

Jun Shao

---

# Mathematical Statistics: Exercises and Solutions

 Springer

# Preface

Since the publication of my book *Mathematical Statistics* (Shao, 2003), I have been asked many times for a solution manual to the exercises in my book. Without doubt, exercises form an important part of a textbook on mathematical statistics, not only in training students for their research ability in mathematical statistics but also in presenting many additional results as complementary material to the main text. Written solutions to these exercises are important for students who initially do not have the skills in solving these exercises completely and are very helpful for instructors of a mathematical statistics course (whether or not my book *Mathematical Statistics* is used as the textbook) in providing answers to students as well as finding additional examples to the main text. Motivated by this and encouraged by some of my colleagues and Springer-Verlag editor John Kimmel, I have completed this book, *Mathematical Statistics: Exercises and Solutions*.

This book consists of solutions to 400 exercises, over 95% of which are in my book *Mathematical Statistics*. Many of them are standard exercises that also appear in other textbooks listed in the references. It is only a partial solution manual to *Mathematical Statistics* (which contains over 900 exercises). However, the types of exercise in *Mathematical Statistics* not selected in the current book are (1) exercises that are routine (each exercise selected in this book has a certain degree of difficulty), (2) exercises similar to one or several exercises selected in the current book, and (3) exercises for advanced materials that are often not included in a mathematical statistics course for first-year Ph.D. students in statistics (e.g., Edgeworth expansions and second-order accuracy of confidence sets, empirical likelihoods, statistical functionals, generalized linear models, nonparametric tests, and theory for the bootstrap and jackknife, etc.). On the other hand, this is a stand-alone book, since exercises and solutions are comprehensible independently of their source for likely readers. To help readers not using this book together with *Mathematical Statistics*, lists of notation, terminology, and some probability distributions are given in the front of the book.

All notational conventions are the same as or very similar to those in *Mathematical Statistics* and so is the mathematical level of this book. Readers are assumed to have a good knowledge in advanced calculus. A course in real analysis or measure theory is highly recommended. If this book is used with a statistics textbook that does not include probability theory, then knowledge in measure-theoretic probability theory is required.

The exercises are grouped into seven chapters with titles matching those in *Mathematical Statistics*. A few errors in the exercises from *Mathematical Statistics* were detected during the preparation of their solutions and the corrected versions are given in this book. Although exercises are numbered independently of their source, the corresponding number in *Mathematical Statistics* is accompanied with each exercise number for convenience of instructors and readers who also use *Mathematical Statistics* as the main text. For example, Exercise 8 (#2.19) means that Exercise 8 in the current book is also Exercise 19 in Chapter 2 of *Mathematical Statistics*.

A note to students/readers who have a need for exercises accompanied by solutions is that they should not be completely driven by the solutions. Students/readers are encouraged to try each exercise first without reading its solution. If an exercise is solved with the help of a solution, they are encouraged to provide solutions to similar exercises as well as to think about whether there is an alternative solution to the one given in this book. A few exercises in this book are accompanied by two solutions and/or notes of brief discussions.

I would like to thank my teaching assistants, Dr. Hansheng Wang, Dr. Bin Cheng, and Mr. Fang Fang, who provided valuable help in preparing some solutions. Any errors are my own responsibility, and a correction of them can be found on my web page <http://www.stat.wisc.edu/~shao>.

Madison, Wisconsin  
April 2005

Jun Shao

# Contents

Preface . . . . .	vii
Notation . . . . .	xi
Terminology . . . . .	xv
Some Distributions . . . . .	xxiii
Chapter 1. Probability Theory . . . . .	1
Chapter 2. Fundamentals of Statistics . . . . .	51
Chapter 3. Unbiased Estimation . . . . .	95
Chapter 4. Estimation in Parametric Models . . . . .	141
Chapter 5. Estimation in Nonparametric Models . . . . .	209
Chapter 6. Hypothesis Tests . . . . .	251
Chapter 7. Confidence Sets . . . . .	309
References . . . . .	351
Index . . . . .	353