

**ELEMENTARY**  

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**MATHEMATICAL and**  

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**COMPUTATIONAL TOOLS**  

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**for ELECTRICAL and**  

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**COMPUTER ENGINEERS**  

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**USING MATLAB®**  

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**Second Edition**

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