exogenous to the country. This is not the case for the countries we will consider, so the endogeneity of those prices must be taken into account. We will discuss this in the theoretical section.

a feature that, as we mentioned, real exchange rates also exhibit. Second, and in spite of the fact that trade in primary commodities has typically been ignored in two-country models of international trade, the numbers on primary commodity world trade are far from trivial: Total trade in a few commodities (10) accounts for 12% to 18% of total world trade in goods, depending on the year chosen.[2] This number clearly underestimates the true share of commodities, since trade data are not value added measures. Thus, when steel is exported, it is fully counted as a manufactured good, while an important component of its cost depends on iron. The same happens when a car is exported. Third, primary commodities are at the bottom of the production chain, so they directly affect final good prices.[3] In addition, they may directly affect the prices of other domestic inputs – like some types of labor and services in general – that are used jointly with primary commodity in the production of intermediate goods, and thus, they may indirectly affect the costs of final goods. Because of the high share on trade of just a few commodities, we only need to focus on just a handful of prices. Therefore, a link can be established between those few prices and final good price indexes, as we show in the theoretical section. Finally, the law of one price on those primary commodities is well known to hold, even at quarterly frequencies, so no ambiguity with respect to the tradeability of them exists. As we will show, it is the assumption of the law of one price in the primary commodity markets that is key in deriving the theoretical relationship between them and the RER.

The exchange rate disconnect puzzle has been widely studied in the literature. Two recent attempts at quantitatively explaining several facts related to the puzzle are Itskhoki and Mukhim (2017), and Eaton, Kortum and Neiman (2015). They provide very good descriptions of the state of the literature. To the best of our knowledge, the connection between RER and primary commodity prices has largely been ignored for the countries we focus on. Our evidence suggest that theoretical models of RER among developed economies that ignore primary commodity markets may fall short of providing a comprehensive explanation of RER movements.

To guide the empirical analysis, in Section 2 we partially spell out the production side of a totally standard model of an open economy that makes explicit the production of commodities and the use of commodities in the production of manufactured goods.[4] We derive an equilibrium condition relating the bilateral real exchange rate between two countries to

[2] It is close to 12% in 1990 and 18% in 2012. The main difference is that the first is a year of low primary commodity prices, while the second is not. The difference is mostly explained by the primary commodity relative prices.

[3] This direct effect is substantial enough for monetary authorities – even in developed countries – to focus attention on measures of "core" inflation, that abstract from the "volatile" effect of primary commodity prices (food and energy).

[4] We generalize the model used in Hevia and Nicolini (2013) to large countries that may have market power on international markets.

primary commodity prices that, in the case of Cobb-Douglas production functions, is linear in the logarithms of the variables. This log-linear relationship is used to perform the empirical analysis, which is the core of the paper, presented in Section 3. A discussion of the implications of the results is presented in a final concluding Section.

2.2 The Model

In this section, we exploit the familiar notion that final good prices can be expressed, in equilibrium, as a combination of factor prices. We get into details, since we want to consider an economy with an input-output matrix slightly more complicated than the ones typically used in macro-trade models, so as to explicitly discuss the role of prices of primary commodities, like oil and wheat, on final goods price indexes.

The discussion is made in the context of a simple Ricardian model, where trade is the result of differences in endowment and productivities. We will not characterize all the equilibrium conditions; rather, we emphasize how final good prices (and therefore real exchange rates) are related to prices of these primary commodities in a competitive equilibrium. Since we want to allow for heterogeneity in labor types and in differentiated intermediate goods, the notation is rather heavy, but the ideas are very simple and well known. The purpose of this section is to obtain an explicit equation that must hold in an equilibrium, that we can take directly to the data. The analysis also highlights under which conditions one should expect the primary commodity prices to be related with the real exchange rate. We would like to emphasize at the outset that we will derive a relationship between prices, all of them endogenous variables. We will explain in detail below how we take this into account in taking the equation to the data.

Specifically, we consider a world with a finite number of countries, each one inhabited by a representative consumer with standard preferences over a final consumption good C_t. This final consumption good should be seen as an aggregate of a very large number of different varieties but, to save in notation, we maintain the assumption of a single final good. We assume the final good to be non-traded to make the model consistent with the overwhelming evidence of lack of the law-of-one price in final goods (Engel, 1999). In this sense, the model below adopts the view of Burstein et. al (2001), who argue that an important share of final good prices have a non-traded component. To motive nominal magnitudes in each country, we assume that a cash-in-advance constraint of the form $P_tC_t \leq M_t$ is imposed on the representative consumer, where P_t is the price of the final good and M_t is the quantity of money.

We will not characterize equilibrium conditions for the household, since all we will exploit is the production structure of the economy. The preferences and the cash-in-advance constraint should be kept on the background, for completeness in terms of thinking how quantities and nominal prices are determined in an equilibrium.

In each country, there are different varieties of labor, intermediate goods, and commodities. In particular, we assume that there are

$$\begin{aligned} j &= 1, 2, ..., J \quad \text{types of labor} \\ i &= 1, 2, ..., N \quad \text{types of intermediate goods} \\ h &= 1, 2, ..., H \quad \text{types of commodities.} \end{aligned}$$

We consider, for simplicity, the case in which there is at least the same number of labor varieties as intermediate goods, $J \geq N$. If this is the case, all varieties will be produced in each country.[5] If not, some varieties may not be produced in some countries and the discussion below should take that possibility into account, making the notation (yet) more cumbersome. This assumption does not affect the result, as it will become apparent. We also assume that for each commodity, there is a non-tradable fixed factor $E_t(h)$, for all h, that is used in the production of the primary commodities.[6] We imagine that the number of labor varieties and intermediate goods is very large, in the order of the tens of thousands. In contrast, in the empirical section below we focus the attention on a handful (four) of primary commodities.

All technologies will be assumed Cobb-Douglas, in spite of the fact that it implies the unrealistic restriction that sector shares are constant over time.[7] The reason is that it makes the algebra very simple and the expressions easy to interpret. The theoretical equation implied by this assumption is linear in logs, with parameters that are time invariant, so it naturally leads to an equation that can be used in the empirical analysis using the simplest techniques. In Appendix B.3 we show that the relationship between the RER and commodity prices that we derived also hold for general constant returns to scale production functions, but it will not be log-linear.

In what follows, we describe in detail the production structure of one of the economies. To fix ideas, consider the economy whose currency is used for international transactions –

[5] Technically, this is only a necessary, but not a sufficient condition. What we need is that the cone for each country is such that all varieties are produced in the countries we will be analyzing.

[6] For instance, in the case of oil, the oil fields are non-tradable; the oil extracted using the oil fields and other inputs is. In the case of wheat, the grain is tradable, the land used to produce it is not.

[7] For instance, in the U.S.A., between 1960 and 2000 the service sector grew from roughly 50 percent of GDP to 65 percent, while manufacturing dropped from 25 to 15 percent of GDP.

the United States in our empirical application. We now describe the environment in this economy, without any country-specific index. Those will be introduced when we consider two countries and construct a measure of their bilateral real exchange rate.

Countries may differ in their endowments of labor, $n_t(j)$ for all j, endowments of primary commodities fixed factors, $E_t(h)$, for all h, and in the parameters of their production function, including the total factor productivity associated to each production function. The fixed factors used in the production of commodities and the different varieties of labor are non-traded, while commodities are internationally traded in perfectly competitive markets. For the expressions that we derive below, we do not need to take a stand on how tradable the intermediate goods are.

Production of all goods (final, intermediate, and commodities) requires, in general, inputs of all types of labor. Labor for the production of each of them is aggregated from all varieties using Cobb-Douglas production functions. The total labor endowment of each variety is equal to L_j, which can be country specific.

The final good is produced according to the technology

$$C_t = Z_t^C \left(\prod_{j=1}^{J} [n_t^C(j)]^{\psi^C(j)} \right)^{\alpha} \left(\prod_{i=1}^{N} q_t(i)^{\varphi(i)} \right)^{1-\alpha}$$

where Z_t^C is productivity, $n_t^C(j)$ is labor of type j, used in the production of final consumption, $q_t(i)$ is the quantity of the intermediate good i used in the production of final consumption, $0 < \alpha < 1$, $\psi^C(j) \geq 0$ for all j, $\varphi(i) \geq 0$ for all i, $\sum_{j=1}^{J} \psi^C(j) = 1$, and $\sum_{i=1}^{N} \varphi(i) = 1$.[8]

Each variety of intermediate good i is produced using labor and primary commodities. The country-specific production function is

$$Q_t(i) = Z_t^Q(i) \left(\prod_{j=1}^{J} [n_t^{Q(i)}(j)]^{\psi^Q(i,j)} \right)^{\beta(i)} \left(\prod_{h=1}^{N} [x_t(i,h)]^{\phi(i,h)} \right)^{1-\beta(i)}, \text{ for all } i,$$

where $Q_t(i)$ is total output of intermediate i, $Z_t^Q(i)$ is productivity, $n_t^{Q(i)}(j)$ is the quantity of labor of type j used in the production of intermediate i, $x_t(i,h)$ is the quantity of primary commodity h used in the production of intermediate i, $\phi(i,h) \geq 0$ for all i and h, $\psi^Q(i,j) \geq 0$ for all i and j, $\sum_{h=1}^{N} \phi(i,h) = 1$, $\sum_{j=1}^{J} \psi^Q(i,j) = 1$, and $0 < \beta(i) < 1$ for all i.

[8] We write the production functions allowing for all possible inputs to be relevant for production in all cases. But we allow for some of the coefficients to be zero.

Finally, in each country there is a technology to produce the commodities given by

$$X_t(h) = Z_t^X(h) \left(\prod_{j=1}^{J} [n_t^{X(h)}(j)]^{\psi^X(h,j)} \right)^{\gamma(h)} E_t(h)^{1-\gamma(h)}, \text{ for all } h$$

where $X_t(h)$ is total output of commodity h, $Z_t^X(h)$ is productivity, $n_t^{X(h)}(j)$ is labor of type j used in the production of commodity h, $E_t(h)$ is the endowment of the fixed factor used in primary commodity h, $\psi^X(i,j) \geq 0$ for all i and j, $\sum_{j=1}^{J} \psi^X(i,j) = 1$ for all i, and $0 < \gamma(h) < 1$ for all h. As the endowment is not traded, as long as $E_t(h) > 0$, a positive amount of the commodity will be produced. Naturally, if $E_t(h) = 0$ for a particular country, production of that commodity will be zero and, as long as some is used in the production of intermediate goods, it will be imported.

2.2.1 Prices

With perfect competition, prices are equal to marginal costs. With Cobb-Douglas production functions, marginal costs are Cobb-Douglas functions of factor prices. Thus, the logarithm of the price level in the numeraire country will be

$$\ln P_t = \ln \left(\frac{\kappa^C}{Z_t^C} \right) + \alpha \sum_{j=1}^{J} \psi^C(j) \ln W_t(j) + (1-\alpha) \sum_{i=1}^{N} \varphi(i) \ln P_t^Q(i), \tag{2.1}$$

where P_t is the price of the final good, $W_t(j)$ is the nominal wage of type-j labor, $P_t^Q(i)$ is the price of the intermediate good i, and κ^C is a constant that depends on the exponents in the Cobb-Douglas production function.

Similarly, the price of intermediate good i is

$$\ln P_t^Q(i) = \ln \left(\frac{\kappa^{Q(i)}}{Z_t^Q(i)} \right) + \beta(i) \sum_{j=1}^{J} \psi^Q(i,j) \ln W_t(j) + (1-\beta(i)) \sum_{h=1}^{H} \phi(i,h) \ln P_t^X(h), \tag{2.2}$$

where $P_t^X(h)$ is the price in domestic currency of primary commodity h, and $\kappa^{Q(i)}$ is a constant depend on parameters of the production functions.

Using (2.2) into (2.1) gives

$$\begin{aligned} \ln P_t &= \ln\left(\frac{\kappa^C}{Z_t^C}\right) + (1-\alpha)\sum_{i=1}^{N}\varphi(i)\ln\left(\frac{\kappa^{Q(i)}}{Z_t^Q(i)}\right) \\ &\quad + \sum_{j=1}^{J}\left[\alpha\psi^C(j) + (1-\alpha)\sum_{i=1}^{N}\varphi(i)\beta(i)\psi^Q(i,j)\right]\ln W_t(j) \\ &\quad + (1-\alpha)\sum_{h=1}^{H}\left[\sum_{i=1}^{N}\varphi(i)(1-\beta(i))\phi(i,h)\right]\ln P_t^X(h) \end{aligned} \tag{2.3}$$

Note that weights on all prices and wages are non-negative, since they are products of exponents in the production functions. They also add up to one, due to the Cobb-Douglas assumption on all production functions.[9]

Summarizing, the log of the aggregate price level is a log-linear function of some constants, productivity shocks in final and intermediate goods, $\ln Z_t^C$ and $\ln Z_t^Q(i)$ for all i, wages for the different types of labor, $\ln W_t(j)$ for all j, and prices of primary commodities, $\ln P_t^X(h)$ for all h.

If we let

$$\begin{aligned} \mathbf{w}_t &= [\ln W_t(1), \ln W_t(2), ..., \ln W_t(J)]', \\ \mathbf{p}_t^X &= \left[\ln P_t^X(1), \ln P_t^X(2), ..., \ln P_t^X(H)\right]', \\ \mathbf{z}_t^Q &= \left[\ln Z_t^{Q(1)}, \ln Z_t^{Q(2)}, ..., \ln Z_t^{Q(N)}\right]', \text{and} \\ z_t^C &= \ln Z_t^C, \end{aligned}$$

we can write (2.3) in vector notation as

$$\ln P_t = a - z_t^C - \Psi_Q \mathbf{z}_t^Q + \Psi_w \mathbf{w}_t + \Psi_X \mathbf{p}_t^X, \tag{2.4}$$

in which Ψ_Q, Ψ_w, Ψ_X are row vectors of coefficients which are functions of the exponents in the Cobb-Douglas production functions. As we argued above, the sum of the components of the vector Ψ_w plus the sum of the components of the vector Ψ_X is equal to 1.

Notice that the dimensions of the vectors $\mathbf{z}_t^Q$ and $\mathbf{w}_t$ are likely to be very large, since they involve all the different types of labor and intermediate goods that are used to produce the final good. On the contrary, as we argue below, with a very low dimension vector $\mathbf{p}_t^X$

[9] The Cobb-Douglas assumption is not required for the property that the final good price is a constant returns to scale function of all factor prices: that property holds as long as technologies are all constant returns to scale. See Appendix B.3.

one can go a long way in accounting for real exchange rate variability.

Now, we use the fact that labor is used to produce commodities to relate the wages to primary commodity prices, and use those relations to replace the wages in equation (2.4). As long as the economy produces some commodity h, cost minimization in that industry implies that the type-j nominal wage is given by

$$\begin{aligned} \ln W_t(j) \;&=\; \ln P_t^X(h) + \gamma(h)\psi^X(h,j)\ln Z_t^X(h) + (1-\gamma(h))\ln\left(\frac{E_t(h)}{n_t^h(j)}\right) \\ &\quad +\gamma(h)\sum_{\tilde{j}=1, j\neq\tilde{j}}^{J}\psi^X\left(h,\tilde{j}\right)\ln\left[\frac{n_t^{X(h)}(\tilde{j})}{n_t^{X(h)}(j)}\right], \end{aligned} \tag{2.5}$$

for all j, as long as $\gamma(h)\psi^X(h,j) > 0$.

Notice that for this equation to hold, it is necessary that the country produces primary commodity h. Thus, if two different countries produce different commodities, the wages will be related to different commodity prices. This heterogeneity is important in order to identify a channel through which commodity prices affect real exchange rates.

Now let $\mathbf{n}_t(h,j)$ be a vector that contains the ratio of inputs in the production of commodity h, all normalized by labor of type j. That is,

$$\mathbf{n}_t(h,j) = \left[\frac{E_t(h)}{n_t^h(j)}, \frac{n_t^{X(h)}(\tilde{j})}{n_t^{X(h)}(j)} \text{ for all } \tilde{j} = 1, ..., J \text{ and } \tilde{j} \neq j.\right]'.$$

Then, we can express equation (2.5) as

$$\ln W_t(j) = \ln P_t^X(h) + \gamma(h)\psi^X(h,j)\ln Z_t^X(h) + \Psi_{\mathbf{n}(h,j)}\mathbf{n}_t(h,j), \tag{2.6}$$

where $\Psi_{\mathbf{n}(h,j)}$ is a vector of constants, also function of the share parameters in the Cobb-Douglas production functions, whose elements also add up to 1.

Using (2.6) to substitute for all wages in equation (2.4), we can write the price level as a log-linear function of constants; productivity shocks in all sectors, $\mathbf{z}_t = [z_t^C, \mathbf{z}_t^{Q\prime}]'$; ratio of input allocations in some primary commodity industry, denoted by $\mathbf{n}_t$; and primary commodity prices, $\mathbf{p}_t^X$,

$$\ln P_t = a + \Gamma_z\mathbf{z}_t + \Gamma_n\mathbf{n}_t + \Gamma_X\mathbf{p}_t^X,$$

where the sum of the coefficients in the row vector Γ_X (the sum of the coefficients on all primary commodity prices) is equal to 1.[10] Note, also, that the vector of primary commodity

[10] See details in Appendix B.4.

prices $\mathbf{p}_t^X$ is country specific despite of being traded goods, since prices are denominated in domestic currency.

Using the U.S. as the benchmark economy, we now make explicit, through a supra-index, that the price level in the U.S. is given by

$$\ln P_t^{USA} = a^{USA} + \Gamma_z^{USA} \mathbf{z}_t^{USA} + \Gamma_n^{USA} \mathbf{n}_t^{USA} + \Gamma_X^{USA} \mathbf{p}_t^{X,USA}. \tag{2.7}$$

Likewise, we can write the price level in a different country, say the United Kingdom, as

$$\ln P_t^{UK} = a^{UK} + \Gamma_z^{UK} \mathbf{z}_t^{UK} + \Gamma_n^{UK} \mathbf{n}_t^{UK} + \Gamma_X^{UK} \mathbf{p}_t^{X,UK}.$$

Notice that, while the log-linear structure is similar, the coefficients in the equation are country-specific, since they depend on each country's production functions. In addition, the shocks are also country specific (they are productivity shocks in the different sectors of each economy), and the way labor is allocated relative to the endowment input into the production of primary commodities, $\mathbf{n}_t^{UK}$, is also country-specific. The vector of commodity prices $\mathbf{p}_t^{X,UK}$ is also different from that in the U.S., $\mathbf{p}_t^{X,USA}$, but only because they are denominated in different currencies.

The law of one price for these primary commodities implies that commodity prices measured in U.S. dollars, $p_t^{X,USA}(h) = \log P_t^{X,USA}(h)$, are related to commodity prices measured in British pounds, $p_t^{X,UK}(h) = \log P_t^{X,UK}(h)$, through

$$p_t^{X,UK}(h) = p_t^{X,USA}(h) + s_t \text{ for all } h, \tag{2.8}$$

where $s_t = \log S_t$ is the logarithm of the nominal exchange rate. Then, if we let $\iota = [1, 1, ..., 1]'$ denote a vector of ones with H elements, we can use (2.8) in the solution for the final good price in the UK as to obtain

$$\ln P_t^{UK} = a^{UK} - \Gamma_z^{UK} \mathbf{z}_t^{UK} + \Gamma_n^{UK} \mathbf{n}_t^{UK} + \Gamma_X^{UK} \mathbf{p}_t^{X,USA} + \Gamma_X^{UK} \iota s_t.$$

But since the sum of the coefficients in the vector Γ_X^{UK} is equal to 1, we can write this expression as

$$\ln P_t^{UK} = a^{UK} + \Gamma_z^{UK} \mathbf{z}_t^{UK} + \Gamma_n^{UK} \mathbf{n}_t^{UK} + \Gamma_X^{UK} \mathbf{p}_t^{X,USA} + s_t \tag{2.9}$$

Subtracting (2.9) from (2.7) we obtain an equation relating the bilateral real exchange rate to the primary commodity prices, productivity shocks, and ratios of labor allocations,

$$\begin{aligned}\ln P_t^{USA} - \ln P_t^{UK} + \log S_t &= \left(a^{USA} - a^{UK}\right) + \left(\Gamma_z^{USA}\mathbf{z}_t^{USA} - \Gamma_z^{UK}\mathbf{z}_t^{UK}\right) \\ &\quad + \left(\Gamma_n^{USA}\mathbf{n}_t^{USA} - \Gamma_n^{UK}\mathbf{n}_t^{UK}\right) + \left(\Gamma_X^{USA} - \Gamma_X^{UK}\right)\mathbf{p}_t^{X,USA}.\end{aligned} \tag{2.10}$$

Equation (2.10) is the one that guides the empirical journey, which is the core of the paper. We will treat the vectors of productivity shocks $\mathbf{z}_t^{USA}$, $\mathbf{z}_t^{UK}$, and the vectors of labor allocations $\mathbf{n}_t^{USA}$, $\mathbf{n}_t^{UK}$ as unobservables.

As long as $\left(\Gamma_X^{USA} - \Gamma_X^{UK}\right) \neq 0$, variations in commodity prices will affect the RER. This will generally be the case if, for instance, the parameters of the production functions or the endowment of commodities $E_t(h)$ differ across countries. Note also that the sum of the coefficients in both Γ_X^{USA} and Γ_X^{UK} is equal to 1, so the sum of the coefficients in $\left(\Gamma_X^{USA} - \Gamma_X^{UK}\right)$ is equal to zero. Thus, we can normalize the commodity prices in $\mathbf{p}_t^{X,USA}$. We choose to do it with the price level in the United States. Thus, the vector $\mathbf{p}_t^{X,USA}$ that we use for our empirical analysis is in constant U.S. dollars.

The first term in the right hand side of (2.10) is a constant and the second is a vector of total factor productivity shocks that, as mentioned, we treat as unobservable. Available estimates of these shocks are much less volatile than the real exchange rates, so they have been disregarded as the main source of their fluctuations. This is one particular example of the exchange rate disconnect puzzle.

This discussion suggests that the volatility of the real exchange rate (left hand side) ought to come from the last two terms of the right hand side. The first of the two is differences in allocation of labor types in the production of commodities, but we have no information on those.

The second term is the one we consider: the primary commodity prices. As we already mentioned, they are an attractive candidate since they are both very volatile and very persistent, a property that will be inherited by real exchange rates, as long as $\left(\Gamma_X^{USA} - \Gamma_X^{UK}\right) \neq 0$. As we argued above, that means that the economies involved must have different production structures.

As a first approximation to the data, discussed in detail below and in Appendix B.1, we show in Tables I.a and I.b the volatility (standard deviation) of monthly data on the bilateral real exchange rate of the United Kingdom, Germany, and Japan, all against the United States for the period 1960 to 2014, as well as for several sub-periods. We also show the average volatility (simple and trade-weighted) of a sample of monthly data on 10 primary commodity prices. These 10 commodities have been chosen to be the ones with largest shares of world trade in the year 1990 and for which price series were available. Direct trade of these account for 12% of world trade in goods in that year.[11] Table I.a presents the volatilities for the raw data. As we will show below, presence of unit roots in some of the series cannot be rejected, so Table I.b presents the volatilities for the high frequency component of the data, computed using the Hodrick-Prescott filter.

Table I: Volatilities of real exchange rates and primary commodity prices

	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
(a) LEVEL					
Real Exchange Rates					
US-UK	0.12	0.06	0.15	0.08	0.08
US-DEU	0.18	0.07	0.21	0.09	0.13
US-JPN	0.37	0.13	0.13	0.12	0.11
Average across commodities					
Simple	0.44	0.17	0.36	0.22	0.36
Trade weighted	0.57	0.13	0.37	0.23	0.46
(b) CYCLE COMPONENT	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
Real Exchange Rates					
US-UK	0.07	0.04	0.10	0.07	0.06
US-DEU	0.07	0.03	0.10	0.07	0.07
US-JPN	0.08	0.03	0.10	0.10	0.07
Average across commodities					
Simple	0.18	0.13	0.21	0.17	0.15
Trade weighted	0.18	0.09	0.22	0.19	0.18

Note: variables are in logs and commodity prices are normalized by US CPI. We use the Hodrick-Prescott filter to compute the cycle component, with the standard smoothing parameter value for monthly data 129,600. Weights are based on the share of total trade in 1990. The set of primary commodities is: oil, fish, meat, aluminum, copper, gold,wheat, maize, timber, and cotton.

As it can be seen, the volatility of primary commodity prices is substantially higher than that of real exchange rates. In addition, it is apparent that when the volatility of commodity prices is high, so is the volatility of the real exchange rates. We find this issue particularly

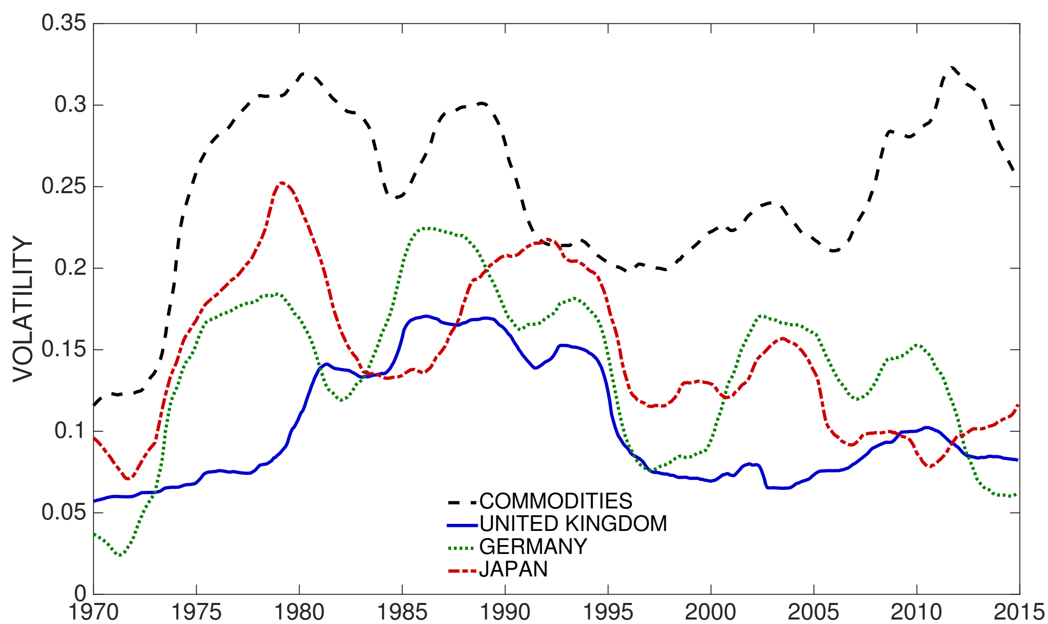
[11] As we mention, there is indirect trade on these commodities, since total trade data is not value added as GDP is.

interesting, since the substantial increase in volatility of real exchange rates after the break down of the Bretton-Woods system of fixed exchange rates is accompanied by an equal substantial increase in volatilities in commodity prices. The conventional interpretation has been that the increase in volatility after 1972 was the result of the regime change, from fixed to flexible exchange rates. An alternative interpretation is that the shocks that move together real exchange rates and commodity prices were more volatile before 1973 than after. To the extent that primary commodity prices are independent of the exchange rate regimes, our evidence points towards an alternative explanation of the increase in volatility post 1972.

Note also that the reduction of real exchange rate volatility that ensued after the mid-eighties is also accompanied with a reduction on commodity prices volatility (although to a lesser extent when considering the filtered data).

As an additional piece of evidence, Figure 2.1 shows rolling volatilities computed using windows of 10 years of data for the three real exchange rates and for the average volatility of the 10 commodity prices. The figure clearly points towards a positive association between volatilities of the real exchange rate and that of the commodity prices.[12]

Figure 2.1: (10-years) Rolling volatilities of real exchange rates and commodity prices



[12]The correlations between the bilateral exchange rate volatilities and the commodity price volatility are 0.40, 0.54, and 0.38 for United Kingdom, Germany, and Japan, respectively.

Real exchange rates are known to be very persistent. Again, so are primary commodity prices. In Table II we show the results of unit root tests for the raw data and for data on differences for three, four and five years. As it can be seen, there is some evidence of unit roots for the raw data, while the evidence vanishes for the four-year differences case. In Table III, we report the autocorrelation for all the series for our benchmark case of four year differences. As it can be clearly seen, the high persistence of real exchange rates is also present in the commodity prices.

Table II: Unit root tests (p-values)

	Level	3-years diff.	4-years diff.	5-years diff.
Real Exchange Rates				
US-UK	0.018	0.001	0.003	0.003
US-DEU	0.117	0.005	0.028	0.027
US-JPN	0.809	0.001	0.018	0.027
Commodities				
Oil	0.485	0.079	0.128	0.356
Fish	0.352	0.001	0.027	0.009
Meat	0.523	0.019	0.047	0.304
Aluminium	0.145	0.001	0.001	0.003
Copper	0.319	0.009	0.025	0.103
Gold	0.508	0.001	0.016	0.025
Wheat	0.226	0.001	0.005	0.009
Maize	0.269	0.001	0.010	0.013
Timber	0.047	0.003	0.018	0.047
Cotton	0.592	0.005	0.016	0.015

Note: variables are in logs and commodity prices are normalized by US CPI. We use the Dickey-Fuller test, in which the p-values are under the null hypothesis that the series follows a unit root process. The lag length is selected according to Ng-Perron test. We assume a trend in the case of Japan.

Before describing the more formal empirical results, we address two issues that are relevant in interpreting them. The first is the endogeneity of the commodity prices. The second is the procedure to select the relevant prices for each of the three real exchange rates we will analyze.

Table III: Autocorrelation of 4-year differences

US-UK	US-DEU	US-JPN	Oil	Fish	Meat	Aluminium
0.98	0.98	0.98	0.97	0.98	0.97	0.98
Copper	Gold	Wheat	Maize	Timber	Cotton	
0.98	0.99	0.98	0.97	0.97	0.98	

Note: variables are in logs and commodity prices are normalized by US CPI.

2.2.2 Endogenous regressor and emphasis on R^2

As we mentioned above, given that we are modeling large economies, one should expect the shocks to productivity and the allocation of different types of labor in the production of commodities to be correlated with the commodity prices. Thus, there is no hope to obtain consistent estimators of $\left(\Gamma_X^{USA} - \Gamma_X^{UK}\right)$ by running a regression of the real exchange rate on a vector of commodity prices.

The purpose of the paper is to document the co-movement between commodity prices and the RER rate, not to estimate those parameters. So we will focus our attention on R^2 and on correlations between fitted values of the RER using regressions of the equation above and observed values. We now discuss why the R^2 does provide relevant information despite the endogeneity problem.

Equation (2.10) must hold in equilibrium. In thinking about a model for the whole world – which we have not fully specified – the equation obtained only describes a relationship between endogenous variables (real exchange rates, primary commodity prices, and labor allocations within two countries) and exogenous variables (productivity shocks in the two countries involved).

Imagine a log-linear approximation to the equilibrium in such a model: If we let ξ_t be a vector of the (orthogonalized) structural shocks (including productivity, preference, policy shocks, among others), mostly unobservable, then the solution for the real exchange rates and the primary commodity prices measured as deviations from their long-run means will be log-linear on those fundamental shocks, or

$$\begin{aligned} \ln P_t^{USA} - \ln P_t^{UK} + \ln S_t &= \chi' \xi_t \\ \mathbf{p}_t^{X,USA} &= \Omega \xi_t \end{aligned}$$

where χ is a vector and Ω is a matrix of constants that determine the effect of each particular shock in the vector ξ_t on the real exchange rate and commodity prices, respectively.

In estimating equation (2.10) we are computing the linear projection of the real exchange rate onto the vector of commodity prices

$$\ln P_t^{USA} - \ln P_t^{UK} + \ln S_t = \delta' \mathbf{p}_t^{X,USA} + v_t$$

where v_t is orthogonal to $\mathbf{p}_t^{X,USA}$ and $\delta = (\Omega\Omega')^{-1}\,\Omega\chi$. It then follows that

$$\ln P_t^{USA} - \ln P_t^{UK} + \ln S_t = \delta'\Omega\xi_t + v_t.$$

Thus, the R^2 of the projection measures how much of the volatility of the real exchange rate can be accounted for by the particular combination of shocks that also affect commodity prices. This is why we focus the discussion on the R^2 that we obtain in each case.

2.2.3 Which commodities?

One thing that remains to be specified is the set of primary commodities that will be used in the right hand side of equation (2.10). The theory discussed above suggests which the commodities that ought to be more relevant are: those for which the relevant entry in the vector $(\Gamma_X^{USA} - \Gamma_X^j)$, for j corresponding to the UK, Germany, and Japan, are significantly different from zero. This is an easy task for countries such as Chile and Norway, since they specialize in the production of a single commodity and are, along that dimension, very different from any other country: exports of Copper in Chile and Oil in Norway have been around 20% of output, on average, in the first decade and a half of this century. As Table IV makes clear, a large fraction of the volatility of their real exchange rates with respect to the US dollar (about 75%) can be accounted for by movements in the corresponding commodity prices.[13] These results are not specific to very small economies: with the prices of the five most exported commodities, one can account for most of the long run movements of the real exchange rate in Brazil.

Table IV: Regressions for Norway and Chile (2000–2014) - R^2

RER (commodity)	US-Norway (oil)		US-Chile (copper)	
	Level	4-year differences	Level	4-year differences
R^2	0.72	0.32	0.75	0.33

Note: variables are in logs and commodity prices are normalized by US CPI.

[13] Using 4-years differences, they explain about 33%. For a more detailed analysis see Hevia and Nicolini (2013).

Pursuing this approach for the countries we focus on in this paper is much more challenging, since the vector $(\Gamma_X^{USA} - \Gamma_X^j)$ is a complicated combination of parameters of the input-output matrices of the two countries involved and there is no obvious asymmetry as there are in the cases of Chile and Norway.

A second, related complication is that to derive equation (2.10), we substituted the value of the nominal wage using equation (2.6), which represents the cost minimization condition for a particular commodity h. But we could have similar conditions for a different commodity, as long as the country produces more than one commodity. Or we could have used a cost minimization condition of an intermediate good produced in the country that also uses an imported commodity as input.[14] In fact, there are multiple ways to write the final good prices as functions of shocks, allocations and alternative commodity prices, where the sum of the parameters multiplying the commodity prices are constant over time and add up to 1.[15] All those must hold in equilibrium. These complications do not arise when a single commodity amounts to such a large share of output on itself and the asymmetry is so remarkable.

Finally, as documented in Table VI, commodity prices are highly correlated among themselves. This multicollinearity is likely to imply that the role of one particular price depends on the whole set of prices used in the regression. Again, this is not a problem when a single commodity is remarkably different from the others.

We therefore opted for an alternative strategy. We first chose the 10 primary commodities whose share on total trade was the largest and for which data on prices were available.[16] We then run an ordinary least squares regression of equation (2.10) and pick the four commodity prices with the largest t-statistics. Throughout the paper we compute the t-statistics using the Newey-West heteroskedasticity-and-autocorrelation-consistent standard errors. We then report the R^2 that results from regressions of the real exchange rate on only those four commodity prices. We also report, in Appendix B.5, the same exercises but choosing three and five commodity prices instead of four.

This strategy has a drawback: we are using the data that we want to explain to select the commodities. In order to deal with this issue, we propose some additional robustness exercises using out-of-sample evidence and a parametric bootstrap procedure that we explain in detail in the next section. Overall, we find the evidence described below quite strong.

[14] Imagine for instance a country that produces cars and imports the iron-ore. Then, one can express the wage as a function of the price of iron-ore and the quantities used.

[15] We formally show this in Appendix B.2.

[16] We excluded natural gas, coal, and iron due to data availability. Otherwise they would be included in the list of top 10 traded primary commodities.

2.2.4 Constant coefficients?

Equation (2.10) has time-invariant coefficients Γ as a consequence of the Cobb-Douglas assumption that was handy for the algebra presented. However, it implies that sector shares over total output are constant, a clearly counterfactual assumption, especially when considering a period longer than half a century. Technical change brings about new goods, even at the bottom of the production chain, where primary commodities are. Developments in the fracking technology changed dramatically the oil sector in the United States: the dependency of the US economy on imported oil is very different now than what it was in the 70's. Our constant shares assumption does not accommodate for those changes.

In spite of all those potential problems, we explore how far one can make progress with a single linear model, with only four commodity prices, and with parameters that do not change over time, from January 1960 to December 2014. In our view, we obtain a decent answer: the R^2 obtained ranges from 34% to 58% for our benchmark case, discussed in detail below. But because of the natural structural changes that occurred in these economies during this period, we will repeat the exercise for four different sub-periods using, for each of them, the same strategy. In these cases, the R^2 obtained range from 71% to 91% in our benchmark exercise. Breaking up the sample in four sub-periods is also useful, as we will show, for the additional out-of-sample exercises that we perform.

As motivational evidence, in Appendix B.1 we provide information regarding the trade positions on the 10 most traded commodities – the ones we chose to begin with – for the four countries considered (US, UK, Germany, and Japan). The evidence shows that there are substantial differences among the four countries in terms of the amounts imported and exported for the 10 commodities we selected.

2.3 Empirical results

We use equation (2.10) for the empirical analysis. We start our analysis by selecting the forty most traded commodities. Table V lists the commodities ranked according to their shares in the value of world trade in 1990 together with their SITC classification.[17] From those, we pick the ten commodities with the largest share of world trade in 1990 and for which we could find complete monthly price data in nominal US dollars since January 1960, and deflate the price data with the US consumer price index (CPI).[18] Our selection of ten commodities includes, in order of trade shares, Petroleum (Oil), Fish, Meat, Aluminum,

[17] The data is discussed in detail in Appendix B.1.

[18] We also repeated the exercise using trade data in 2000, and the results remain the same. The set of commodities is very similar: maize, sugar and cotton are replaced by platinum, coffee and soybeans.

Copper, Gold, Wheat, Maize, Timber, and Cotton. The bilateral real exchange rates are defined as the nominal exchange rate between Germany (DEU), Japan (JPN), and the United Kingdom (UK) all against the US dollar multiplied by the ratio of CPIs. In the case of Germany, we use the Mark until 2000 and the Euro since then. We use monthly data over the period 1960M1–2014M12.

In Table VI we report the correlation between each pair of commodity prices together with the correlation between commodity prices and RERs for the whole period (1960-2014). The table shows that the simple correlation between real exchange rates and commodity prices is far from trivial, and that some commodity prices are highly correlated, for example maize and cotton.[19]

[19] These correlations also change substantially across sub-periods. For instance, the correlation between meat and oil is positive in the second sub-period.

Table V: Commodity List

Commodity	(%) share of world trade in 1990	SITC (rev.3)	Commodity	(%) share of world trade in 1990	SITC (rev.3)
(1) Petroleum	7.22	33	(21) Oranges	0.17	057.1+059.1
(2) Fish	1.05	03	(22) Tobacco	0.15	121
(3) Meat	0.89	011+012	(23) Platinum	0.15	681.2
(4) Natural Gas	0.58	343	(24) Milk and butter	0.14	022.1+023+024
(5) Coal	0.54	321	(25) Rubber	0.13	231
(6) Aluminium	0.49	285.1+684.1	(26) Rice	0.09	042
(7) Copper	0.45	283.1+682.1	(27) Nickel	0.09	284.1+683.1
(8) Gold	0.42	971.01	(28) Bananas	0.09	057.3
(9) Wheat	0.35	041	(29) Nuts	0.09	057.7
(10) Maize	0.28	044	(30) Other base metals	0.08	287.8+287.9+689.1
(11) Iron	0.28	281	(31) Grapes	0.08	057.5+059.93
(12) Timber	0.26	24	(32) Apples	0.08	057.4+059.94
(13) Cotton	0.22	263	(33) Barley	0.08	043
(14) Sugar	0.22	061.1+061.2	(34) Leguminous vegetables	0.07	054.2
(15) Hides and skins	0.21	21	(35) Silver	0.07	681.1
(16) Coffee	0.20	071.1+071.2	(36) Tomatos	0.07	054.4+059.92
(17) Wool	0.19	268	(37) Tea and mate	0.06	074
(18) Other vegetables	0.19	054.5	(38) Lead	0.06	287.4+685.1
(19) Soya beans	0.19	222.2	(39) Palm Oil	0.06	422.2
(20) Zinc	0.19	287.5+686.1	(40) Cocoa	0.06	072.1+072.2

Source: Comtrade

Notes: SITC (rev3) stands for Standard International Trade Classification (revision 3).

Table VI: Correlations (1960–2014)

	Oil	Fish	Meat	Alum.	Copper	Gold	Wheat	Maize	Timber	Cotton
RER										
US-UK	-0.47	0.00	0.30	0.11	0.09	-0.53	0.26	0.36	-0.40	0.30
US-DEU	-0.51	-0.24	0.16	0.08	-0.08	-0.62	0.06	0.14	-0.58	0.11
US-JPN	-0.49	0.25	0.59	0.52	0.41	-0.63	0.59	0.63	-0.44	0.55
Commodities										
Oil	1.00									
Fish	0.28	1.00								
Meat	-0.17	0.45	1.00							
Alum.	-0.20	0.36	0.73	1.00						
Copper	0.07	0.72	0.57	0.52	1.00					
Gold	0.88	0.25	-0.16	-0.22	0.03	1.00				
Wheat	-0.05	0.57	0.78	0.70	0.60	-0.07	1.00			
Maize	-0.11	0.58	0.81	0.70	0.62	-0.13	0.94	1.00		
Timber	0.39	0.10	0.18	0.17	0.10	0.56	0.21	0.15	1.00	
Cotton	-0.23	0.39	0.83	0.78	0.43	-0.21	0.84	0.86	0.24	1.00

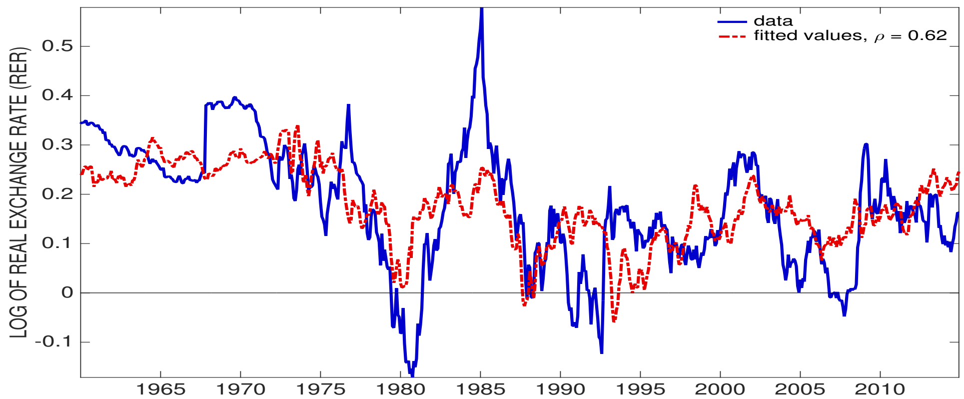
Note: variables are in logs and commodity prices are normalized by US CPI.

We start by running a simple regression of the level of the real exchange rates on the level of ten commodity prices, and pick the four commodities with the highest t-statistics. We next run the regression again with the four relevant commodity prices and report the R^2 in Table VII. We perform this exercise for the entire sample period and for the four sub-periods. In Appendix B.5, Tables A.8–A.13, we show the estimated coefficients of each regression.

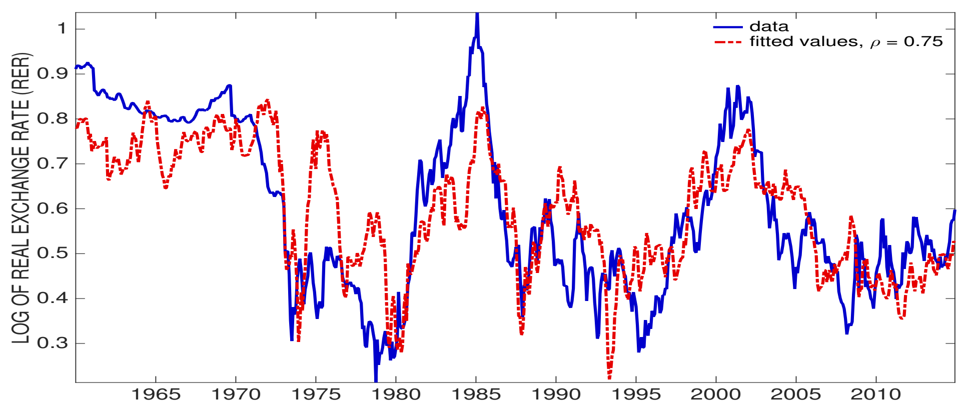
The R^2 of the regressions for the entire period, 1960-2014, are 0.39, 0.56, and 0.79 in the case of UK, Germany, and Japan respectively. The other columns of the table show that the R^2s are larger when we consider each sub-period separately, reaching values as high as 0.85 in the case of Japan. Figure 2.2 displays the real exchange rates and the fitted values of the regressions for the entire sample period. The match is remarkably good in all three cases. In the case of Germany, for example, the correlation between the fitted values using only four commodities and the actual real exchange rate is about 0.75.

Figure 2.2: Real exchange rates and fitted values, in levels (1960–2014)

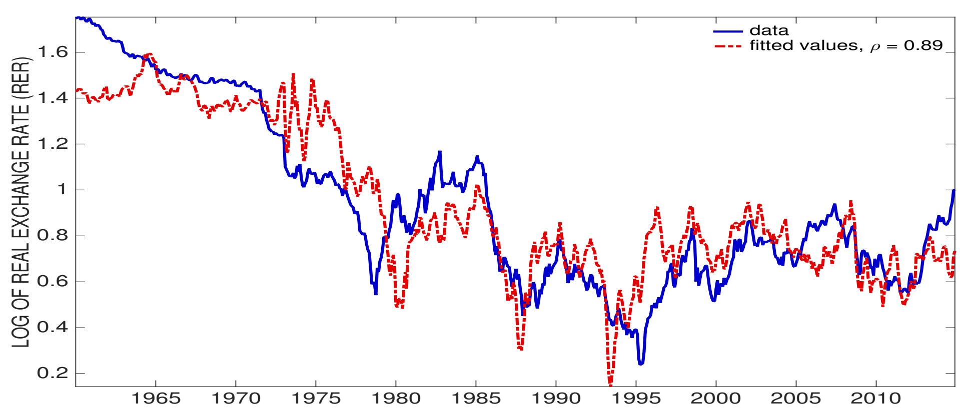
(a) United Kingdom



(b) Germany



(c) Japan



However, since real exchange rate and commodity price data are very persistent, and for many of them we cannot reject the null hypothesis of a unit root (Table II), the results in Table VII and Figure 2.2 may be spurious. For that reason, we repeat the regressions differencing the data over a 4-year period which, as shown in Table II, produces stationary series.[20] The results, displayed in Table VIII and Figure 2.3, confirm the previous finding: with just four commodities we obtain a remarkable fit of the bilateral real exchange rates for the entire sample period and for the different sub-periods. Furthermore, the fit of the regressions for the different sub-periods using the differenced data is better than that of the regressions in levels, with many R^2s higher than 0.8. The exceptions are the regressions using the entire sample, which give somewhat smaller R^2s. But this should not be too surprising given that we are imposing constant coefficients over a period of more than 50 years.

Table VII: Regressions in levels - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.39	0.66	0.70	0.51	0.51
Germany	0.56	0.77	0.83	0.38	0.66
Japan	0.79	0.85	0.57	0.67	0.66

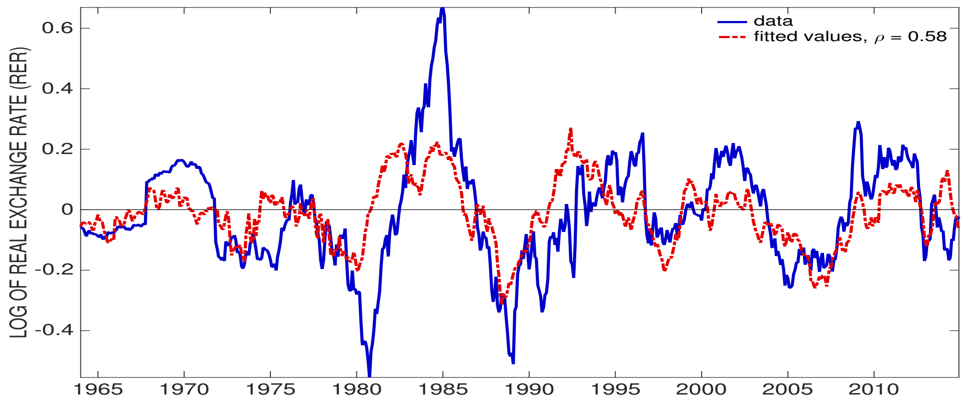
Table VIII: Regressions in 4-year differences - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.33	0.72	0.82	0.63	0.58
Germany	0.56	0.84	0.87	0.81	0.74
Japan	0.48	0.88	0.76	0.86	0.80

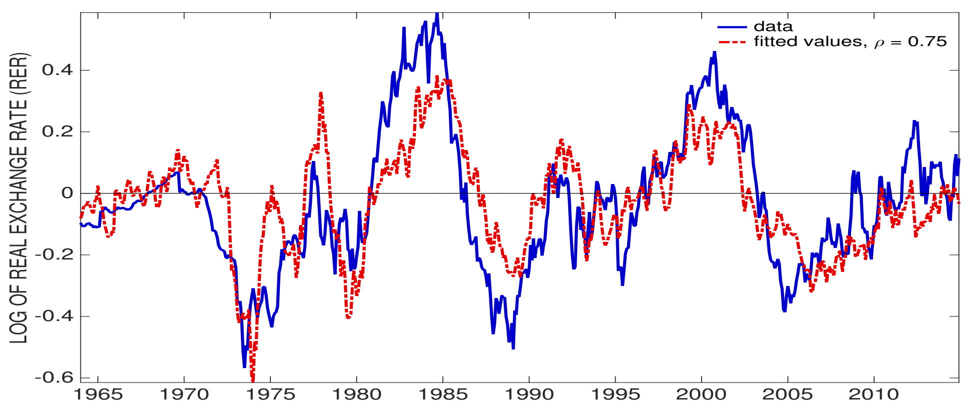
[20] Results for the 3-year and 5-year differences are shown in Appendix B.5.

Figure 2.3: Real exchange rates and fitted values, in 4-year differences (1960–2014)

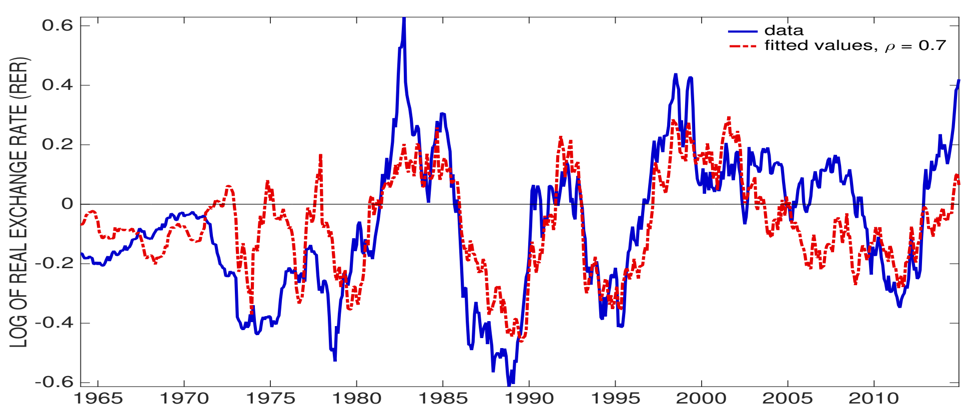
(a) United Kingdom



(b) Germany



(c) Japan



In the previous regressions we chose the commodities that obtain a good fit of the real exchange rates. One could argue that the regressors have been chosen precisely to match the data. Even in this case, we find it remarkable that a linear combination of such a small number of variables co-moves so well with the real exchange rate. In contrast, if one were to consider a more realistic case that does not imply constant sector shares over time, then neither the parameters nor the regressors would be the same over time. As an example, consider Norway before 1978, when the oil industry started developing. The clear correlation between the price of oil and the real exchange rate of Norway that is evident in the post-1980 data should not hold for the pre-1980 data. However, to the extent that those changes happen only infrequently, one should expect the relevant regressors to change slowly over time.

To further explore this idea, we run regressions for each sub-period, select the four commodities with largest t-statistics in that sub-period, and then run regressions of the real exchange rates on those commodities for the following sub-period. As long as the input-output matrix and the production structures of the two involved economies do not change much over time, those regressors should do a good job in explaining the real exchange rate in the following period.

The results of this exercise are shown in Table IX using data in 4-year differences. For example, in column 1973-1985, in the case of Germany, we run the regression for the period 1960-1972, pick the commodities that are statistically significant, and then run the regression for the period 1973-1985 using this set of commodities. The R^2 of the regression is 0.68, smaller than the 0.87 obtained using contemporaneous data (Table VIII), but still a sizable number. Table IX shows that our previous results are robust to the selection of commodities based on data from the previous sub-period.

Table IX: Regressors from previous sub-period - R^2

	1973-1985	1986-1998	1999-2014
United Kingdom	0.54	0.49	0.39
Germany	0.68	0.58	0.66
Japan	0.55	0.67	0.42

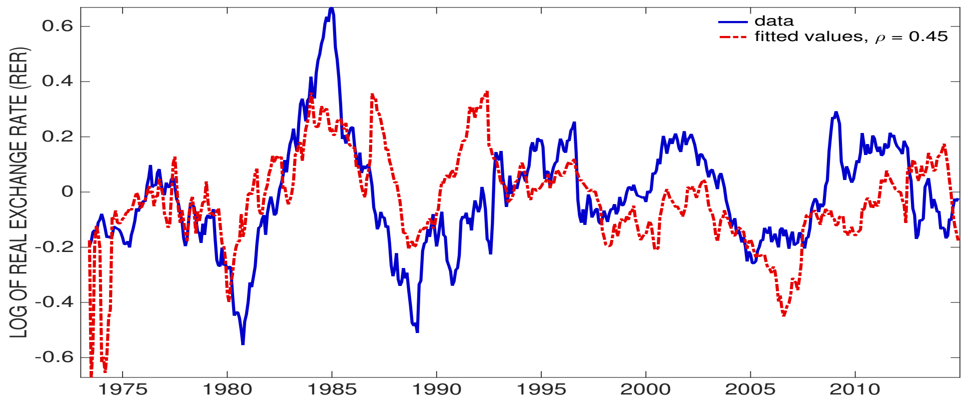
One could go one step further and fit the value of the real exchange rate for a particular period using not only the regressors but also the estimated parameter obtained using data from a previous period. This "out-of-sample" fitting exercise is what we do next.[21]

[21] We emphasize that this is not a forecasting exercise since we use information on current commodity prices to fit the current real exchange rate.

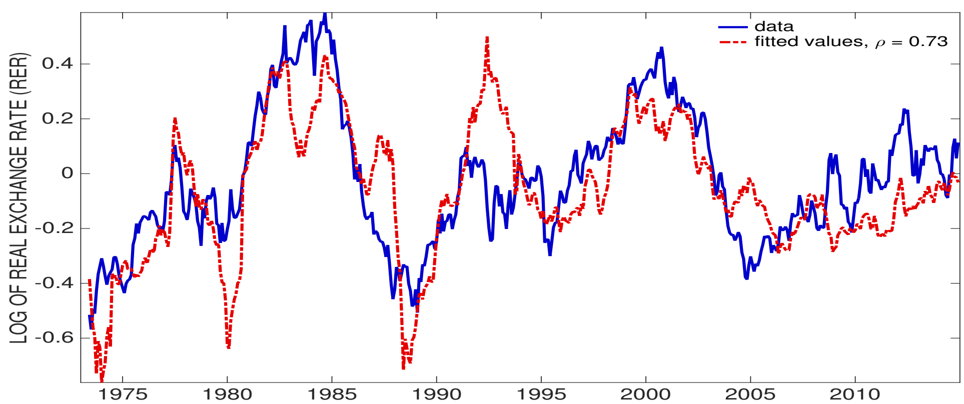
We start by running a regression (using data in 4-year differences) over the period 1960-1972. We drop the six commodities with the lowest t-statistics using the Newey-West standard errors and re-run the regression. Based on the four commodities selected by this procedure and their estimated coefficient, we use observed commodity prices over the following R periods to fit the real exchange rate and store the R fitted values. We next add one observation to the sample and repeat previous regressions to fit the real exchange rates over the following R periods. Repeating this procedure until the end of the sample, we construct R time series of out-of-sample fitted real exchange rates over the following $r = 1, 2, .., R$ periods. For example, Figure 2.4 shows the actual and fitted real exchange rates for the case $r = 6$ months ahead. The out-of-sample fit is remarkable, with a correlation between fitted and actual values of 0.45 for the United Kingdom, 0.73 for Germany, and 0.64 for Japan.

Figure 2.4: Out-of-sample fit 6 months ahead

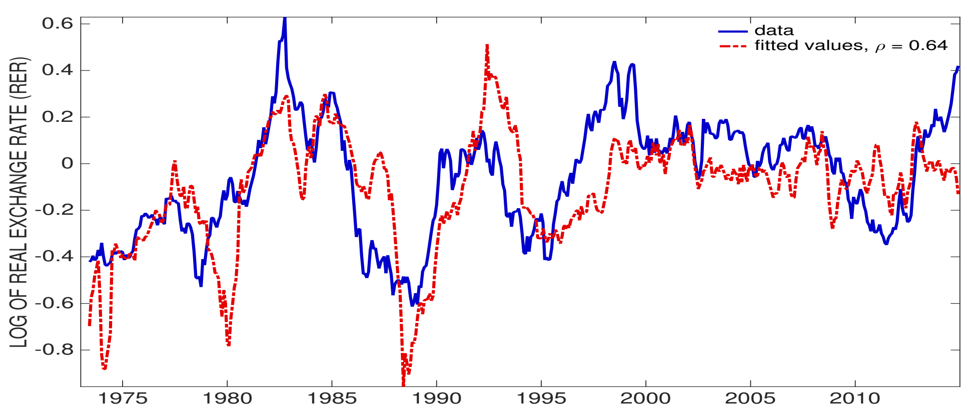
(a) United Kingdom



(b) Germany

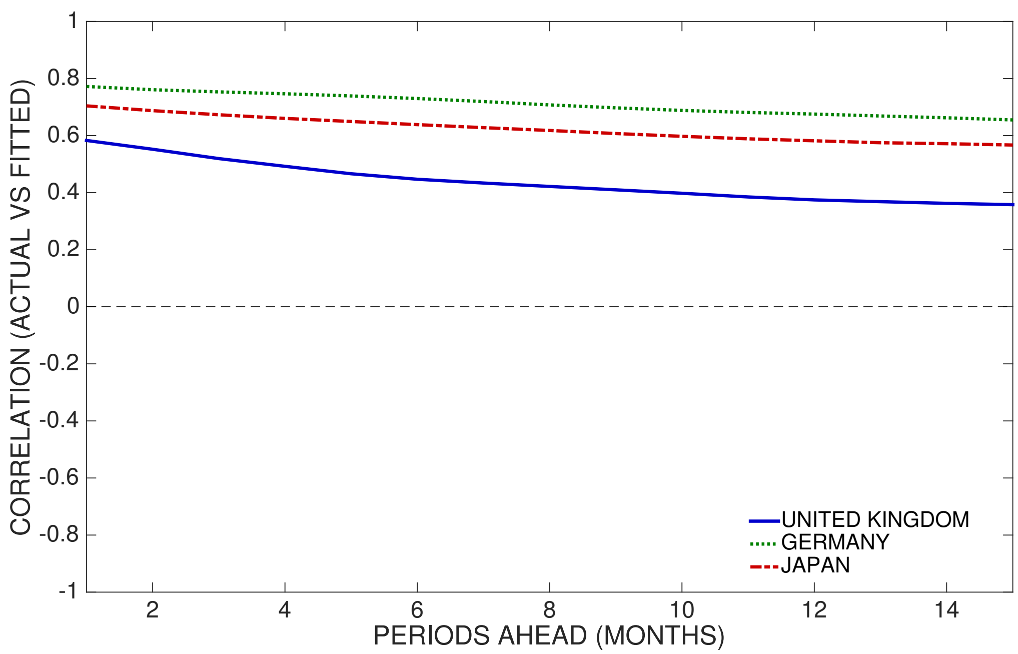


(c) Japan



We summarize the results in Figure 2.5, where we show the correlation between fitted and actual real exchange rates as we vary the forward window from $r = 1$ through $r = 15$ months ahead. Although the correlations decrease as the fitting horizon increases, they decrease slowly. There is a good out-of-sample fit even using data that is 15 months old to select the commodities and coefficients to fit real exchange rates today. Overall, we interpret these results as supporting our initial findings that shocks that affect just four commodity prices account for a substantial fraction of real exchange rate movements.

Figure 2.5: Out-of-sample fit – correlations as a function of r (months ahead)



2.3.1 Are the results spurious?

A concern with the previous regressions is to what extent the results could be due to a problem of small sample size. It is well known that, even with stationary series, regressing two orthogonal, but highly persistent series, could lead to a spurious correlation for moderate sample sizes. To explore this issue, we perform small sample inference by using a parametric bootstrap procedure that generates real exchange rate and commodity price data under the null hypothesis that commodity prices are orthogonal to real exchange rates.

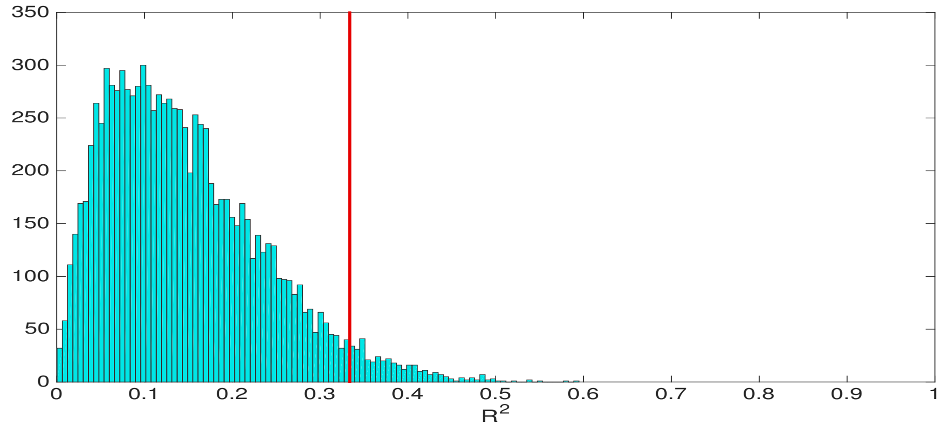
Consider the regressions using data in 4-year differences displayed in Table VIII. Take, for example, the R^2 of 0.56 of Germany-US real exchange rate regression on the four commodity prices with the highest t-statistics. In this case, the bootstrap procedure under the null hypothesis that real exchange rates are orthogonal to commodity prices is as follows.

We first estimate an autoregressive (AR) process for the the Germany-US real exchange rate and an independent vector autoregression (VAR) with the four commodity prices used in the regression. In both cases, we use data in 4-year differences and choose the lag-length of the estimated processes according to the Schwarz information criterion. By construction, commodity prices and real exchange rates are orthogonal. Hence, if we draw artificial time series from the estimated processes, a regression of the real exchange rate on the commodity prices delivers an R^2 that converges to zero as the artificial sample size grows towards infinity. But for a sample size such as ours, the R^2 of the regression is positive. The question is, how common is to obtain an R^2 of 0.56 under the null hypothesis of orthogonality given an artificial sample of data with the same number of observations that we have?

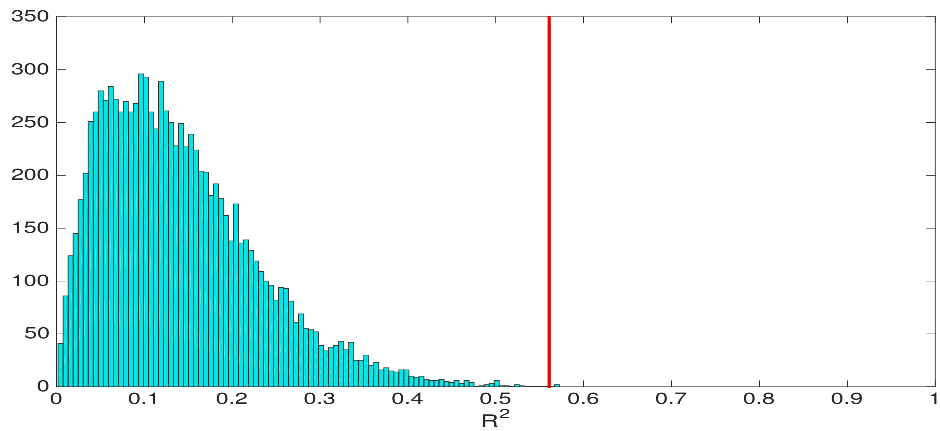
To compute the small sample distribution of the R^2, we draw 10000 samples of length 660 (the number of months between January 1960 and December 2014) by resampling from the residuals of the estimated AR and VAR processes and computing artificial real exchange rate and commodity price data. Then, for each artificial sample, we run a regression of the real exchange rate on the four commodity prices and store the associated R^2. Finally, we compare the estimated R^2 of 0.56 with the small sample distribution of R^2s computed with the bootstrap procedure. We use the same procedure to compute small sample distributions of the R^2 for each regression and sub-sample in Table VIII.

Figure 2.6: Small sample distribution of the R^2 over the period 1960–2014

(a) United Kingdom



(b) Germany



(c) Japan

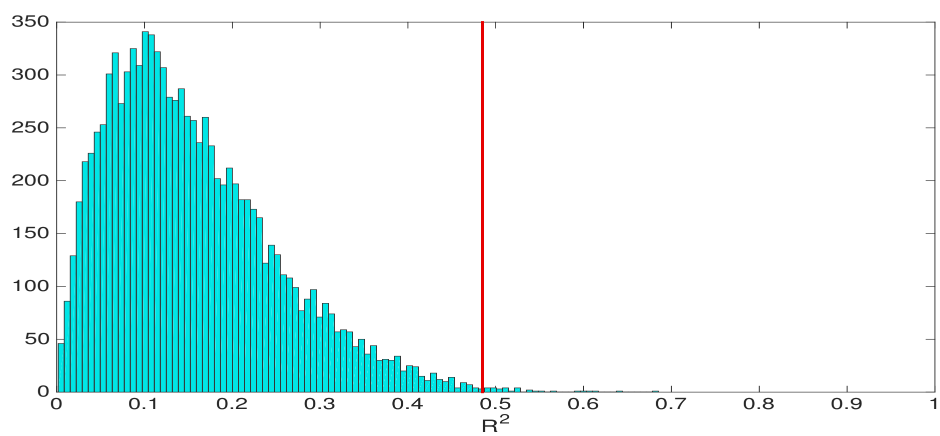


Figure 2.6 displays the small sample distributions of the R^2 under the null hypothesis of

a spurious correlation over the entire sample period (1960-2014) for the three real exchange rates. The vertical lines are the estimated R^2 using the actual data. In all cases, the probability of obtaining an R^2 as large as that estimated in Table VIII is smaller than 5 percent and as low as 0 percent for the case of Germany. The three distributions under the null hypothesis are positively skewed with a mode of about 0.1, much smaller than the estimated R^2 in the table.

Table X: Bootstrapped distributions of the R^2 under the null hypothesis of orthogonality

		Percentiles distribution of R^2				
	$\hat{R}^2$	Median	75	90	95	$\Pr(R^2 \geq \hat{R}^2)$
United Kingdom						
1964-2014	0.33	0.13	0.20	0.27	0.31	0.037
1964-1972	0.72	0.52	0.66	0.75	0.80	0.143
1973-1985	0.82	0.37	0.52	0.64	0.70	0.004
1986-1998	0.63	0.37	0.50	0.61	0.67	0.077
1999-2014	0.58	0.29	0.41	0.53	0.59	0.059
Germany						
1964-2014	0.56	0.13	0.19	0.26	0.31	0.000
1964-1972	0.84	0.56	0.69	0.79	0.83	0.032
1973-1985	0.87	0.49	0.63	0.73	0.78	0.005
1986-1998	0.81	0.40	0.54	0.65	0.71	0.007
1999-2014	0.74	0.30	0.43	0.55	0.61	0.007
Japan						
1964-2014	0.48	0.14	0.21	0.29	0.34	0.003
1964-1972	0.88	0.59	0.72	0.81	0.85	0.022
1973-1985	0.76	0.46	0.60	0.70	0.75	0.045
1986-1998	0.86	0.41	0.55	0.66	0.71	0.001
1999-2014	0.80	0.33	0.46	0.57	0.63	0.002

Table X shows statistics of the small sample distributions under the null of orthogonality for the three bilateral real exchange rates and for the five sub-periods, together with the probability of observing an R^2 as large as that estimated under the null of orthogonality. For comparison, the table also includes the estimated R^2s from Table VIII for each country and sub-period. Overall, these results suggest that the estimated correlations are robust for every sub-period and bilateral real exchange rate. Of course, for some sub-periods and countries, the small sample distributions are more dispersed and it is not uncommon to observe a relatively large R^2 under the null of orthogonality, specially for smaller sample sizes.

2.4 Concluding comments

In this paper we argue that commodity price fluctuations are an important source of volatility in real exchange rates among developed economies. In particular, we study the bilateral exchange rates between the United States, Japan, Germany and the United Kingdom.

Using a standard model that explicitly accounts for commodities, we derive a relationship between real exchange rates on one hand and productivity shocks and commodity prices on the other. Productivity shocks cannot be observed, but they can and have been widely estimated using the Solow residual methodology. All measures available imply a volatility of productivity shocks that is orders of magnitude lower than the one observed in commodity price shocks. One could be naturally lead to think therefore that most of the volatility of RER should be accounted by the more volatile component. In addition, one would expect that periods - say decades - with lower commodity price volatility should therefore lead to periods with lower RER volatility. Is it the case that the post Bretton-Woods period lead to more RER volatility because of the prevailing free floating era or just because the turmoil that followed the oil price shock of the seventies lead to higher commodity price volatility? Does it matter at which frequency we measure the volatility? We just begun to explore a few possibilities.

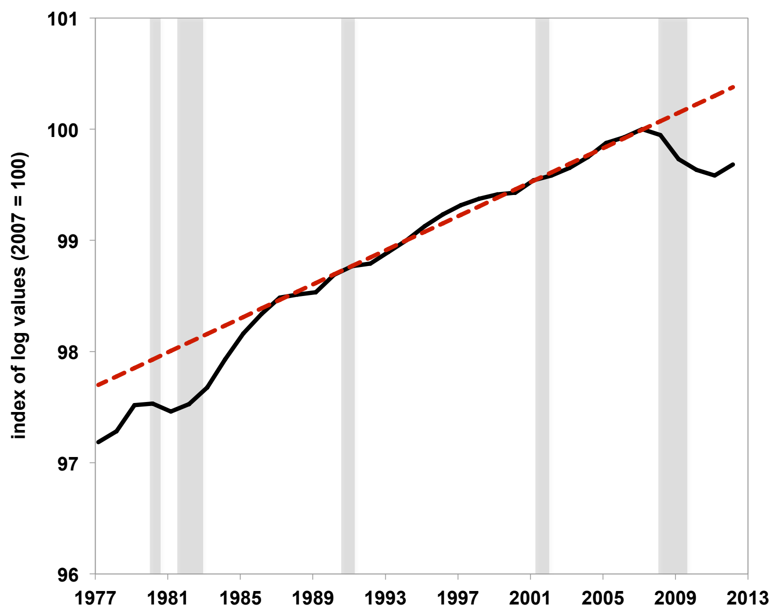
Chapter 3

Lack of Firm Entry and the Slow Recovery of the U.S. Economy After the Great Recession

3.1 Introduction

This paper studies firm entry in the United States during the Great Recession and its subsequent years. We show that besides the slow recovery of employment, the recovery after the Great Recession is also characterized by the slow recovery of firm entry. Using data from the Business Dynamics Statistics (U.S. Census), we show that both the Double-Dip Recession in the 1980s and the Great Recession featured a substantial drop in the number of firms (see Figures 3.1 and 3.2).[1] However, when comparing the recovery in the number of firms, we verify that it has been remarkably slower after the Great Recession.

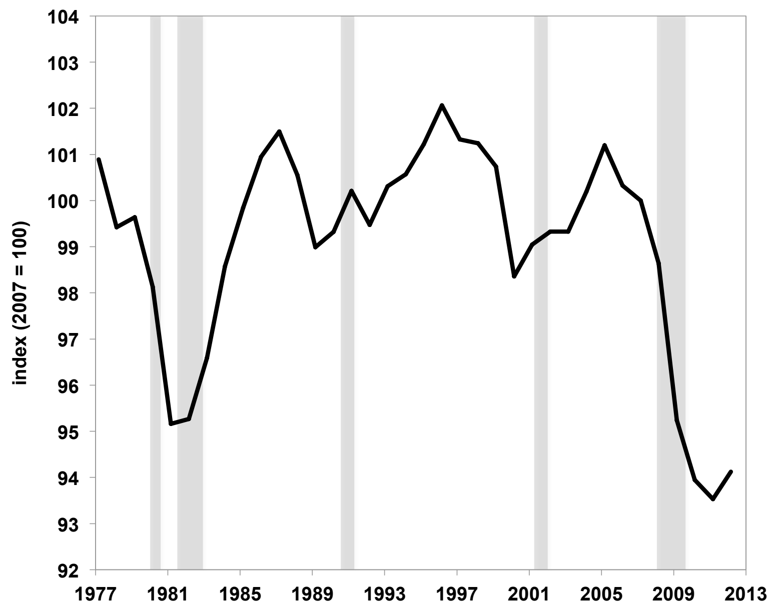
Figure 3.1: Number of firms



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey.*

[1]There were two recessions in the early 1980s, but we are treating them as one. We compare the Great Recession to the Double-Dip Recession because both feature a similar drop in the magnitude of output and employment.

Figure 3.2: Number of firms per labor force participant



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

We then link the slow recovery of firm entry to the slow recovery of employment, which has been the focus of many researchers in recent years.[2] We consider a counterfactual exercise where we quantify the effect of lack of firm entry on employment, and show that it accounts for 22 percent of the difference between the actual level of employment per labor force participant in March 2012 and its pre-recession level, in March 2007. This result is consistent with recent findings in the labor economics literature, which shows that low probabilities of finding jobs play an important role in explaining fluctuations in unemployment in the business cycle frequency (e.g., Shimer (2008)).

Motivated by the empirical facts mentioned above, we then extend the industry equilibrium framework of Hopenhayn (1992) by adding aggregate uncertainty in productivity and assess how firm entry reacts to a negative supply shock and a negative demand shock. Our results show that a negative aggregate productivity shock does not generate a drop in firm entry, while a negative demand shock does. The latter causes a significant drop in firm entry, similar to the one observed during the Great Recession. However, the demand shock alone does not generate a slow recovery.

[2] E.g. Elsby et al. (2011), Jaimovich and Siu (2014), and Haltiwanger, Jarmin and Miranda (2013).

Finally, we empirically assess alternative explanations for the slow recovery. These explanations include financial constraints, offshoring, increased uncertainty at the firm level and transfers to self-employment. The empirical evidence that we provide contradicts such explanations.

Related Literature: our work is related to studies that have focused on firm dynamics and the Great Recession. Siemer (2014) argues that in a model with firm entry and financial constraints, a large financial shock curtails young small firms more relative to large old ones. The author then argues that a financial shock can result in a long lasting recession caused by a “missing generation" of entrants. The same distinction between young and old firms is made in Mehrotra and Sergeyev (2015) and Dyrda (2014), but they do not analyze entry and exit of firms. These studies are different from ours because we do not focus on the drop of firm entry, but on the slow recovery of firm entry. As we show, a standard model that generates a drop in firm entry would predict that firm entry should increase above trend after few periods following the recession.

The paper proceeds as follows: in Section 2 we provide the empirical evidence; in Section 3 we describe the counterfactual exercise, where we quantify the effect of the lack of firm entry on the slow recovery of employment; in Section 4 we analyze firm entry over the business cycle using a frictionless model of firm dynamics facing aggregate uncertainty; in Section 5 we empirically assess different hypotheses for the slow recovery of firm entry; Section 6 concludes.

Our work is preliminary and incomplete. The goal of this project is to provide both empirical evidence and a theoretical understanding on the sources driving the lack of firm entry after the Great Recession.

3.2 Empirical Evidence

The primary dataset used in our analysis is the Business Dynamics Statistics (BDS), published by the Center of Economic Studies in the U.S. Census Bureau. It is a publicly available dataset containing annual (mid-March) information on private businesses in the United States from 1977 to 2012 (see Haltiwanger, Jarmin and Miranda (2009) and Jarmin and Miranda (2002) for a complete description of the data). It is based on administrative records and covers most of the private non-agricultural sector of the economy. The main exclusions are self-employed individuals, employees of private households, agricultural production employees, and most government employees. BDS includes only employer firms, i.e., for a firm to be included in the BDS it must have at least one employee in its payroll.

Information is available both at the firm and establishment levels. An establishment is defined as a single physical location where production takes place, while a firm corresponds to a group of establishments linked to each other by ownership status, i.e., they operate under the control of the same firm. We consider the firm as the main economic unit, since it is the one who makes the relevant decisions about the economic activities of its own establishments.[3] Finally, labor force series is based on the Current Population Survey (CPS) from the Bureau of Labor Statistics (BLS).

Drop in the number of firms and its slow recovery after the Great Recession

In Figure 3.2, we plot number of firms per labor force participant from 1977 until 2012. As pointed out in Luttmer (2010), the number of firms and the number of labor force participants share a common trend during the period we are covering. We can see in Figure 3.2 that once we normalize the number of firms by the number of labor force participants we have a stationary series.[4]

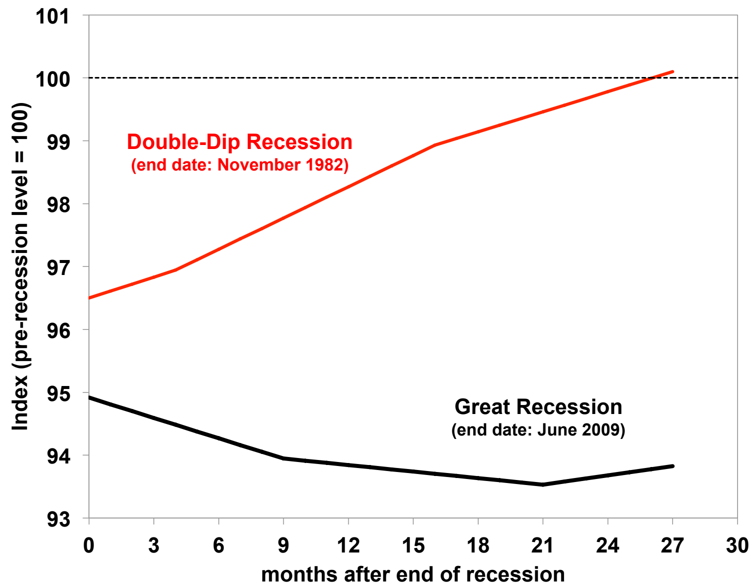
It can be seen in Figure 3.2 that the number of firms per labor force participant dropped significantly during both the Double-Dip Recession in the early 1980s and during the Great Recession (2007-2009). From March 1979 to March 1981, the number of firms per labor force participant dropped 4.5 percent and from March 2007 to March 2010, it dropped 6 percent. The latter is the largest variation observed in the period for which we have data.

Although the Double-Dip Recession and the Great Recession have in common the fact that they both featured a significant drop in the number of firms per labor force participant (more than 4 percent each), the recovery periods following them are remarkably different. The recovery of the number of firms per labor force participant after the Great Recession has been much slower than the one observed in the early 1980's. This fact is illustrated in Figure 3.3.

[3] Our results wouldn't change if we carried out the same analysis using establishment as the main economic unit instead, because in our data, cyclical variations in the number of establishments is driven mainly by cyclical variations in the number of firms.

[4] Luttmer (2010) shows that the trend is similar for the last 80 years.

Figure 3.3: Slow recovery in the number of firms per labor force participant after the Great Recession



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

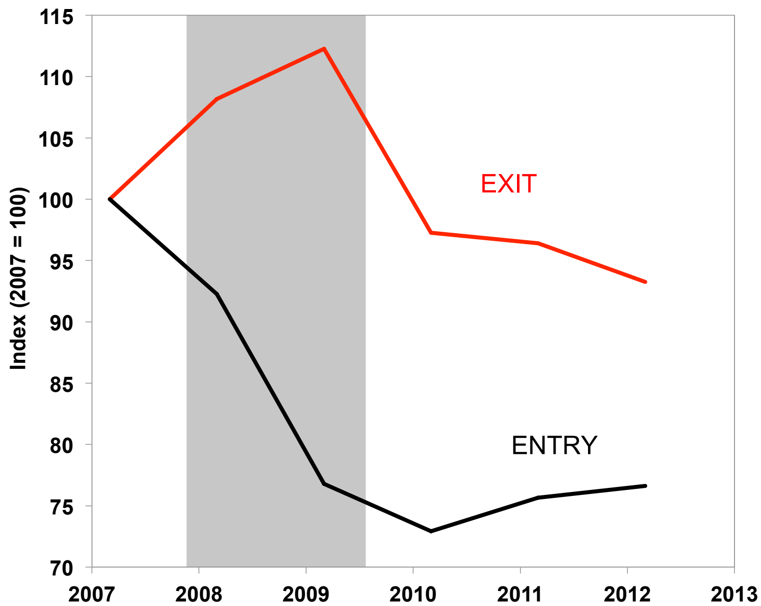
According to the recession dates of the National Bureau of Economic Research (NBER), the Double-Dip Recession started in January 1980 and ended in November 1982, and the Great Recession started in December 2007 and ended in June 2009. Since we have annual data, we take the values in March 1979 and March 2007 as the pre-recession levels.

Figure 3.3 shows that the number of firms per labor force participant increased immediately after the end of the Double-Dip Recession. After 15 months, it was only 1 percent lower than its pre-recession level, and after 25 months it had recovered completely. On the other hand, the number of firms per labor force participant continued to drop in the months after the Great Recession. Only after 20 months, it started to increase again. In March 2012, 27 months after the end of the recession, the number of firms to labor force is still 6% below trend. Therefore, even after taking into account the fact that the drop in the number of firms per labor force participant was larger during the Great Recession than during the Double Dip Recession, the recovery of the number firms per labor force participant after the Great Recession seems to be remarkably slow.

Lack of Firm entry as the main driver

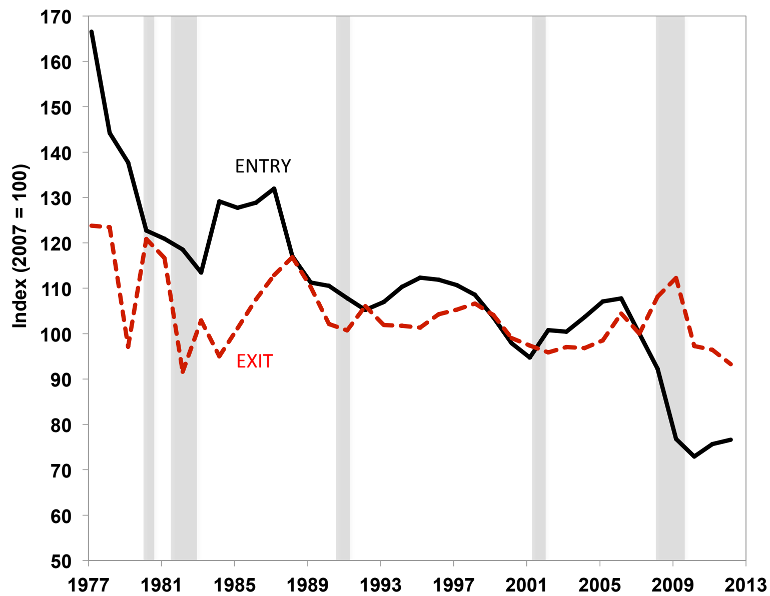
The drop in the number of firms per labor force participant observed during the Great Recession can be accounted for by either variations in the number of entering firms per labor force participant or by variations in the number of exiting firms per labor force participant. In figure 3.4, we show how these two series evolved in the past years.

Figure 3.4: Number of entering and exiting firms per labor force participant



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

Figure 3.5: Number of entering and exiting firms per labor force participant (1977:2012)



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

Between March 2007 and March 2009, the number of exiting firms per labor force participant increased 13 percent and the number of entering firms per labor force participant decreased 23 percent, indicating that firm entry contributed more to the initial drop in the number of firms per labor force participant during the Great Recession.

Furthermore, lack of firm entry plays a major role in accounting for the slow recovery of the number of firms per labor force participant after the Great Recession. The number of exiting firms per labor force participant returned to its pre-recession level in March 2010. However, the number of entering firms per labor force participant continued to fall, and after March 2010 it has been recovering slowly. In Figure 3.5 we plot the time series for firm entry and exit as a fraction of the size of the labor force. It shows that firm entry also dropped in the Double-Dip Recession in the early 1980s, but it recovered much faster when compared to the Great Recession.

Therefore, we can conclude that the drop in the number of entering firms per labor force participant is the main force driving both the drop in the number of firms per labor force participant during the Great Recession and its slow recovery thereafter.

3.3 Counterfactual: quantifying the impact of lack of firm entry on the slow recovery of employment

In the previous section we could see that the period following the Great Recession is also characterized by the slow recovery in the number of entering firms per labor force participant. In this section we will link the slow recovery of firm entry to the slow recovery of employment per labor force participant, a subject of much debate in the recent years (Elsby et al. (2011), Jaimovich and Siu (2014), and Haltiwanger, Jarmin and Miranda (2013)).

Despite the fact that on average younger firms have fewer employees and face lower survival rates (see Table I), job creation from young firms, specially startups (age 0), is very important for the net job creation in the economy (Decker et al. (2014)). Conditional on survival, young firms show on average much higher growth rates than the more established firms (Haltiwanger et al. (2012) and Decker et al. (2014)).

Table I: Survival and growth rates of young firms

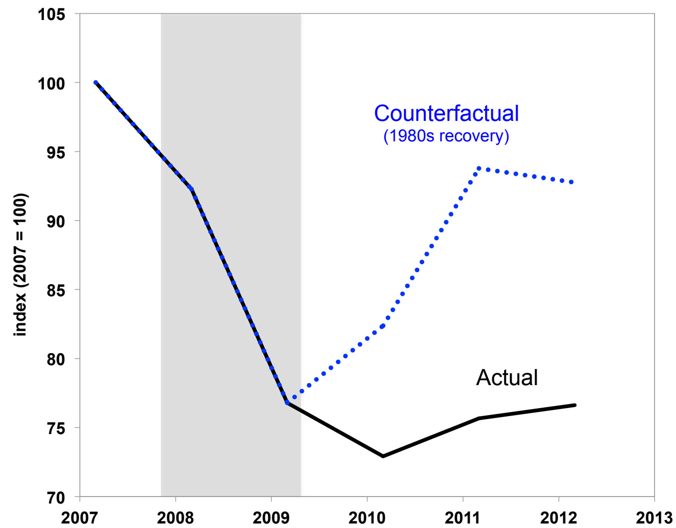
firm age	average employment per firm			exit rate (%)	
(years)	2010	2011	2012	2010-2011	2011-2012
0	6.3	5.8	5.8	25.7	23.0
1		7.5	7.3		11.8
2			8.5		

Sources: U.S. Census *Business Dynamics Statistics*

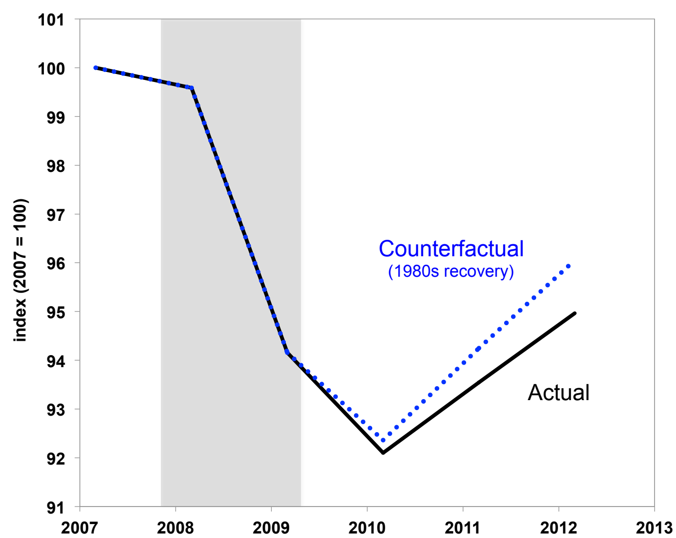
Therefore, in this section we quantify the impact of the slow recovery of firm entry on the slow recovery of employment by doing a counterfactual, where we calculate how the U.S. economy would have recovered after the Great Recession if the number of entering firms per labor force participant had recovered as it did after the Double-Dip Recession. As discussed above, since the Double-Dip Recession is the most comparable to the Great Recession in terms of magnitude, we use the recovery of the number of entering firms per labor force participant after the Double Dip Recession to discipline our analysis.

Figure 3.6: Counterfactual

(a) number of entering firms per labor force participant



(b) employment per labor force participant



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

Figures 3.6a and 3.6b illustrate our exercise. Let $entry_t^{CF}$ denote the number of entering firms in period t, where we use subscripts CF and AC to denote the counterfactual series and the actual series, respectively, and let LF_t denote the number of labor force participants in t. We define:

$$\begin{aligned} entry\,_t^{CF}/LF_t &= entry\,_t^{AC}/LF_t\ , \qquad\qquad t = 2007, 2008, 2009 \\ \frac{entry\,_{2009+i}^{CF}/LF_{2009+i}}{entry\,_{2007}^{AC}/LF_{2007}} &= \frac{entry\,_{1982+i}^{AC}/LF_{1982+i}}{entry\,_{1979}^{AC}/LF_{1979}}\ , \qquad i = 1, 2, 3 \end{aligned}$$

For the years from 2007 to 2009, we choose the counterfactual series $entry\,_t^{CF}/LF_t$ to be equal to the series that we actually observe in the data. Since our focus is on the recovery of firm entry, we decided to take the drop as given. For the periods after 2009, we assume that the number of entering firms per labor force participant recovers as it did in the early 1980s, using the pre-recession values in 1979 and 2007 as reference.

Next, we calculate the counterfactual series of employment that would result from this new series of firm entry. In order to do that, we need to take into account the differences in growth rates and survival rates between firms of different age profiles in different periods. We use the inputs in Table I. For example, the average number of employees of an entrant firm (age 0) was 6.3 in 2010. Between 2010 and 2011, 25.7% of these entrant firms exited, so the survival rate of firms of age 0 in 2010 was 74.3%.

Let $l(x,t)$ denote the average number of employees of a firm that is x years old in period t. In our example we used $l(0, 2010) = 6.3$. Let $s(x,t)$ be the cumulative survival rate that a firm of age x in t faced between $t-1$ and t. In our example, we used $s(1, 2011) = 0.743$. We define $s(0,t) = 1$. The counterfactual series of employment per labor force participant is given by,

$$\begin{aligned} emp_t^{CF}/LF_t &= emp_t^{AC}/LF_t\ , \qquad\qquad t = 2007, 2008, 2009 \\ emp_t^{CF}/LF_t &= emp_t^{AC}/LF_t + \\ &\quad \sum_{i=0}^{t-2010} (entry_{t-i}^{CF} - entry_{t-i}^{AC}) \frac{l(i,t)}{LF_t} \prod_{j=0}^{i} s(i-j, t-j) \end{aligned}$$

for $t = 2010, ..., 2012$.

Note that we are assuming that the firms that did not enter would behave exactly as the ones that did enter. Figure 3.6b shows the counterfactual series for employment. In 2010, the lack of firm entry could explain only 3 percent of the difference between the actual value of the employment per labor force participant and its pre-recession level. However, by 2013, it could explain 22 percent of the difference. The reason for the divergence between the two series is exactly the cumulative effect of firm entry. For example, in 2012, besides taking into account the employment level of entering firms, we also need to account for the employment levels of the previous cohorts (2010 and 2011), adjusted by the survival and growth rates according to their respective age profiles.

The facts presented above lead us to analyze how a simple model of firm dynamics is capable of explaining the behavior of firm entry in the business cycle.

3.4 Model

We extend the industry equilibrium framework of Hopenhayn (1992) by adding aggregate uncertainty in productivity and aggregate uncertainty in the marginal rate of substitution between consumption and labor. We add aggregate uncertainty in productivity because we want to first study what happens to firm entry when there is a negative supply shock, which is represented by the aggregate productivity shock. We add aggregate uncertainty in the marginal rate of substitution between consumption and labor because we want to study what happens to firm entry when there is a negative demand shock, which is represented by the negative preference shock affecting the marginal rate of substitution between consumption and labor.

Firm's Problem

Upon entry, firms draw idiosyncratic productivity s from a distribution $G(s)$ after paying sunk entry cost c_e, in units of labor. After that, idiosyncratic productivity shocks s follow a log $AR(1)$ process:

$$\log s_{t+1} = \rho_s \log s_t + \epsilon^s_{t+1}, \quad \epsilon \sim N(0, \sigma^2_{\epsilon^s})$$

A firm is then characterized by its idiosyncratic productivity s. Let Ω be the distribution of firms. The aggregate state of the economy is given by aggregate productivity Z^A, aggregate preference shock Z^D, and the distribution of firms Ω, over s. Let $S = (Z^S, Z^D, \Omega)$ denote the aggregate state of the economy.

A firm maximizes the expected discounted value of profits, which are then passed on to households who own the firms. In our setting, this is equivalent to the firm facing a sequence of static problems. Given a decreasing returns to scale technology, the firm chooses labor in order to maximize current profits. The current profits are given by,

$$\pi(s,S) = \max_{l_f(s,S)} \quad sZ^A l_f(s,S)^\theta - w(S) l_f(s,S)$$

where $0 < \theta < 1$.

Firms die exogenously with probability η, where $0 < \eta < 1$. The value of a firm with idiosyncratic productivity s is given by

$$V^f(s,S) = \pi(s,S) + \beta(1-\eta) E_{S'} m(S') V(s', S')$$

where $m(S') = \frac{U_c(S')}{U_c(S)}$ is the stochastic discounting factor of the representative household.

Household Problem

A representative household faces a sequence of static problems where it chooses consumption and leisure, given Z^D, $w(S)$, and $\Pi(S)$.

$$\begin{gathered} \max_{C(S),L(S)} \quad U(C(S), 1-L(S); Z^D) \\ s.t. \\ C(S) = w(S)L(S) + \Pi(S) \\ C(S) \geq 0;\ L(S) \in [0.1] \end{gathered}$$

Recursive Competitive Equilibrium

Given initial aggregate state (Z_0^A, Z_0^D, Ω_0), an *equilibrium* is wage function $w(S)$, mass of entrants function $\mu(S)$, value functions for the firm $V^f(s,S)$, policy functions for the household $C(S)$, $L(S)$ and for the firms $l_f(s,S)$ such that

- given $w(S)$, the policy functions $C(S)$, $L(S)$ solve the household problem;
- given $w(S)$, $V^f(s,S)$, the policy function $l_f(s,S)$ solves the firm's problem;
- the zero-profit condition holds

$$\int V^f(s,S) G(s) ds = w(S) c_e;$$

- markets clear,

$$C(S) = \int sZ^A l_f(s,S)^\theta(\Omega(s) + \mu(S))ds$$
$$L(S) = \int l_f(s,S)(\Omega(s) + \mu(S)G(s))ds + \mu(S)c_e$$

- the distribution of firms Ω evolves according to

$$\Omega'(B) = (1-\eta) \int_{1\{s'\in B\}} \int f(s,s')(\Omega(s) + \mu(S)G(s))dsds'.$$

for all $B \subset S$.

Remarks: We assume that firms die exogenously with probability η mainly because we are focusing on lack of firm entry. Exit in this model can be endogenized by adding a fixed operating cost as in Hopenhayn (1992). While the assumption of exogenous exit doesn't drive our results, it reduces the computational burden of solving for equilibrium (see Appendix).

Quantitative Analysis

For the functional form of the utility function, we choose

$$U(c, 1-l; Z^D) = Z^D \log c + \psi \log(1-l).$$

Note that in this case Z^D works as a labor wedge (Chari et al. (2008)). Aggregate preference and productivity shocks follow log $AR(1)$ processes,

$$\begin{aligned} \log Z^A_{t+1} &= \rho_A \log Z^A_t + \epsilon^A_{t+1}, \\ \log Z^D_{t+1} &= \rho_D \log Z^D_t + \epsilon^D_{t+1}, \end{aligned}$$

where $\epsilon^A_{t+1} \sim N(0, \sigma^2_{\epsilon^A})$ and $\epsilon^D_{t+1} \sim N(0, \sigma^2_{\epsilon^D})$.

The labor preference parameter ψ is chosen such that the Frisch elasticity of labor with respect to the wage rate is 2.65. This is in the range used in the macro literature (Rogerson and Wallenius, 2009). The death rate η is chosen to be .08, which is the average exit rate of firms in the data (1977 to 2007). We set $c_e = 0.11$ so that entrants' share of aggregate employment is equal to 3%. The rest of the parameters are standard.

Table II: Parameter values

labor share	θ	0.64
discount factor	β	0.96
death rate	η	0.08
cost of entry	c_e	1.36
preference for leisure	ψ	1.75
idiosyncratic shock persistence	ρ_s	0.70
idiosyncratic shock standard deviation	σ_{ϵ^s}	0.22
persistence of aggregate stochastic processes	$\rho_A = \rho_D$	0.80
standard deviation of aggregate stochastic processes	$\sigma_{\epsilon^A} = \sigma_{\epsilon^D}$	0.01

In these kinds of models where we have firm heterogeneity and aggregate uncertainty, we have to keep track of the firm distribution to solve for prices. However, that is an object with infinitely many dimensions. This leads to an algorithm similar to that used in Krussell and Smith (1998), Khan and Thomas (2003), and Clementi and Palazzo (2014). The algorithm is discussed in more detail in the Appendix.

In Figure 3.7 we show the impulse response functions resulting from a 4 percent drop in aggregate productivity. The drop in aggregate productivity leads to a similar drop in wages, which leads to an increase in firm entry. As can be seen in 3.7 , output falls by approximately 4 percent and firm entry increases by almost 8 percent. We can also observe that firm entry recovers quickly after the recession. Since entry is a flow, it makes sense that it falls back to a level below trend after the recession so that the mass of firms in the economy also recovers back to trend. Therefore, we could see that in this simple model of firm dynamics, a negative productivity shock actually generates an increase in firm entry, contrary to what we observed in Great Recession.

In Figure 3.8 we show the impulse response functions resulting from a 4 percent drop in Z^D. The negative demand shock leads to a small increase in wages, which leads to a drop in firm entry. As can be seen in 3.7 , output falls by approximately 1 percent and firm entry falls by almost 20 percent. We can also observe that firm entry recovers quickly after the recession. Therefore, we could see that in this simple model of firm dynamics, a negative demand shock generates a drop in firm entry similar to the one observed in the Great Recession. However, it does not generate persistence. After three periods, firm entry is already above its steady-state level.

This leads us to the next section, where we rule out possible hypotheses that have been often suggested in the literature. We do it based on empirical evidence .

Figure 3.7: Impulse responses of a 4 percent drop in aggregate productivity

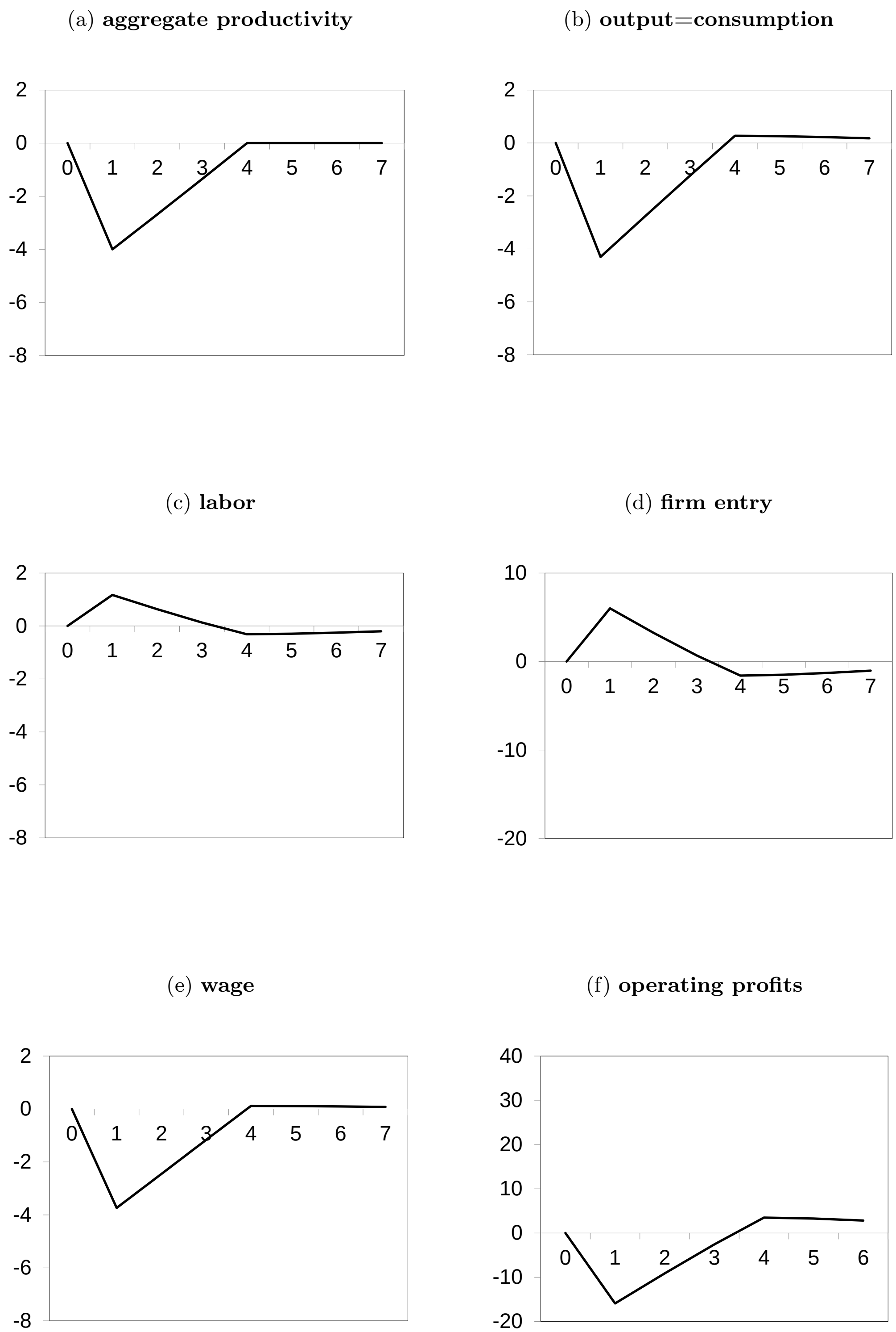
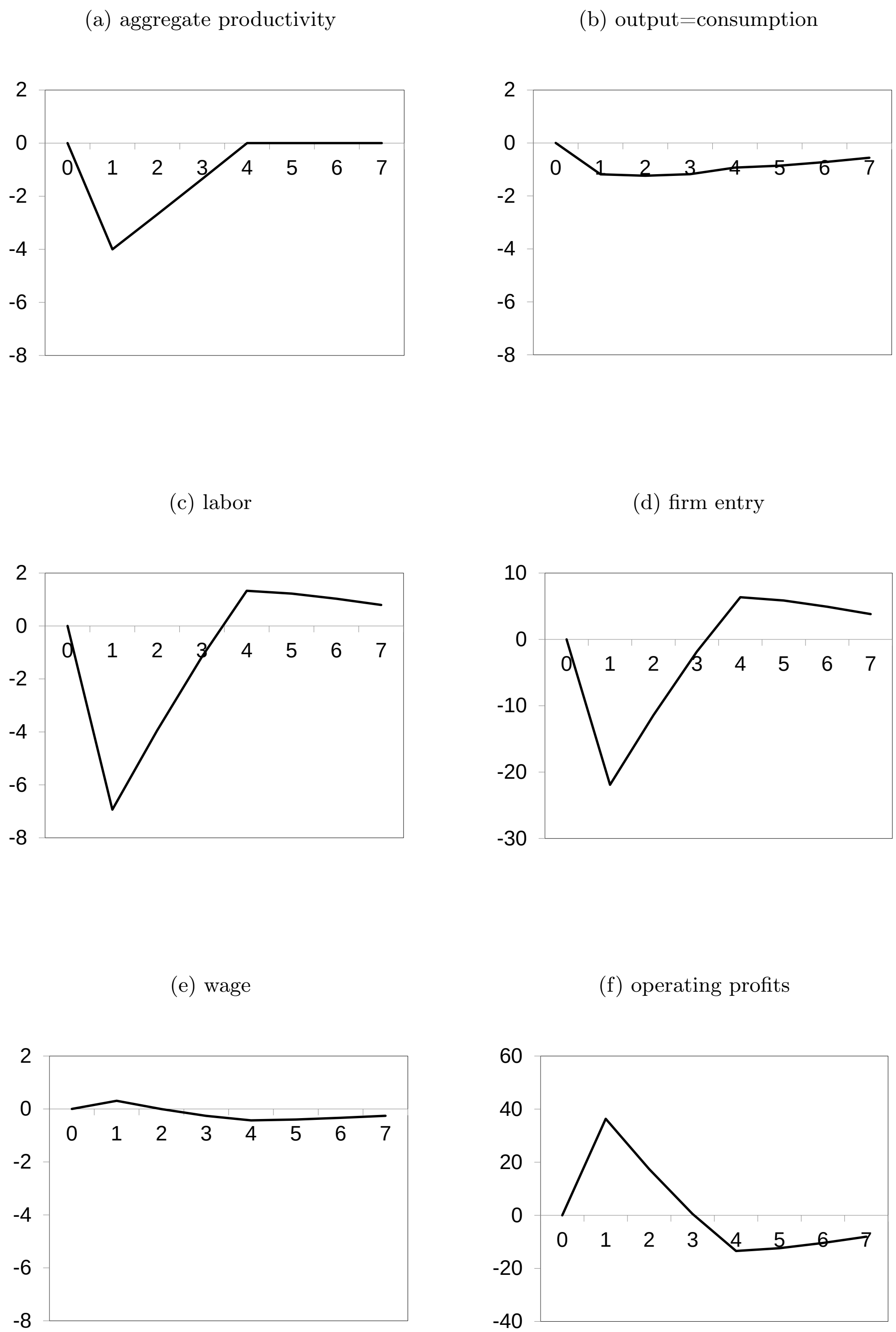


Figure 3.8: Impulse responses of a 4 percent negative demand shock



3.5 Assessing Alternative Hypotheses

Financial Constraints

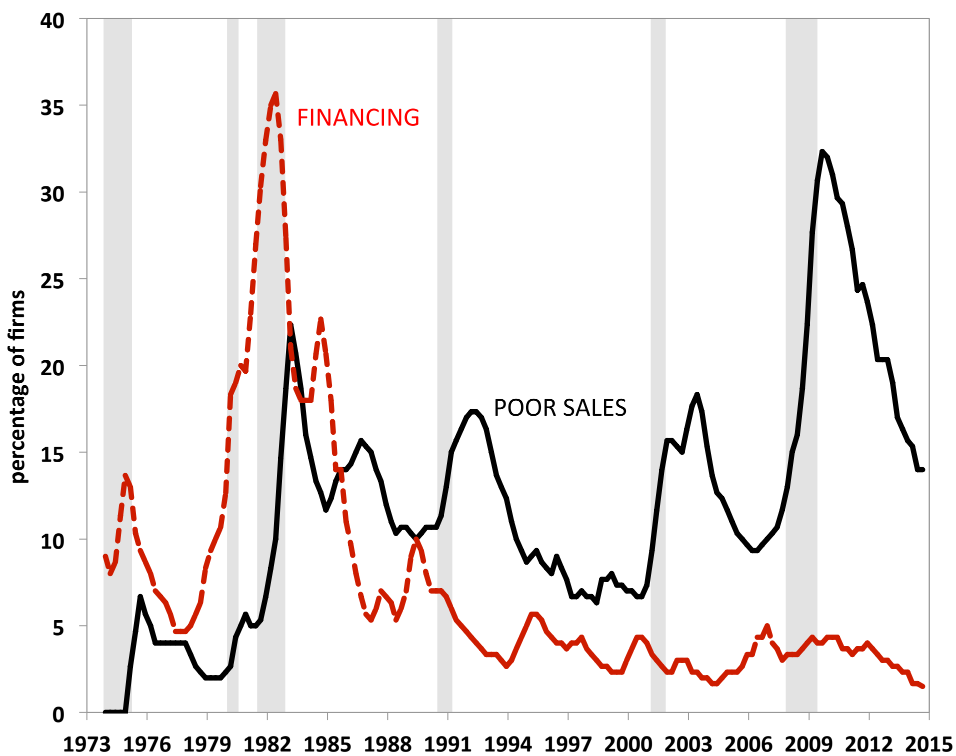
Given the financial aspect of the Great Recession, many models that try to account for it rely on financial frictions. Following this line, Mehrotra and Sergeyev (2015) and Siemer (2014), both featuring firm dynamics, model the crisis as financial shocks. They focus on the financial needs of young firms, that borrow from commercial banks in order to finance their initial investment (Robb and Robinson (2014)). Mehrotra and Sergeyev (2015) consider that young firms use real state as collateral in order to finance investment, so the drop in housing prices observed in the data could represent a tightening of the borrowing constraint. Siemer (2014), on the other hand, explain the slow recovery in the number of entering firms as the result of a credit crunch that followed the crisis, which reduced bank lending to new business.

In order to assess the financial constraint channel, we use the survey conducted by the National Federation of Independent Business, the Small Business Economic Trends. In this survey, small business owners are asked what is the single most important problem they are facing. The alternatives are: taxes, inflation, poor sales, financing and interest rates, cost of labor, government regulation, competition from large businesses, quality of labor, cost of insurance, and others.

In Figure 3.9 we plot the time series for two of the alternatives: financing and interest rates, and poor sales.[5] Despite the fact that financing seemed to be a major issue during the Double-Dip Recession, it does not show a similar pattern in the recent crisis. We take this result as evidence that the financial constraint channel, at least in the way it has been proposed so far, is not the main driver of the slow recovery in firm entry.

[5] These are the series that show higher cyclicality during recessions.

Figure 3.9: Single most important problem



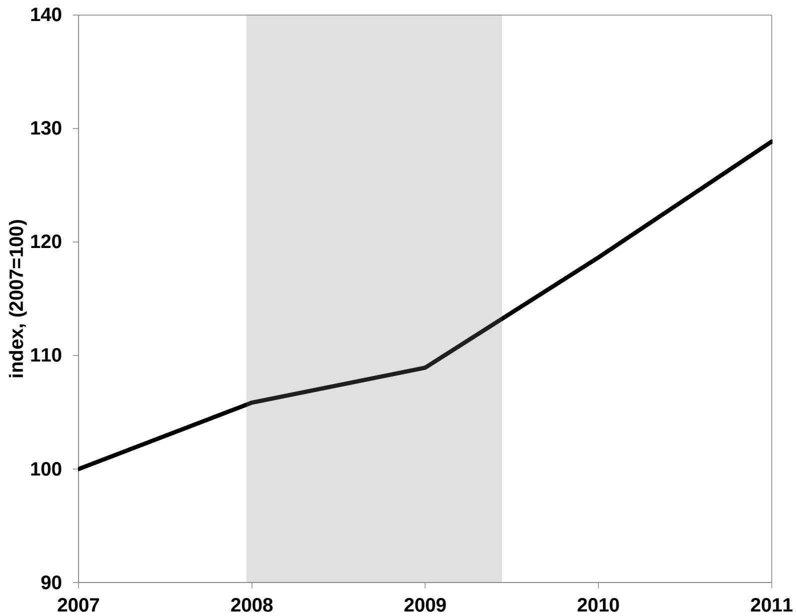
Sources: National Federation of Independent Business (NFIB) *Small Business Economic Trends.*

Openness and Offshoring

Since the 1980s, the U.S. economy has become more open and off shoring of jobs by companies operating in the U.S. has increased. This is a factor that has been popular in explaining jobless recoveries (e.g., Waddle (2013)). We question if off shoring and increased openness is contributing to the slow recovery of firm entry. We consider two mechanisms through which offshoring and increased openness might contribute to the slow recovery of firm entry.

First, we consider a direct mechanism, where we would observe less foreign firms entering the U.S. market. However, in Figure 3.10, we plot the number of tax returns filed by foreign corporations operating in the U.S. which shows that it has continued to increase since 2007.

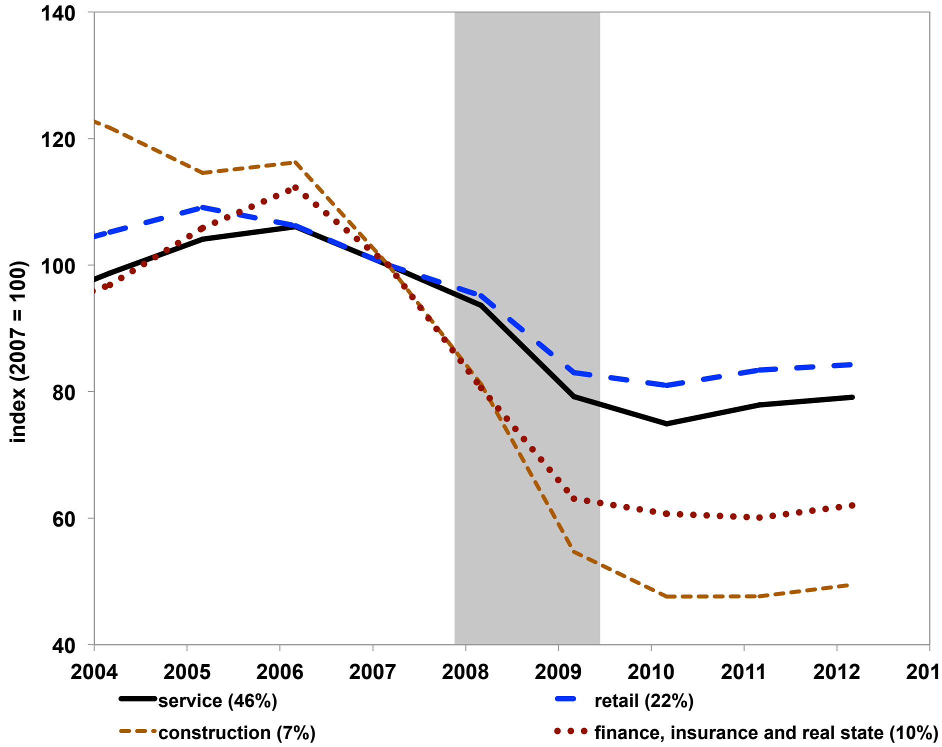
Figure 3.10: Number of tax returns filed by foreign corporations in the U.S.



Sources: Internal Revenue Services.

Second, we consider an indirect mechanism, where large firms in the U.S. substituting inputs from domestic firms by foreign inputs. In this case, the lower demand for domestic inputs might reduce the incentives of new firms to enter the market. However, we observe lack of firm entry in all sectors, including sectors which are highly nontradable (e.g., construction and retail services). This can be seen in Figure 3.11, where we plot the number of entering firms per labor force participants for the sectors that account for most of entering firms in the economy: service; construction; retail; finance, insurance and real estate. They account for 46%, 7%, 22%, 10% of entering firms, respectively, and together they account for 85% of firm entry.

Figure 3.11: Number of entering firms per labor force participant, by sector



Sources: U.S. Census *Business Dynamics Statistics* and Bureau of Labor Statistics *Current Population Survey*.

Uncertainty at the firm level

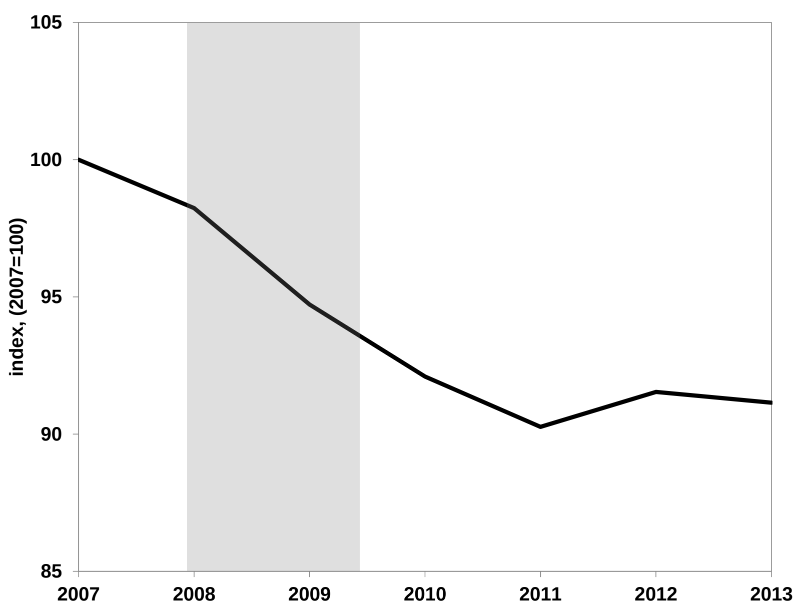
Suppose firms face idiosyncratic time varying productivity shocks as in Hopenhayn (1992). Bloom et al. (2011) study manufacturing establishments for the U.S. economy and show that the variance of idiosyncratic shocks increases during recessions. The literature refers to it as increased uncertainty at the firm level. Arellano et al. (2012) argue that increased uncertainty along with labor adjustment costs and financial frictions can generate a significant decline in output and labor, but not in labor productivity, similar to what was observed in the Great Recession.

However, in Bloom et al. (2012), we can see that in the Doble-Dip Recession in the 1980s, uncertainty increased to approximately 85 percent of the level observed during the Great Recession. Therefore, an explanation to the slow recovery in firm entry that relies on increased uncertainty at the firm level must account for the Double-Dip Recession in the 1980s, when firm entry recovered relatively quick. It is then a challenge to explain why increased uncertainty would generate a slow recovery in the recent recession as compared to the Double-Dip Recession in the 1980s, unless there was some structural change that complements the increased uncertainty.

Self employed

In the data we use, BDS, self-employment is not included. So it might be the case that more people are becoming self-employed, which might explain the drop in the number of new employer firms and its slow recovery. However, Figure 3.12 shows that the recovery in the number of self-employed after the Great Recession has been slow, which contradicts the hypothesis.

Figure 3.12: Number of self-employed in the U.S.



Sources: Bureau of Labor Statistics *Current Population Survey*

3.6 Conclusion

Besides the slow recovery of output and employment, we showed that lack of firm entry is another feature of the Great Recession and its subsequent years. We have shown that the number of firms per labor force participant dropped significantly during the Great Recession and has been recovering slowly ever since, and that lack of firm entry is the main force driving it.

We quantified the effect of the lack of firm entry on the slow recovery of employment, where we showed that it accounts for 22 percent of the lack of employment by 2012.

We then investigate how firm entry reacts to negative supply and demand shocks in a simple firm dynamics model. The supply shock does not generate a drop in firm entry, while the demand shock does. The latter causes a significant drop in firm entry, similar to the one observed during the Great Recession. However, the demand shock alone does not generate a slow recovery.

Finally, we showed how empirical evidence contradicts common explanations for the slow recovery. These explanations include financial constraints, offshoring, increased uncertainty at the firm level and transfers to self-employment.

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Appendix A

Appendix to Chapter 1

A.1 Model of international knowledge flows through multinational corporations

In this section, I solve the planner's problem with respect to the deterministic model presented in Section (1.3). Let ψ, $0 < \psi < 1$, denote the weight given to households of Country 1. The planner chooses the sequence fo allocations $(c_{i,t}, n_{i,t}, a_{i,t}, b_{i,t}, n^{y}_{ii,t}, n^{y}_{ij,t}, n^{m}_{ii,t}, k^{y}_{ii,t+1}, k^{y}_{ij,t+1}, k^{m}_{ii,t+1}, m_{i,t+1})$ for $i, j = 1, 2$, $j \neq i$, in order to solve the following problem:

$$\max \psi \sum_{t=0}^{\infty} \beta^t U(c_{1,t}, n_{1,t}) + (1-\psi) \sum_{t=0}^{\infty} \beta^t U(c_{2,t}, n_{2,t}) \tag{A.1}$$

subject to the following constraints:[1]

$$
\begin{aligned}
(\lambda_{1,t}) &: \quad c_{1,t} + k^y_{11,t+1} + k^y_{21,t+1} + k^m_{11,t+1} = G_1(a_{1,t}, b_{1,t}) + (1-\delta^k)(k^y_{11,t} + k^y_{21,t} + k^m_{11,t}),\\
(\lambda_{2,t}) &: \quad c_{2,t} + k^y_{22,t+1} + k^y_{12,t+1} + k^m_{22,t+1} = G_2(a_{2,t}, b_{2,t}) + (1-\delta^k)(k^y_{22,t} + k^y_{12,t} + k^m_{22,t}),\\
(\lambda_{3,t}) &: \quad a_{1,t} + a_{2,t} = z_{1,t} F(k^y_{11,t}, m_{1,t}, n^y_{11,t}) + z_{2,t} F(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}),\\
(\lambda_{4,t}) &: \quad b_{1,t} + b_{2,t} = z_{1,t} F(k^y_{21,t}, \theta m_{2,t}, n^y_{21,t}) + z_{2,t} F(k^y_{22,t}, m_{2,t}, n^y_{22,t}),\\
(\lambda_{5,t}) &: \quad m_{1,t+1} = z_{1,t} F(k^m_{11,t}, m_{1,t}, n^m_{11,t}) + (1-\delta^m) m_{1,t},\\
(\lambda_{6,t}) &: \quad m_{2,t+1} = z_{2,t} F(k^m_{22,t}, m_{2,t}, n^m_{22,t}) + (1-\delta^m) m_{2,t},\\
(\lambda_{7,t}) &: \quad n_{1,t} = n^y_{11,t} + n^y_{21,t} + n^m_{11,t},\\
(\lambda_{8,t}) &: \quad n_{2,t} = n^y_{22,t} + n^y_{12,t} + n^m_{22,t}.
\end{aligned}
$$

The terms $\lambda_{i,t}$ denote the respective Lagrange multiplier of each constraint. The first-order conditions are:

$$(c_{1,t}) \quad : \quad \beta^t \psi U_c(c_{1,t}, n_{1,t}) = \lambda_{1,t} \tag{A.2}$$

$$(c_{2,t}) \quad : \quad \beta^t (1-\psi) U_c(c_{1,t}, n_{1,t}) = \lambda_{2,t} \tag{A.3}$$

$$(n_{1,t}) \quad : \quad -\beta^t \psi U_n(c_{1,t}, n_{1,t}) = \lambda_{7,t} \tag{A.4}$$

$$(n_{2,t}) \quad : \quad -\beta^t (1-\psi) U_n(c_{1,t}, n_{1,t}) = \lambda_{8,t} \tag{A.5}$$

$$(a_{1,t}) \quad : \quad \lambda_{1,t} G_{1a}\left(a_{1,t}, b_{1,t}\right) = \lambda_{3,t} \tag{A.6}$$

$$(a_{2,t}) \quad : \quad \lambda_{2,t} G_{2a}\left(a_{2,t}, b_{2,t}\right) = \lambda_{3,t} \tag{A.7}$$

$$(b_{1,t}) \quad : \quad \lambda_{1,t} G_{1b}\left(a_{1,t}, b_{1,t}\right) = \lambda_{4,t} \tag{A.8}$$

$$(b_{2,t}) \quad : \quad \lambda_{2,t} G_{2b}\left(a_{2,t}, b_{2,t}\right) = \lambda_{4,t} \tag{A.9}$$

[1] And also subject to the non-negativity constraints.

$$(n^y_{11,t}) \quad : \quad \lambda_{3,t} z_{1,t} F_n(k^y_{11,t}, m_{1,t}, n^y_{11,t}) = \lambda_{7,t} \tag{A.10}$$

$$(n^y_{12,t}) \quad : \quad \lambda_{3,t} z_{2,t} F_n(k^y_{12,t}, \theta m_{1,t}, n^y_{12,t}) = \lambda_{8,t} \tag{A.11}$$

$$(n^y_{21,t}) \quad : \quad \lambda_{4,t} z_{1,t} F_n(k^y_{21,t}, \theta m_{2,t}, n^y_{21,t}) = \lambda_{7,t} \tag{A.12}$$

$$(n^y_{22,t}) \quad : \quad \lambda_{4,t} z_{2,t} F_n(k^y_{22,t}, m_{2,t}, n^y_{22,t}) = \lambda_{8,t} \tag{A.13}$$

$$(n^m_{11,t}) \quad : \quad \lambda_{5,t} z_{1,t} F_n(k^m_{11,t}, m_{1,t}, n^m_{11,t}) = \lambda_{7,t} \tag{A.14}$$

$$(n^m_{11,t}) \quad : \quad \lambda_{6,t} z_{2,t} F_n(k^m_{22,t}, m_{2,t}, n^m_{22,t}) = \lambda_{8,t} \tag{A.15}$$

$$(k^y_{11,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{3,t+1} z_{1,t+1} F_n(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) + \lambda_{1,t+1}(1-\delta^k) = 0 \tag{A.16}$$

$$(k^y_{12,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{3,t+1} z_{2,t+1} F_n(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) + \lambda_{2,t+1}(1-\delta^k) = 0 \tag{A.17}$$

$$(k^y_{21,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{4,t+1} z_{1,t+1} F_n(k^y_{21,t+1}, \theta m_{2,t+1}, n^y_{21,t+1}) + \lambda_{1,t+1}(1-\delta^k) = 0 \tag{A.18}$$

$$(k^y_{22,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{4,t+1} z_{2,t+1} F_n(k^y_{22,t+1}, m_{2,t+1}, n^y_{22,t+1}) + \lambda_{2,t+1}(1-\delta^k) = 0 \tag{A.19}$$

$$(k^m_{11,t+1}) \quad : \quad -\lambda_{1,t} + \lambda_{5,t+1} z_{1,t+1} F_n(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + \lambda_{1,t+1}(1-\delta^m) = 0 \tag{A.20}$$

$$(k^m_{11,t+1}) \quad : \quad -\lambda_{2,t} + \lambda_{6,t+1} z_{2,t+1} F_n(k^m_{22,t+1}, m_{2,t+1}, n^m_{22,t+1}) + \lambda_{2,t+1}(1-\delta^m) = 0 \tag{A.21}$$

$$(m_{1,t+1}) \quad : \quad \begin{array}{c} -\lambda_{5,t} + \lambda_{3,t+1} z_{1,t+1} F_m(k^y_{11,t+1}, m_{1,t+1}, n^y_{11,t+1}) \\ +\lambda_{3,t+1} \theta z_{2,t+1} F_m(k^y_{12,t+1}, \theta m_{1,t+1}, n^y_{12,t+1}) \\ +\lambda_{5,t+1} z_{1,t+1} F_m(k^m_{11,t+1}, m_{1,t+1}, n^m_{11,t+1}) + \lambda_{5,t+1}(1-\delta^m) \end{array} = 0 \tag{A.22}$$

$$(m_{2,t+1}) \quad : \quad \begin{array}{c} -\lambda_{6,t} + \lambda_{4,t+1} z_{2,t+1} F_m(k^y_{22,t+1}, m_{2,t+1}, n^y_{22,t+1}) \\ +\lambda_{4,t+1} \theta z_{2,t+1} F_m(k^y_{21,t+1}, \theta m_{2,t+1}, n^y_{21,t+1}) \\ +\lambda_{6,t+1} z_{2,t+1} F_m(k^m_{22,t+1}, m_{2,t+1}, n^m_{22,t+1}) + \lambda_{6,t+1}(1-\delta^m) \end{array} = 0 \tag{A.23}$$

Together with the transversality conditions.

A.1.1 Relation to competitive equilibrium

The mapping from competitive prices used in Section (1.3) to the Lagrange multipliers is the following:[2]

[2] I do not provide a full proof of the equivalence between the competitive equilibrium allocation and the solution to the planner's problem.

$$w_{1,t} = \frac{\lambda_{7,t}}{\lambda_{1,t}}, \quad q_{a,t} = \frac{\lambda_{3,t}}{\lambda_{1,t}}, \quad p^m_{1,t} = \frac{\lambda_{5,t}}{\lambda_{1,t}}, \quad p_{2,t} = \frac{\lambda_{2,t}}{\lambda_{1,t}},$$

$$w_{2,t} = \frac{\lambda_{8,t}}{\lambda_{1,t}}, \quad q_{b,t} = \frac{\lambda_{4,t}}{\lambda_{1,t}}, \quad p^m_{2,t} = \frac{\lambda_{6,t}}{\lambda_{1,t}}.$$

A.1.2 Symmetric Steady State

I am solving the planner's problem in which the planner gives equal weight to households of both countries. In this case, the steady state equilibrium is symmetric. The following conditions must hold:[3]

$$\frac{U_n(c_1, n_1)}{U_c(c_1, n_1)} = w, \tag{A.24}$$

$$G_{1a}(a_1, b_1) = q, \tag{A.25}$$

$$G_{1b}(a_1, b_1) = q, \tag{A.26}$$

$$qF_n\left(\frac{k^y_{11}}{m_1}, 1, \frac{n^y_{11}}{m_1}\right) = w, \tag{A.27}$$

$$qF_n\left(\frac{k^y_{12}}{m_1}, \theta, \frac{n^y_{12}}{m_1}\right) = w, \tag{A.28}$$

$$p^m F_n\left(\frac{k^m_{11}}{m_1}, 1, \frac{n^m_{12}}{m_1}\right) = w, \tag{A.29}$$

$$qF_k\left(\frac{k^y_{11}}{m_1}, 1, \frac{n^y_{11}}{m_1}\right) = \frac{1 - \beta(1 - \delta^k)}{\beta}, \tag{A.30}$$

$$qF_k\left(\frac{k^y_{12}}{m_1}, \theta, \frac{n^y_{12}}{m_1}\right) = \frac{1 - \beta(1 - \delta^k)}{\beta}, \tag{A.31}$$

$$p^m F_k\left(\frac{k^m_{11}}{m_1}, 1, \frac{n^m_{12}}{m_1}\right) = \frac{1 - \beta(1 - \delta^k)}{\beta}, \tag{A.32}$$

[3] I suppress the subscript t for steady-state variables, and use the fact that $z_{1,t} = z_{2,t} = 1$.

$$\frac{q}{p^m}\left[F_m\left(\frac{k^y_{11}}{m_1},1,\frac{n^y_{11}}{m_1}\right)+\theta F_m\left(\frac{k^y_{12}}{m_1},\theta,\frac{n^y_{12}}{m_1}\right)\right]+F_m\left(\frac{k^m_{11}}{m_1},1,\frac{n^m_{12}}{m_1}\right)=\frac{1-\beta(1-\delta^m)}{\beta}, \quad \text{(A.33)}$$

$$c_1+\delta^k\left(k^y_{11}+k^y_{12}+k^m_{11}\right)=G_1\left(a_1,b_1\right), \quad \text{(A.34)}$$

$$n^y_{11}+n^y_{12}+n^m_{11}=n_1, \quad \text{(A.35)}$$

$$a_1+b_1=m_1\left[F\left(\frac{k^y_{11}}{m_1},1,\frac{n^y_{11}}{m_1}\right)+F\left(\frac{k^y_{12}}{m_1},\theta,\frac{n^y_{12}}{m_1}\right)\right], \quad \text{(A.36)}$$

$$\delta^m=F\left(\frac{k^m_{11}}{m_1},1,\frac{n^m_{11}}{m_1}\right), \quad \text{(A.37)}$$

where $q=q_a=q_b$, $w=w_1=w_2$, $p^m=p^m_1=p^m_2$, and I used the fact that $n^y_{12}=n^y_{21}$, $k^y_{12}=k^y_{21}$, and $a_2=b_1$. Above we have a system of 14 equations and 14 variables $(c_1,n_1,a_1,b_1,n^y_{11},n^y_{12},n^m_{11},k^y_{11},k^y_{12},k^m_{11},m_1,w,q,p^m)$.

Next, I use the following functional forms to compute the steady state:

$$\begin{aligned} G_1(a,b) &= \left(\omega a^{\frac{\sigma-1}{\sigma}}+(1-\omega)b^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \\ F(k,m,n) &= \left(k^\alpha n^{1-\alpha}\right)^{1-\phi}m^\phi, \\ U(c,n) &= \left(c^\mu\left(1-n\right)^{1-\gamma}\right)^\gamma/\gamma. \end{aligned}$$

Under these functional forms, the steady-state can be computed as follows:

Step 1: Prices

$$q = \left(\omega^\sigma+(1-\omega)^\sigma\right)^{\frac{1}{\sigma-1}}, \quad \text{(A.38)}$$

$$p^m = q\frac{\delta^m}{\kappa_1}\left(\kappa_2\kappa_3\right)^{-\alpha(1-\phi)}\left((1-\alpha)(1-\phi)\right)^{\alpha(1-\phi)}, \quad \text{(A.39)}$$

$$w = \left(p^m\right)^{\frac{1}{1-\alpha}}\left(\delta^m\right)^{\frac{-\phi}{(1-\alpha)(1-\phi)}}\left(\kappa_3\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)(1-\phi), \quad \text{(A.40)}$$

$$\kappa_1 \equiv \frac{1-\beta(1-\delta^m)-\delta^m\phi\beta}{\phi\beta(1+\theta)} \quad \text{(A.41)}$$

$$\kappa_2 \equiv \frac{1-\beta\left(1-\delta^k\right)}{\beta}\frac{1-\alpha}{\alpha} \quad \text{(A.42)}$$

$$\kappa_3 \equiv \frac{\beta\alpha\left(1-\phi\right)}{1-\beta\left(1-\delta^k\right)} \quad \text{(A.43)}$$

Step 2: Ratios

$$\frac{n_{11}^m}{m_1} = \frac{p^m}{w}(1-\alpha)(1-\phi)\,\delta^m, \quad (A.44)$$

$$\frac{k_{11}^m}{m_1} = p^m\delta^m\kappa_3, \quad (A.45)$$

$$\frac{k_{11}^y}{m_1} = (w)^{-\frac{(1-\alpha)(1-\phi)}{\phi}}\left(q(1-\alpha)(1-\phi)\right)^{\frac{1}{\phi}}(\kappa_2)^{\frac{(1-\alpha)(1-\phi)-1}{\phi}}, \quad (A.46)$$

$$\frac{n_{11}^y}{m_1} = \kappa_2\frac{1}{w}\frac{k_{11}^y}{m_1}, \quad (A.47)$$

$$\frac{k_{12}^y}{m_1} = \theta\frac{k_{11}^y}{m_1}, \quad (A.48)$$

$$\frac{n_{12}^y}{m_1} = \theta\frac{n_{11}^y}{m_1}, \quad (A.49)$$

$$\frac{a_1}{m_1} = \left(1+\left(\frac{1-\omega}{\omega}\right)^{\sigma}\right)^{-1}(1+\theta)\left(\frac{k_{11}^y}{m_1}\right)^{\alpha(1-\phi)}\left(\frac{n_{11}^y}{m_1}\right)^{(1-\alpha)(1-\phi)}, \quad (A.50)$$

where all the ratios are with respect to the stock of knowledge m_1.

Step 3: I compute the stock of knowledge using the following equation

$$m_1 = w\frac{\mu}{1-\mu}\left[\begin{array}{c} \frac{a_1}{m_1}\left(\omega+(1-\omega)\left(\frac{1-\omega}{\omega}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \\ +\frac{\mu}{1-\mu}w\left((1+\theta)\frac{n_{11}^y}{m_1}+\frac{n_{11}^m}{m_1}\right)-\delta^k\left((1+\theta)\frac{k_{11}^y}{m_1}+\frac{k_{11}^m}{m_1}\right)\end{array}\right]^{-1}.$$

Step 4: Finally, I compute $(a_1, n_{11}^y, n_{12}^y, n_{11}^m, k_{11}^y, k_{12}^y, k_{11}^m)$ using the ratios, and (b_1, c_1, n_1) using the following relations:

$$n_1 = n_{11}^y + n_{12}^y + n_{11}^m, \quad (A.51)$$

$$c_1 = \frac{\mu}{1-\mu}w(1-n_1), \quad (A.52)$$

$$b_1 = \left(\frac{1-\omega}{\omega}\right)^{\sigma}a_1. \quad (A.53)$$

Steady state moments: Given the solution to the steady state equilibrium derived above, the following conditions hold:

$$
\begin{aligned}
\frac{w(n_{11}^y + n_{11}^m)}{qF(k_{11,t}^y, m_{1,t}, n_{11,t}^y)} &= (1-\alpha)(1-\phi)\left(1 + \frac{\delta^m \phi \beta (1+\theta)}{1-\beta+\beta\delta^m(1-\phi)}\right), \\
\frac{wn_{12}^y}{qF(k_{12,t}^y, \theta m_{1,t}, n_{12,t}^y)} &= (1-\alpha)(1-\phi), \\
\frac{\delta^k(n_{11}^y + n_{11}^m)}{w(n_{11}^y + n_{11}^m)} &= \frac{\delta^k \beta}{1-\beta(1-\delta^k)} \frac{\alpha}{1-\alpha}, \\
\frac{wn_{12}^y}{w(n_{11}^y + n_{11}^m)} &= \theta \frac{1-\beta(1-\delta^m) - \delta^m \phi \beta}{1-\beta(1-\delta^m) + \delta^m \phi \beta \theta}.
\end{aligned}
$$

Appendix B

Appendix to Chapter 2

B.1 Data

We used the official monthly series (end of period) for the nominal exchange rates, and the monthly Consumer Price Index (CPI) of each country for the price indices. These series were downloaded from Global Financial Data.

Regarding the commodity price series, most series are from the World Bank *Commodity Price Data* (Pink Sheet) and United Nations (UNCTAD*stat*). Below we list the data source for the price series of each commodity:

(1) Petroleum - Brent crude oil. Source: Global Financial Data, Ticker: BRT_ D.

(2) Fish - price of fish meal. Source: UNCTAD*stat*.

(3) Meat - price of beef. Source: World Bank *Commodity Price Data*.

(4) Aluminium - Source: World Bank *Commodity Price Data*.

(5) Copper: - Source: World Bank *Commodity Price Data*.

(6) Wheat - US, $n°1$, hard red winter. Source: World Bank *Commodity Price Data*.

(7) Maize - Source: World Bank *Commodity Price Data*.

(8) Timber - Logs, Malaysia. Source: World Bank *Commodity Price Data*.

(9) Cotton - Cotton Outlook A index. Source: World Bank *Commodity Price Data*.

(10) Sugar - International Sugar Agreement. Source: World Bank *Commodity Price Data*.

Finally, the trade data was obtained from the United Nations *Comtrade* Database. World trade (exports + imports) for each commodity and its total were computed as the sum of trade over all the countries in the dataset.

B.2 Multiple representations of the real exchange rate

In this section we show that the model admits many representations of the bilateral real exchange rate. All of these representations hold simultaneously in equilibrium and are derived by substituting different equilibrium conditions into the definition of the real exchange rate. To make the discussion as simple as possible, we develop our arguments using an economy with one non-traded final good, Y_t, one traded intermediate good, q_t, two commodities, $x_t(1)$ and $x_t(2)$, and one type of labor. In terms of the previous notation, $J = N = 1$ and $H = 2$. We use the symbol "$\sim$" to represent variables from an arbitrary foreign country.

The non-traded final good is produced with a Cobb-Douglas technology

$$Y_t = \bar{\alpha} Z_t^y \left(n_t^y\right)^{\alpha} \left(q_t\right)^{1-\alpha},$$

where $\bar{\alpha} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. The technology to produce intermediate goods is

$$Q_t = \bar{\beta} Z_t^q \left(n_t^q\right)^{\beta_n} x_t(1)^{\beta_1} x_t(2)^{\beta_2},$$

where $\bar{\beta} = \beta_n^{-\beta_n} \beta_1^{-\beta_1} \beta_2^{-\beta_2}$ and $\beta_n + \beta_1 + \beta_2 = 1$. Finally, commodity $h = 1, 2$ is produced with the production function

$$X_t(h) = Z_t^x(h) \left(n_t^x(h)\right)^{\eta_h} E(h)^{1-\eta_h}.$$

As mentioned above, we set $E(h) = 0$ if the economy is unable to produce commodity h.

Competitive markets imply that nominal prices equal marginal costs. In the final and intermediate goods sectors these conditions are, respectively,

$$P_t = (W_t)^{\alpha} (P_t^q)^{1-\alpha} / Z_t^y \tag{B.1}$$

$$P_t^q = (W_t)^{\beta_n} \left(P_t^x(1)\right)^{\beta_1} \left(P_t^x(2)\right)^{\beta_2} / Z_t^q, \tag{B.2}$$

where $P_t^x(h)$ denotes the nominal price of commodity $h = 1, 2$.

The two commodities and the intermediate good can be internationally traded, possibly with some frictions represented by trade taxes – these could be actual taxes or any other implicit impediment to trade that drives a wedge between home and foreign prices. If we let S_t denote the nominal exchange rate, defined as foreign currency per unit of domestic currency, the laws of one price in the intermediate goods and commodities sectors are respectively,

$$S_t P_t^x(1) T_t^x(1) = \tilde{P}_t^x(1) \quad \text{(B.3)}$$

$$S_t P_t^x(2) T_t^x(2) = \tilde{P}_t^x(2) \quad \text{(B.4)}$$

$$S_t P_t^q T_t^q = \tilde{P}_t^q, \quad \text{(B.5)}$$

where T_t^q and $T_t^x(h)$ for $h = 1, 2$ are (gross) taxes on foreign trade.

We now consider two representations of the bilateral real exchange rate in terms of commodity prices and nominal wages measured in a common currency. The two representations differ in whether we replace or not the law of one price in intermediate goods (B.5) into the price indices (B.1). In particular, introducing the intermediate good price index (B.2) into the final good price index (B.1) and taking logs gives[1]

$$p_t = (\alpha + (1-\alpha)\beta_n) w_t + (1-\alpha)\beta_1 p_t^x(1) + (1-\alpha)\beta_2 p_t^x(2) - (1-\alpha) z_t^q - z_t^y.$$

Using the equivalent expression for the foreign country and the laws of one price (B.3) and (B.4) we derive the first representation of the bilateral real exchange rate in terms of commodity prices and nominal wages,

$$\begin{aligned} p_t + s_t - \tilde{p}_t = {} & [(1-\alpha)\beta_1 - (1-\tilde{\alpha})\tilde{\beta}_1] p_t^x(1) + [(1-\alpha)\beta_2 - (1-\tilde{\alpha})\tilde{\beta}_2] p_t^x(2) \\ & [\alpha + (1-\alpha)\beta_n] w_t - [\tilde{\alpha} + (1-\tilde{\alpha})\tilde{\beta}_n](\tilde{w}_t - s_t) + \varepsilon_{1t}, \end{aligned} \quad \text{(B.6)}$$

where ε_{1t} is a bundle of productivity shocks and trade taxes defined as

$$\varepsilon_{1t} = \tilde{z}_t^y - z_t^y + (1-\tilde{\alpha})\tilde{z}_t^q - (1-\alpha) z_t^q - (1-\tilde{\alpha})[\tilde{\beta}_1 t_t^x(1) + \tilde{\beta}_2 t_t^x(2)].$$

Equation (B.6) is one representation of the real exchange rate in terms of commodity prices and wages. We can also use the law of one price (B.2) to obtain a similar representation of the real exchange rate. In particular, using equations (B.2), (B.3) and (B.4) the law of one price for intermediate goods (B.5) implies the following relation between home and foreign wages,

$$\beta_n w_t + (\beta_1 - \tilde{\beta}_1) p_t^x(1) + (\beta_2 - \tilde{\beta}_2) p_t^x(2) + t_t^q - z_t^q = \tilde{\beta}_n(\tilde{w}_t - s_t) + \tilde{\beta}_1 t_t^x(1) + \tilde{\beta}_2 t_t^x(2) - \tilde{z}_t^q.$$

[1] We use lowercase letters to denote the natural logarithm of the corresponding uppercase letters. In particular, $p_t = \log P_t$, $t_t^x(h) = \log T_t^x(h)$, and so forth.

Therefore, equation (B.6) can also be written as

$$\begin{aligned} p_t + s_t - \tilde{p}_t &= (\tilde{\alpha}\tilde{\beta}_1 - \alpha\beta_1)p_t^x(1) + (\tilde{\alpha}\tilde{\beta}_2 - \alpha\beta_2)p_t^x(2) \\ &\quad + \alpha(1-\beta_n)w_t - \tilde{\alpha}(1-\tilde{\beta}_n)(\tilde{w}_t - s_t) + \varepsilon_{2t}, \end{aligned} \tag{B.7}$$

where ε_{2t} is given by

$$\varepsilon_{2t} = \tilde{z}_t^y - z_t^y + \alpha z_t^q - \tilde{\alpha}\tilde{z}_t^q + \tilde{\alpha}\tilde{\beta}_1 t_t^x(1) + \tilde{\alpha}\tilde{\beta}_2 t_t^x(2) - t_t^q.$$

The two representations (B.6) and (B.7) emphasize the relation between the bilateral real exchange rate, nominal wages measured in a common currency, productivity shocks, and trade taxes. Yet, the interpretation of the parameters multiplying wages and commodities changes depending on what equation we use to represent the real exchange rate.

It is also possible to express the real exchange rate in terms of commodity prices, allocations, shocks, and trade taxes. Suppose first that the home and foreign countries produce the same commodity, let us say $x_t(1)$. The first order conditions for the optimal choice of labor in the commodity sector $h = 1$ in the home and foreign countries are given, respectively, by

$$\begin{aligned} w_t &= p_t^x(1) + z_t^x(1) - (1-\eta_1)\log n_t^x(1) + \log(\eta_1 E(1)^{1-\eta_1}) \\ \tilde{w}_t &= \tilde{p}_t^x(1) + \tilde{z}_t^x(1) - (1-\tilde{\eta}_1)\log \tilde{n}_t^x(1) + \log(\tilde{\eta}_1 \tilde{E}(1)^{1-\tilde{\eta}_1}). \end{aligned}$$

Using the law of one price (B.3) into the foreign first order condition and replacing the two wage equations into (B.7) delivers the following expression for the real exchange rate in terms of commodity prices, labor allocations, productivity shocks, and trade taxes,

$$p_t + s_t - \tilde{p}_t = (\alpha\beta_2 - \tilde{\alpha}\tilde{\beta}_2)(p_t^x(1) - p_t^x(2)) + \varepsilon_{3t}, \tag{B.8}$$

where ε_{3t} depends on productivity shocks, trade taxes, and labor allocations as follows,

$$\begin{aligned} \varepsilon_{3t} &= \varepsilon_{2t} + \alpha(1-\beta_n)[z_t^x(1) - (1-\eta_1)\log n_t^x(1) + \log(\eta_1 E(1)^{1-\eta_1})] \\ &\quad - \tilde{\alpha}(1-\tilde{\beta}_n)[t_t^x(1) + \tilde{z}_t^x(1) - (1-\tilde{\eta}_1)\log \tilde{n}_t^x(1) + \log(\tilde{\eta}_1 \tilde{E}(1)^{1-\tilde{\eta}_1})]. \end{aligned}$$

There is, of course, a symmetric expression if both countries produce the commodity $x_t(2)$.

Suppose instead that the home country produces commodity $x_t(1)$ and the foreign country produces commodity $x_t(2)$. A similar algebra delivers the following expression for the real exchange rate

$$p_t + s_t - \tilde{p}_t = \left(\tilde{\alpha}\tilde{\beta}_1 + \alpha\left(1 - \beta_n - \beta_1\right)\right)\left(p_t^x\left(1\right) - p_t^x\left(2\right)\right) + \varepsilon_{4t}, \tag{B.9}$$

where ε_{4t} is now given by

$$\begin{aligned} \varepsilon_{4t} &= \varepsilon_{2t} + \alpha\left(1 - \beta_n\right)\left[z_t^x\left(1\right) - \left(1 - \eta_1\right)\log n_t^x\left(1\right) + \log(\eta_1 E\left(1\right)^{1-\eta_1})\right] \\ &\quad -\tilde{\alpha}(1 - \tilde{\beta}_n)[t_t^x\left(2\right) + \tilde{z}_t^x\left(2\right) - \left(1 - \tilde{\eta}_2\right)\log \tilde{n}_t^x\left(2\right) + \log(\tilde{\eta}_2 \tilde{E}\left(2\right)^{1-\tilde{\eta}_2})]. \end{aligned}$$

Again, there is a symmetric representation if the home country produces $x_t\left(2\right)$ and the foreign country produces $x_t\left(1\right)$.

In obtaining the real exchange rate representations (B.8) and (B.9) we introduced the labor first order conditions in the commodities sector into the representation in terms of wages (B.7). We could have used instead the cost minimization conditions in the final good's sector

$$\frac{W_t}{P_t^q} = \frac{\alpha}{1-\alpha}\frac{q_t}{n_t^y} \text{ and } \frac{\tilde{W}_t}{\tilde{P}_t^q} = \frac{\tilde{\alpha}}{1-\tilde{\alpha}}\frac{\tilde{q}_t}{\tilde{n}_t^y}, \tag{B.10}$$

to replace nominal wages in equation (B.7). Inserting these expressions into (B.7) gives a representation of the real exchange rate in terms of commodity prices, intermediate good prices measured in a common currency, productivity shocks, trade taxes, and allocations,

$$\begin{aligned} p_t + s_t - \tilde{p}_t &= (\tilde{\alpha}\tilde{\beta}_1 - \alpha\beta_1)p_t^x\left(1\right) + (\tilde{\alpha}\tilde{\beta}_2 - \alpha\beta_2)p_t^x\left(2\right) \\ &\quad +\alpha\left(1 - \beta_n\right)p_t^q - \tilde{\alpha}(1 - \tilde{\beta}_n)\left(\tilde{p}_t^q - s_t\right) + \varepsilon_{5t}, \end{aligned} \tag{B.11}$$

where

$$\begin{aligned} \varepsilon_{5t} &= \varepsilon_{2t} + \alpha\left(1 - \beta_n\right)\log\left(q_t/n_t^y\right) - \tilde{\alpha}(1 - \tilde{\beta}_n)\log\left(\tilde{q}_t/\tilde{n}_t^y\right) \\ &\quad + \log\left(\frac{\alpha}{1-\alpha}\right)\left(\alpha\left(1 - \beta_n\right) - \tilde{\alpha}(1 - \tilde{\beta}_n)\right). \end{aligned}$$

Finally, we show a representation of the real exchange rate that is independent of commodity prices. The cost minimization conditions (B.10), the price index (B.2), and the law of one price in commodities allows us to write

$$\begin{aligned} w_t(1 - \beta_n) &= \beta_1 p_t^x\left(1\right) + \beta_2 p_t^x\left(2\right) - z_t^q + \log\left(\frac{\alpha}{1-\alpha}\frac{q_t}{n_t^y}\right) \\ \left(\tilde{w}_t - s_t\right)(1 - \tilde{\beta}_n) &= \tilde{\beta}_1\left[p_t^x\left(1\right) + t_t^x\left(1\right)\right] + \tilde{\beta}_2\left[p_t^x\left(2\right) + t_t^x\left(2\right)\right] - \tilde{z}_t^q + \log\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\frac{\tilde{q}_t}{\tilde{n}_t^y}\right). \end{aligned}$$

Replacing these expressions into (B.7) delivers the representation real exchange rate that is independent of commodity prices,

$$p_t + s_t - \tilde{p}_t = \tilde{z}_t^y - z_t^y - t_t^q + \alpha \log \left(q_t / n_t \right) - \tilde{\alpha} \log \left(\tilde{q}_t / \tilde{n}_t \right) + \kappa, \tag{B.12}$$

where $\kappa = \alpha \log \left(\alpha / \left(1 - \alpha \right) \right) - \tilde{\alpha} \log \left(\tilde{\alpha} / \left(1 - \tilde{\alpha} \right) \right)$ is a constant.

B.3 General Technologies

In this section we show that the simple case with Cobb-Douglas technologies generalizes, in an appropriate fashion, to general constant returns to scale production functions. The equilibrium prices in the final good sector and the intermediate good sector can be represented by the marginal cost function

$$P_t = C^y \left(W_t, P_t^q \right) / Z_t^y \tag{B.13}$$

$$P_t^q = C^q \left(W_t, P_t^x \left(1 \right), P_t^x \left(2 \right) \right) / Z_t^q \tag{B.14}$$

Replacing (B.14) into (B.13) and using that the cost function is homogeneous of degree one in factor prices we can write the price of the final good as

$$P_t = P_t^x \left(1 \right) c \left(\frac{W_t}{P_t^x \left(1 \right)}, \frac{P_t^x \left(2 \right)}{P_t^x \left(1 \right)} \right) / Z_t^y$$

with a similar expression for the foreign country,

$$\tilde{P}_t = \tilde{P}_t^x \left(1 \right) \tilde{c} \left(\frac{\tilde{W}_t}{\tilde{P}_t^x \left(1 \right)}, \frac{\tilde{P}_t^x \left(2 \right)}{\tilde{P}_t^x \left(1 \right)} \right) / \tilde{Z}_t^y.$$

The real exchange rate is then defined as

$$rer_t = \frac{c \left(\frac{W_t}{P_t^x (1)}, \frac{P_t^x (2)}{P_t^x (1)} \right)}{\tilde{c} \left(\frac{\tilde{W}_t}{\tilde{P}_t^x (1)}, \frac{\tilde{P}_t^x (2)}{\tilde{P}_t^x (1)} \right)} \frac{\tilde{Z}_t^y}{Z_t^y} \frac{P_t^x \left(1 \right) S_t}{\tilde{P}_t^x \left(1 \right)}$$

Using the law of one price for the commodities,

$$rer_t = \frac{c \left(\frac{W_t}{P_t^x (1)}, \frac{P_t^x (2)}{P_t^x (1)} \right)}{\tilde{c} \left(\frac{\tilde{W}_t}{\tilde{P}_t^x (1)}, \frac{P_t^x (2)}{P_t^x (1)} \frac{T_t^x (2)}{T_t^x (1)} \right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x \left(1 \right)} \tag{B.15}$$

Let the technology to produce commodity $i = 1, 2$ be given by $X_t(i) = Z_t^x(i) F_i(n_t^x(i))$. The first order condition with respect to labor is then given by

$$W_t / P_t^x(i) = Z_t^x(i) F'(n_t^x(i)).$$

Suppose that both countries produce commodity $X_t(1)$. Then replacing the previous condition evaluated at $i = 1$ into (B.15) gives the first representation of the real exchange rate in terms of commodity prices and labor allocations,

$$rer_t = \frac{c\left(Z_t^x(1) F_1'(n_t^x(1)), \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{Z}_t^x(1) F_1'(\tilde{n}_t^x(1)), \frac{P_t^x(2)}{P_t^x(1)} \frac{T_t^x(2)}{T_t^x(1)}\right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x(1)}. \tag{B.16}$$

Suppose instead that the home country produces commodity $X(1)$ but the foreign country produces commodity $X(2)$. The real exchange rate can then be written as

$$rer_t = \frac{c\left(Z_t^x(1) F_1'(n_t^x(1)), \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{Z}_t^x(2) F_2'(\tilde{n}_t^x(2)) \frac{P_t^x(2)}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x(2)} \tag{B.17}$$

which is our second representation of the real exchange rate.

To obtain our third representation of the real exchange rate we use the cost minimization condition in the final goods' sector, which can be written as

$$\frac{W_t}{P_t^q} = h\left(\frac{n_t^y}{q_t}\right) \tag{B.18}$$

where h is a decreasing function. Using this expression into (B.15) delivers

$$rer_t = \frac{c\left(h\left(\frac{n_t^y}{q_t}\right) \frac{P_t^q}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{h}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}\right) \frac{\tilde{P}_t^q}{\tilde{P}_t^x(1)}, \frac{T_t^x(2)}{T_t^x(1)}\right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x(1)}. \tag{B.19}$$

This expression relates the real exchange rate to relative commodity prices, to the relative price of the intermediate good in terms of commodity $X(1)$, and to the allocation.

Finally, we obtain an expression independent of commodity prices. To that end, we use that the equilibrium price in the intermediate goods' sector (B.14) can be written as

$$\frac{P_t^q}{P_t^x(1)} = \phi\left(\frac{W_t}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right) / Z_t^q.$$

But (B.18) implies

$$\frac{W_t}{P_t^x(1)} = \frac{P_t^q}{P_t^x(1)} h\left(\frac{n_t^y}{q_t}\right)$$

which, using the previous expression, becomes

$$\frac{W_t}{P_t^x(1)} = \phi\left(\frac{W_t}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right) h\left(\frac{n_t^y}{q_t}\right) / Z_t^q.$$

Using the properties of the cost function,

$$1 = \phi\left(1, \frac{P_t^x(2)/P_t^x(1)}{W_t/P_t^x(1)}\right) h\left(\frac{n_t^y}{q_t}\right) \frac{1}{Z_t^q}.$$

This is an implicit function which delivers

$$\frac{W_t}{P_t^x(1)} = \frac{P_t^x(2)}{P_t^x(1)} \kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right).$$

The equivalent expression for the foreign country (using the law of one price for the commodities) is

$$\frac{\tilde{W}_t}{\tilde{P}_t^x(1)} = \frac{P_t^x(2)}{P_t^x(1)} \frac{T_t^x(2)}{T_t^x(1)} \tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right).$$

Using the latter two expressions into (B.15) gives the last representation of the real exchange rate

$$rer_t = \frac{c\left(\frac{P_t^x(2)}{P_t^x(1)} \kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right), \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\frac{P_t^x(2)}{P_t^x(1)} \frac{T_t^x(2)}{T_t^x(1)} \tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right), \frac{P_t^x(2)}{P_t^x(1)} \frac{T_t^x(2)}{T_t^x(1)}\right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x(1)}$$

The properties of the cost function implies that the relative prices $P_t^x(2)/P_t^x(1)$ disappear,

$$rer_t = \frac{c\left(\kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right), 1\right)}{\tilde{c}\left(\frac{T_t^x(2)}{T_t^x(1)} \tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right), \frac{T_t^x(2)}{T_t^x(1)}\right)} \frac{\tilde{Z}_t^y / Z_t^y}{T_t^x(1)}. \quad \text{(B.20)}$$

B.4 Sum of coefficients

In this appendix, we show that the sum of the coefficients in prices and wages in expression (2.3), repeated here for convenience,

$$\begin{aligned}
\ln P_t &= \ln \frac{\kappa^C}{z_t^y} + (1-\alpha) \sum_{i=1}^{N} \varphi(i) \frac{\kappa^{Q(i)}}{z_t(i)} \\
&+ \sum_{j=1}^{J} \left[\alpha \psi^C(j) + \sum_{i=1}^{N} (1-\alpha) \varphi(i) \beta(i) \psi^Q(i,j) \right] \ln W_t(j) \\
&+ (1-\alpha) \sum_{i=1}^{N} \varphi(i) (1-\beta(i)) \sum_{h=1}^{N} \gamma(i,h) \ln P_t(h),
\end{aligned}$$

add up to 1. The sum of the coefficients is

$$\sum_{j=1}^{J} [\alpha \psi^C(j) + (1-\alpha) \sum_{i=1}^{N} \beta(i) \varphi(i) \psi^Q(i,j)] + (1-\alpha) \sum_{h=1}^{N} \sum_{i=1}^{N} (1-\beta(i)) \varphi(i) \gamma(i,h)$$

which can be written

$$\alpha \sum_{j=1}^{J} \psi^C(j) + (1-\alpha) \sum_{i=1}^{N} \beta(i) \varphi(i) \sum_{j=1}^{J} \psi^Q(i,j) + (1-\alpha) \sum_{i=1}^{N} (1-\beta(i)) \varphi(i) \sum_{h=1}^{N} \gamma(i,h)$$

or

$$\begin{aligned}
& \alpha + (1-\alpha) \sum_{i=1}^{N} \beta(i) \varphi(i) + (1-\alpha) \sum_{i=1}^{N} (1-\beta(i)) \varphi(i) \\
= \;& \alpha + (1-\alpha) \sum_{i=1}^{N} \beta(i) \varphi(i) + (1-\alpha) - (1-\alpha) \sum_{i=1}^{N} \beta(i) \varphi(i) = 1.
\end{aligned}$$

B.5 Tables

B.5.1 Trade data

Table A.1: Share of imports and exports in each country (% average in 1990–1999)

	United States		United Kingdom		Germany		Japan	
	imp.	exp.	imp.	exp.	imp.	exp.	imp.	exp.
Petroleum	8.5	1.2	3.4	6.0	5.2	0.7	13.1	0.4
Fish	1.0	0.6	0.7	0.5	0.5	0.2	5.0	0.2
Meat	0.3	0.9	0.7	0.6	1.1	0.4	2.4	0.0
Aluminium	0.4	0.2	0.3	0.2	0.5	0.1	1.5	0.0
Copper	0.2	0.2	0.3	0.0	0.4	0.1	1.2	0.1
Gold	0.3	0.9	0.0	0.1	0.3	0.2	0.8	0.1
Wheat	0.0	0.8	0.1	0.3	0.1	0.2	0.4	0.0
Maize	0.0	1.1	0.1	0.0	0.1	0.0	0.8	0.0
Timber	0.9	1.0	0.8	0.0	0.5	0.2	3.5	0.0
Cotton	0.0	0.5	0.0	0.0	0.1	0.0	0.3	0.0
SUM	**11.8**	**7.2**	**6.5**	**7.7**	**8.7**	**2.1**	**29.0**	**0.8**

Source: Comtrade

B.5.2 Selecting different number of commodities

Table A.2: Regressions in levels, selecting 3 commodities - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.37	0.66	0.70	0.51	0.51
Germany	0.54	0.66	0.81	0.37	0.65
Japan	0.78	0.78	0.42	0.66	0.38

Table A.3: Regressions in 4-year differences, selecting 3 commodities - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.24	0.68	0.78	0.63	0.39
Germany	0.53	0.84	0.80	0.72	0.72
Japan	0.46	0.81	0.58	0.73	0.69

Table A.4: Regressors from previous sub-period, selecting 3 commodities - R^2

	1973-1985	1986-1998	1999-2014
United Kingdom	0.45	0.56	0.39
Germany	0.68	0.57	0.66
Japan	0.27	0.31	0.42

Table A.5: Regressions in levels, selecting 5 commodities - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.45	0.74	0.70	0.65	0.52
Germany	0.57	0.78	0.85	0.49	0.66
Japan	0.80	0.86	0.58	0.71	0.71

Table A.6: Regressions in 4-year differences, selecting 5 commodities - R^2

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
United Kingdom	0.41	0.86	0.89	0.75	0.59
Germany	0.61	0.91	0.87	0.82	0.74
Japan	0.54	0.90	0.83	0.89	0.80

Table A.7: Regressors from previous sub-period, selecting 5 commodities - R^2

	1973-1985	1986-1998	1999-2014
United Kingdom	0.85	0.66	0.47
Germany	0.72	0.62	0.66
Japan	0.76	0.68	0.57

B.5.3 Regression coefficients

Table A.8: Regression coefficients, in levels - United Kingdom

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil	-0.065***	-0.317***			
Fish				-0.256***	
Meat			-0.430***		
Aluminium	-0.122***	-0.045		0.157***	
Copper			0.173***	-0.312***	-0.179***
Gold			-0.247***		0.218***
Wheat				0.251***	-0.072
Maize	0.162***	-0.280***	0.558***		0.090***
Timber	-0.150***	-0.010			
Cotton					

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

Table A.9: Regression coefficients, in levels - Germany

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil		-0.349***			
Fish	-0.209***				
Meat		-0.152***		0.156**	
Aluminium		0.301***	-0.533***	0.096*	0.056
Copper				-0.229***	-0.215***
Gold	-0.101**		-0.021		0.130**
Wheat		0.061	-0.339**		-0.088
Maize	0.284***		0.166		
Timber	-0.386***			-0.121***	
Cotton					

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

Table A.10: Regression coefficients, in levels - Japan

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil	-0.143***	-0.331***		0.115**	
Fish					0.133***
Meat		-0.313**	-0.136	0.176**	
Aluminium		0.932***	-0.400***		
Copper		-0.124**		-0.182**	
Gold	-0.103**				
Wheat	0.723***				0.219***
Maize			0.248***		
Timber	-0.669***		0.094	-0.369***	-0.497***
Cotton					-0.179***

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

Table A.11: Regression coefficients, in 4-year differences - United Kingdom

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil	-0.126**	-0.374***			
Fish	-0.308***	-0.206***		-0.224***	
Meat			-0.275***	-0.307***	
Aluminium		-0.596***	-0.216***		
Copper				-0.215***	-0.235***
Gold		0.035***	-0.208***		0.203***
Wheat				0.243***	
Maize	0.253***		0.389***		0.009
Timber					-0.066
Cotton	0.032				

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

Table A.12: Regression coefficients, in 4-year differences - Germany

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil		-0.274***			
Fish	-0.293***	-0.064***	-0.194**		-0.435***
Meat			-0.005		
Aluminium			-0.541***		
Copper				-0.203***	
Gold	-0.068**			-0.061***	
Wheat			-0.233***		-0.239***
Maize	0.165*	-0.071***		0.280***	0.240**
Timber	-0.310***			-0.160***	-0.159**
Cotton		-0.192***			

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

Table A.13: Regression coefficients, in 4-year differences - Japan

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
Oil	0.177***	-0.236***		0.270***	
Fish		-0.039**		0.241***	
Meat			-0.231***		
Aluminium	-0.159*		-0.581***	-0.260***	0.209***
Copper			0.343***		
Gold	-0.068*				-0.247***
Wheat			-0.325***		0.196***
Maize		-0.222***			
Timber	-0.401***	-0.187***		-0.352***	-0.228***
Cotton					

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5% and 1%, respectively.

B.6 Figures

Figure A.1: Out-of-sample fit – selecting 3 commodities

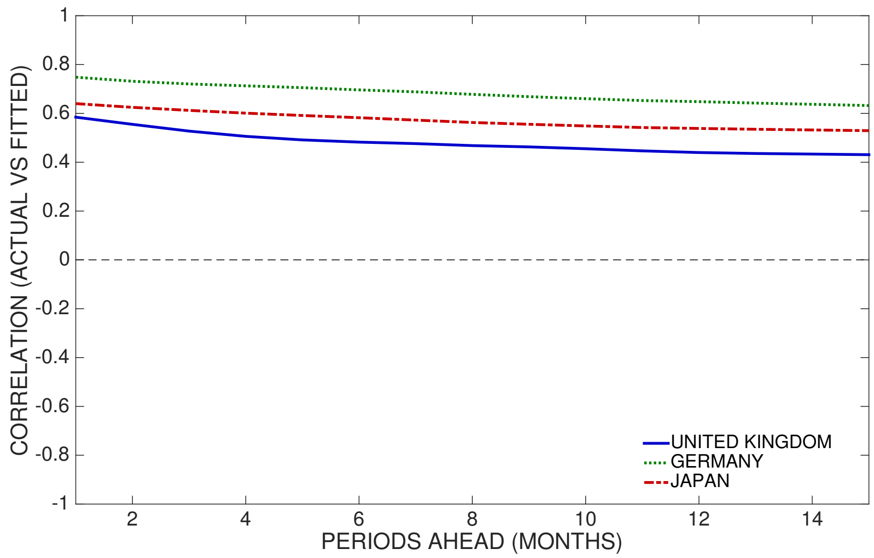
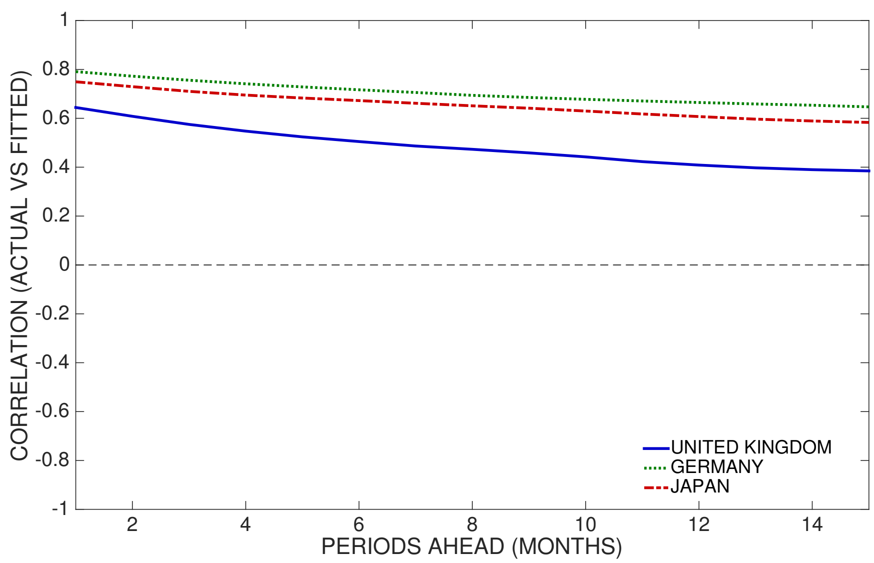


Figure A.2: Out-of-sample fit – selecting 5 commodities



Appendix C

Appendix to Chapter 3

C.1 Numerical Solution

Since it is not possible to keep track of the firm distribution Ω, we follow Clementi and Palazzo (2014) and assume the following forecasting rules for w' where

$$\log w' = \alpha_w + \beta_w \log w + \beta_{Z'} \log Z + \beta_{Z'} \log Z'.$$

C.1.1 Firm's Approximated Problem

Given an initial guess for the $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$, the firm uses the law of motion for w' and solves the following problem,

$$\tilde{V}^f(s, Z, w) = \pi(s, Z, w) + \beta(1-\eta) E_Z \tilde{V}^f(s, Z', w')$$

where $\pi(s, Z, w)$ is given by

$$\max_{l_f(s,Z,w)} \quad sZl_f(s, Z, w)^\theta - w(Z) l_f(s, Z, w)$$

C.1.2 Algorithm

1. given a guess for $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$, approximate the firms' value function
2. simulate the economy with TFP shocks
3. in the simulation, for every period, we have to solve for $\{w, \mu\}$ such that the labor market clears and zero profit condition holds in equilibrium. Nelder-Mead along with Newton's method is used to clear the market and ensure that the zero profit condition

holds in equilibrium. Note that when the values for $\{w, \mu\}$ are updated, we have to re-optimize decision rules for both the firm and household. However, we still use the laws of motion and value function from step 2 to forecast in the firm's problem.

4. Given the results of the simulation, we can use OLS to get new estimates for $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$. Then we go back to step 2 until parameters converge.

The initial guess for α_w is $\log w$ where w is the wage rate in the economy without aggregate uncertainty. The initial guesses for $\{\beta_w, \beta_Z, \beta_{Z'}\}$ are set to 0. We get an r-square of .999999 for the law of motion. We first start by simulating the economy for 750 periods where we ignore the first 250 periods for OLS estimates of $\{\alpha_w, \beta_w, \beta_Z, \beta_{Z'}\}$. Once the parameters have converged, we do a check where we simulate the economy for 3,500 periods where we ignore the first 500 periods.