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Research Paper

Unconventional monetary policy, financial frictions, and the equity tandem[☆]

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ABSTRACT

A key feature of many DSGE frameworks designed to model Quantitative Easing (QE) is that net worth only plays a relevant role on bank's balance sheets. In reality, however, net worth of borrowers *and* lenders plays a relevant role in financing investment projects. I show that this *equity tandem* has important implications. Net worth of non-financial firms acts as a *first line of defense*, since non-financial firm's balance sheets are hit in the first place by real sector shocks. Modeling the equity tandem increases the resilience of the model and, therefore, implies smaller gains of unconventional monetary policy. A novel insight from the simultaneous modeling of borrowers and lenders net worth is that by decreasing the cost of external finance a QE policy is redistributing net worth from banks to non-financial firms. Additionally, considering the reverse operation, a credibly announced Quantitative Tightening (QT), helps to stabilize the spread between the return to capital and the deposit rate during the zero lower bound period. However, different anticipated QT paths are shown to have little consequences for output and inflation.

1. Introduction

A significant amount of research has been done to study unconventional monetary policy and there is an ongoing discussion in the literature about the effectiveness of Quantitative Easing (QE), forward guidance, and negative interest rate policies (NIRPs).[1] This paper contributes to the theoretical literature about QE policies. Because the interaction between financial markets and the real economy is essential for the understanding of unconventional monetary policy, the literature about macro-financial linkages is closely intertwined with the literature about unconventional monetary policy. In the interest of parsimony and tractability, many theoretical contributions to both strands of the literature limit the financial structure to a single agent's balance sheet who serves as supplier of funds or has demand for funds.[2] This implies that net worth—if at all—becomes a relevant state variable only on the supply *or* demand side of the intermediation process.[3]

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[1] Reviews of the literature can be found in, e.g., Joyce et al. (2012), Kuttner (2018) and Bernanke (2020). The first joint comparison of all three policies within a single dynamic stochastic general equilibrium (DSGE) model can be found in Sims and Wu (2021).

[2] For example, Claessens and Kose (2017) survey the literature about macro-financial linkages and classify studies by whether they consider the demand or supply side of finance or, in other words, by whether they focus on balance sheets of borrowers or lenders.

[3] A prominent example from the macro-finance literature is the work by Bernanke et al. (1999) where borrower's net worth is a relevant proxy for their default risk and intermediaries act as a simple pass-through of funds from households to firms. In other words, the modeling of credit intermediation does not involve bank's net worth.

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The key contribution of this paper is to analyze QE type policies within a monetary DSGE model featuring a financial structure which explicitly takes net worth of banks and non-financial firms into account. Once I consider this *equity tandem*, the impact of QE on output and inflation is more muted and the influence of the zero lower bound (ZLB) becomes relatively small. Compared with a conventional interest rate cut, a large scale asset purchase—calibrated to match the initial increase in output from a decline in the policy rate by one annualized percentage point—leads to a very short-lived increase in output and a smaller increase in inflation compared to a conventional rate cut. Though it seems reasonable to conduct unconventional asset purchases when conventional measures are unavailable, they are rather imperfect substitutes. This findings relate to the discussion about the irrelevance of the ZLB constraint and the substitutability of conventional and unconventional monetary policy instruments (e.g. Debortoli et al., 2020; Sims and Wu, 2020, 2021; Karadi and Nakov, 2021). Additionally, considering the reverse operation shows that the influence of Quantitative Tightening (QT) under the equity tandem is rather symmetric to a QE policy within a ZLB environment. Since there is an interaction between QE and subsequent QT, a credibly announced unwinding of the central bank balance sheet helps to stabilize the spread between the return to capital and the deposit rate during the ZLB period. However, different anticipated QT paths are shown to have little consequences for output and inflation.

Many contributions to the literature of unconventional monetary policy use a supply side approach to capture certain aspects of the Great Financial Crisis where banks had a leading role. By imposing an agency problem on banks, their balance sheets become subject to a binding or occasionally-binding leverage constraint (e.g. Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013; Carlstrom et al., 2017; Sims and Wu, 2020, 2021; Cardamone et al., 2023; Sims et al., 2023; Karadi and Nakov, 2021). Therefore, in many previous papers, bank's net worth is a state variable of interest, but the financial structure is often kept at a low level of complexity by neglecting net worth or equity of non-financial firms. For instance, in Karadi and Nakov (2021) bank equity is a key state variable when they warn against the potential addictiveness of quantitative easing and show that QE may not always be a substitute for conventional policy.

The main contribution of my research is to analyze unconventional balance sheet policies within a quantitative DSGE model with a richer financial structure.[4] By modeling financial frictions at different layers of the intermediation process, both net worth of borrowers and lenders becomes relevant. This *equity tandem* yields some interesting insights. First, net worth of non-financial firms can be thought of as *first line of defense*, once an adverse shock located in the real sector hits the economy, since losses are covered by non-financial firms in the first place. Second, modeling net worth on both sides of the financial contract increases the resilience of the model. Because non-financial firms are less leveraged, relative to financial firms, the propagation and amplification of macroeconomic shocks is muted compared to a *supply side only* approach used by many previous papers. Third, the improvement of resilience implies that there are smaller gains of unconventional monetary policy. Therefore, neglecting the role of net worth on the demand side, as many papers do, might lead to an overestimate of the effectiveness of QE policies. Fourth, a novel insight from the simultaneous modeling of borrowers and lenders net worth is that an aggressive credit/QE policy is shifting net worth from the banking system to the balance sheets of non-financial firms.[5]

From a theoretical point of view it was less clear cut whether or not such a policy can have real macroeconomic effects, since the Wallace (1981) irrelevance proposition basically states that such a policy should be neutral. Eggertsson and Woodford (2003) restate this irrelevance result for open market operations within a representative agent framework. Due to this lack of theoretical justification Ben Bernanke quipped: "Well, the problem with QE is it works in practice, but it doesn't work in theory".[6]

In order to create a rationale for the effectiveness of QE, many papers deviate from the assumption of perfect substitutability between assets and introduce limits for arbitrage. Following Andrés et al. (2004), Chen et al. (2012) consider segmented markets with frictions between short- and long-term bond markets and portfolio adjustment costs to model a term premium. Ellison and Tischbirek (2014) assume that investors perceive bonds with different maturities as imperfect substitutes and implement a preferred habitat channel similar to the mechanism studied by Vayanos and Vila (2021). Another set of assumptions to break Wallace (1981) irrelevance proposition is a combination of limited participation of households in bond markets and agency frictions within banks due to limited enforcement (e.g. Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013). This paper is closely related to many papers which, following Gertler and Karadi (2011, 2013), introduce a banking sector with a permanently binding balance sheet constraint (e.g. Carlstrom et al., 2017; Sims et al., 2023; Sims and Wu, 2020, 2021; Cardamone et al., 2023; Boehl et al., 2022). Consequently, *by construction*, QE policies have real macroeconomic effects via excess return of long-term assets. Karadi and Nakov (2021) departs from this assumption and study optimal policy in a model where banks face occasionally binding balance sheet constraints and can raise new equity similar to Akinci and Queralto (2022). Karadi and Nakov (2021) find that in times of financial distress QE can be quite effective even if the policy rate is not constrained by the ZLB. However, as long as balance sheet constrains are slack QE policies turn out to be ineffective since there is a perfect crowding out of private lending activity. Paries and Kühl (2017) study optimal asset purchases from a timeless perspective in an estimated model for the euro area and Harrison (2017) studies optimal QE under discretion. Cardamone et al. (2023) analyze QE and direct lending policies in an economy where, similar to the present paper, frictions appear within banks and non-financial firms. However, their non-financial firms are subject to cash flow constraints and do not accumulate net worth. In contrast, non-financial firms in my model accumulate net worth to overcome

[4] The idea to integrate multiple financial frictions into a DSGE model is not new and already previous papers considered frictions à la Gertler and Karadi (2011, 2013) and Bernanke et al. (1999) at the same time (see Clerc et al., 2015; Massari and Tirelli, 2022). However, to the best of my knowledge, there is no other paper in the literature explicitly considering the implications of the resulting equity tandem for credit or QE policy.

[5] In the following, I will use the terms credit and QE policy interchangeably.

[6] See the transcript in Ahamed and Bernanke (2014, p. 12).

a costly state verification friction. Also related, but not discussed within this paper, are macroprudencial policy considerations (e.g. Woodford, 2016).

The course of investigation will be as follows. Section 2 presents the model. For comparability reasons, the key idea is to merge two prominent approaches of modeling net worth of borrowers and lenders. The supply side of intermediation is modeled as in Gertler and Karadi (2011) and the demand side follows the exposition of Bernanke et al. (1999). Section 3 discusses the quantitative results. Here, the baseline model from Gertler and Karadi (2011) is compared to the extended model including the BGG contracting problem to allow for a dual role of net worth in the economy. Section 4 concludes.

2. Model

The model is a synthesis of the frameworks of Gertler and Karadi (2011) and Bernanke et al. (1999).[7] The core is a conventional New Keynesian model with nominal frictions and capital accumulation. On the supply side, the financial intermediation process is characterized by financial intermediaries (banks) who obtain funds from households and lend to non-financial firms. An agency problem in the form of an incentive compatibility constraint or run-away friction limits the ability of intermediaries to obtain funds. Consequently, banks face an endogenous leverage constraint which imposes a friction on the banking channel. Additionally, following Massari and Tirelli (2022), there is a non-bank financial intermediary offering financing via bond markets to introduce a market channel into the model.[8] On the demand side, non-financial firms are modeled in the spirit of Bernanke et al. (1999) and borrow funds from banks and non-banks in order to finance investments into the capital stock. Non-financial firm's investments are subject to two sources of risk. Beyond aggregate risk a firm specific shock to each project creates a source of idiosyncratic risk that is perfectly diversifiable in aggregate. Banks ability to supply credit is linked to their stock of net worth and a conventional costly state verification problem links the cost of external finance of non-financial firms to their net worth stock. Compared to Gertler and Karadi (2011) there are two frictions that create a potential role for unconventional monetary policy.[9]

The economic environment is characterized by households, banks, non-banks, intermediate firms, retailers, final goods producers, capital goods producers, and a government sector. The latter is mainly concerned with the conduct of conventional and unconventional monetary policy. For the sake of completeness, and in order to make this paper selfcontained, I shall discuss the microfoundations of each agent in Appendix A.4. In the following subsections, I briefly explain the financial structure of the model characterized by the relation between households, intermediaries, intermediate firms, and the central bank.

2.1. Households

The household derives utility from consumption C_t, real holdings of liquid assets B_{t+1} and disutility from labor supply, L_t. The expected lifetime utility function of a household is described by

$$E_t \sum_{i=0}^{\infty} \beta^i \left(\log(C_{t+i} - \Xi C_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} + \frac{s_{t+i}}{1-\sigma_b} B_{t+i}^{1-\sigma_b} \right) \tag{1}$$

where $\beta \in (0,1)$ is the household discount factor, $\Xi \in (0,1)$ determines the degree of internal habit formation, φ is the inverse of the Frisch elasticity of labor supply, χ is a scaling parameter of disutility from labor supply and s_t is a preference shock for holding liquid assets.[10] Each household can save by holding one period real deposits on accounts with banks, not belonging to their own household, or by holding riskless one period real government bonds. By construction of the equilibrium, both saving vehicles are perfect substitutes and pay the predetermined gross real return R_t from $t-1$ to t. Overall, the total amount of the household's real holding of one period real bonds and real deposits at banks, acquired at $t-1$ and due in t, is given by B_t. Additionally, the household can save by holding real deposits at non-bank intermediaries D_t^{NB} that pay $R_t^{d,\text{NB}}$. The household faces a sequence of budget constraints given by

$$C_t + B_{t+1} + D_{t+1}^{\text{NB}} = W_t L_t + R_t B_t + R_t^{d,\text{NB}} D_t^{\text{NB}} + \Pi_t^{\text{Prof.}} - T_t \tag{2}$$

where W_t, $\Pi_t^{\text{Prof.}}$ and T_t denote the real wage, real profits from the ownership of financial and non-financial firms net of start-up funds for new bankers and lump sum taxes at time t, respectively.

[7] Massari and Tirelli (2022) embed a very similar financial structure into the New York Fed DSGE model and focus on the analysis of liquidity shocks. However, they do not consider unconventional monetary policy nor do they focus on the different degrees of leverage on the supply and demand side of financial markets.

[8] As suggested by an anonymous reviewer, the introduction of this agent allows to realistically capture U.S. capital markets, where the market channel dominates over the bank channel. However, results obtained in an earlier version of this paper without non-bank intermediaries are qualitatively similar.

[9] As shown below, the overall spread in the economy can be decomposed into the individual spreads induced by each of the frictions.

[10] The modeling of preferences for liquid assets follows Fisher (2015). Fisher (2015) shows that the resulting Euler consumption equation is similar to the one in Smets and Wouters (2007), if s_t has a zero steady state as assumed here.

2.2. Financial intermediaries

There are two types of financial intermediaries which channel household savings to intermediate firms. Banks are modeled in three layers similar to Gerali et al. (2010).[11] Banks face finite horizons, because there is an exogenous probability of death equal to $1-\theta$. They combine household deposits and their own net worth, $N_t(j)$, to finance loans to intermediate firms. A bank that operates today (from t to $t+1$) will discontinue its operations and make a stochastic dividend payment to the household it belongs to in period $t+1+k$ with probability $(1-\theta)\theta^k$ and $k \geq 0$. Its objective is to maximize the discounted stream of dividend payments

$$V_t(j) = \max E_t \sum_{k=0}^{\infty} (1-\theta)\theta^k \Lambda_{t,t+1+k} N_{t+1+k}(j) \tag{3}$$

where $\Lambda_{t,t+1}$ is the household's stochastic discount factor (SDF). Following Gertler and Karadi (2011), an agency problem is introduced to impose a leverage constraint on banks. That is, each period the management board of a bank can decide to divert a fraction λ of assets. If it does so, this comes at the cost of being forced into bankruptcy by depositors that recover the remaining fraction $1-\lambda$. Only if there is no incentive to divert funds from the balance sheet households are willing to lend to banks. This implies that the following incentive compatibility constraint

$$V_t(j) \geq \lambda B_t^{Retail,MB}(j) \tag{4}$$

must be satisfied at any time. The left hand side indicates that the bank would lose its entire franchise value in the event of diverting funds. The right hand side indicates that the bank would gain the amount $\lambda B_t^{Retail,MB}(j)$ by cheating depositors. As long as this gains do not exceed losses there is no incentive for fraud and depositors are willing to lend to banks. From the maximization problem (3) subject to (4) one obtains in the symmetric equilibrium

$$\nu_t = E_t\left[(1-\theta)\Lambda_{t,t+1}(R_{t+1}^{Retail} - R_{t+1}) + \Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1}\right] \tag{5}$$

$$\eta_t = E_t\left[(1-\theta) + \Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\right] \tag{6}$$

$$z_{t,t+1} = \left[(R_{t+1}^{Retail} - R_{t+1})\phi_t + R_{t+1}\right] \tag{7}$$

$$x_{t,t+1} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1} \tag{8}$$

$$B_t^{Retail,P} = \phi_t N_t \tag{9}$$

$$N_t = \theta\left[(R_t^{Retail} - R_t)\phi_{t-1} + R_t\right] N_{t-1}\exp(\mathcal{E}_t^{NS}) + \omega B_{t-1}^{Retail,P} \tag{10}$$

where R_{t+1}^{Retail} is the real gross return earned on a bank's loan portfolio, ν_t is the marginal value of a bank loan, η_t is the marginal value of bank net worth, $z_{t,t+1}$ is the gross growth rate of bank net worth, $x_{t,t+1}$ is the gross growth rate of bank loans and ϕ_t is the leverage ratio of each bank. Eq. (9) shows that capacity of banks to offer loans is tied to their level of net worth and Eq. (10) determines the law of motion of bank net worth in the aggregate, where $\mathcal{E}_t^{NS}$ is an exogenous shock to bank net worth.

In addition to banks, non-banks are introduced as a second type of intermediary in order to allow intermediate firms to obtain funds from the corporate bond market as a second source of external finance. Different from banks, non-banks face no agency problem when issuing deposits and do not accumulate net worth. A representative non-bank intermediary creates a loan

$$B_t(i)^{\mathrm{NB}} = \left(D_{t+1}^{\mathrm{NB}}(i)\right)^{\alpha_{NB}}, \quad \alpha_{NB} < 1 \tag{11}$$

subject to a decreasing returns to scale technology. The aspect that non-banks have no agency problem, in contrast to banks, makes them the dominant providers of external funds. Under perfect competition, the returns earned by the non-bank intermediary R_{t+1}^{NB} equal R_{t+1}^{Retail}. Since a non-bank remunerates household deposits, D_{t+1}^{NB}, with $R_{t+1}^{d,\mathrm{NB}}$ and due to profit maximization, in a symmetric equilibrium, the total supply via the market channel is given by

$$B_t^{\mathrm{NB,P}} = \left(\frac{R_{t+1}^{Retail}}{R_{t+1}^{d,\mathrm{NB}}}\right)^{\frac{\alpha_{NB}}{1-\alpha_{NB}}}. \tag{12}$$

2.3. Financial contracts and firms

Competitive intermediate firms produce output from labor and capital inputs. The capital stock, $K_t(f)$, is repurchased at the relative price, Q_t, each period and enters into the production function in the subsequent period. Capital purchases are financed from firm's net worth, $N_t^{int}(f)$, and external borrowing, $B_t^{int}(f)$. Hence, the balance sheet identity applies

$$Q_t K_t(f) = N_t^{int}(f) + B_t^{int}(f). \tag{13}$$

External finance comes from two sources. An intermediate firm can negotiate a loan contract with a bank or issue a one period bond and sell it to a non-bank intermediary. Since both intermediaries offer the firm the same contract terms, the bank lending channel and

[11] See the appendix for further details.

the market channel are perfect substitutes for the intermediate firm. Following Bernanke et al. (1999), there is an agency problem in the borrower lender relationship arising from asymmetric information. That is, beyond aggregate risk, the capital investment of a firm is subject to idiosyncratic risk which is unobservable for the lender. In case of a defaulting loan, the lender must pay a monitoring cost to observe the state of the borrower's investment. This monitoring cost, $\mu\omega^f R^k_{t+1} Q_t K_t(f)$, is proportional to the realized project outcome with a proportionality factor, μ, determining the degree of the friction. This is basically the costly state verification problem in the spirit of Townsend (1979). A key assumption is that intermediate firms are risk neutral and bear all the aggregate risk involved in the investment. Since both types of intermediaries are able to diversify the idiosyncratic risk, they earn a predetermined return on their loan portfolios. Similar to banks, there is a constant survival probability, γ, to prevent intermediate firms to become fully self-financing. Operating intermediate firms, in aggregate, accumulate $\zeta^{\text{int}}(\bar{\omega}_{t+1}) R^k_{t+1} \phi^{\text{int}}_t N^{int}_t$ from t to $t+1$. Exiting firms make a dividend payment to households and are replaced with new firms that obtain start-up funds $N^{\text{int,new}}_t = X$ from their respective household. Aggregate net worth of intermediate firms is therefore given by

$$N^{\text{int}}_{t+1} = \gamma \zeta^{\text{int}}(\bar{\omega}_{t+1}) R^k_{t+1} \phi^{\text{int}}_t N^{int}_t + X. \tag{14}$$

2.4. Unconventional policy and capital market clearing

The central bank conducts both conventional and unconventional monetary policy. Conventional interest rate policy is implemented via a Taylor rule, which determines the riskless rate.[12] To introduce unconventional monetary policy, the central bank assists in the process of creating loans. That is, it purchases composite bonds, $\tilde{B}^{CB}_t$, from the continuum of bank retail branches and non-bank intermediaries.[13] These purchases are financed by issuing government debt, $B^{CB}_t = \tilde{B}^{CB}_t$, to households which pay the riskless return and are perfect substitutes for deposits at banks.[14] Note that there is no agency problem between households and government. Therefore, in contrast to banks, the government is not constrained by obtaining funds. Also the central bank, in contrast to non-banks, has constant returns to scale. Let the total amount of loans created by banks and non-banks and demanded by intermediate firms be $B^{Total}_t = \int_0^1 B^{int}_t(f)df = \int_0^1 B^{Retail}_t(j)dj + \int_0^1 B^{\text{NB}}_t(i)di$. By aggregating over all bank retail branches and non-banks one obtains

$$B^{Total}_t = B^{Retail,P}_t + B^{\text{NB},P}_t + B^{CB}_t. \tag{15}$$

The total amount of loans is due to private lending and lending under government assistance, where the former is constrained by the aggregate stock of banks net worth and decreasing returns to scale of non-banks. Unconventional monetary policy is implemented by assuming that the central bank finances a fraction, ψ_t, of the total value of intermediated loans, B^{Total}_t.

$$B^{CB}_t = \psi_t B^{Total}_t \tag{16}$$

By combining Eqs. (15), (16), (12) and (9), I obtain

$$B^{Total}_t = \phi^{\text{total}}_t N_t + \frac{1}{1-\psi_t}\left(\frac{R^{Retail}_{t+1}}{R^{d,NB}_{t+1}}\right)^{\frac{\alpha_{NB}}{1-\alpha_{NB}}} \tag{17}$$

where $\phi^{\text{total}}_t = \phi_t/(1-\psi_t)$ is the total leverage applied to bank's net worth taking account of central bank intermediation. The fraction, ψ_t, of the total value of intermediated loans, intermediated under central bank assistance, follows a feedback rule of the form

$$\psi_t = \psi + \upsilon E_t\left(\log R^k_{t+1} - \log R_{t+1} - \log\frac{R^k}{R}\right) + \mathcal{E}^{\psi}_t. \tag{18}$$

According to (18), the central bank credit policy is characterized by a steady state fraction ψ of loans intermediated under public assistance, a non-negative response coefficient υ and an exogenous credit policy shock ϵ^{ψ}_t. The idea is that the central bank uses the overall spread between the aggregate return to capital and the safe rate, $\log R^k_{t+1} - \log R_{t+1}$, in deviation from its steady state, $\log\frac{R^k}{R}$, in order to proxy for the degree of credit scarcity in the economy. In an economic downturn external finance becomes more expensive for non-financial firms as their net worth deteriorates. To stabilize aggregate demand the central bank expands its credit policy as the spread increases. This will increase the leverage ratio, ϕ^{total}_t, and therefore the total amount of loans.

[12] See the appendix for details.

[13] At this point my modeling differs from Gertler and Karadi (2011). Their model considers central bank's direct lending to non-financial firms financed via government debt issued to households or equivalently interest bearing reserves issued to financial intermediaries. In my framework the central bank channels household's savings to the retail branches in order to bypass the agency friction within management boards. In contrast to direct lending, the costly state verification friction between non-financial firms and intermediaries still applies (see the appendix for further details).

[14] Within the steady state, and as long as there are no liquidity shocks, also non-bank deposits are perfect substitutes for government bonds.

3. Quantitative results

3.1. Calibration

There are thirty-eight parameters in the model. For the sake of comparability, most of the calibration follows Gertler and Karadi (2011). Table 1 summarizes the baseline calibration for the twenty-six parameters related to the endogenous propagation mechanisms. The calibration of the remaining twelve parameters related to exogenous shock processes can be found in Table 7 in Appendix A.5. Among the twenty-six parameters related to the endogenous propagation mechanisms, seventeen are conventional, three are particular to the design of banks along the lines of Gertler and Karadi (2011), $(\theta, \omega, \lambda)$, four are particular to the introduction of firm's net worth and the BGG-type borrower lender relationship, $(X, \sigma_\omega, \gamma, \mu)$, and two are related to unconventional policy, (ψ, υ).

With respect to households, the discount factor, β, the habit formation parameter, Ξ, and the inverse of the Frisch elasticity of labor supply, φ, are set to conventional values. The value for the scaling parameter of disutility from labor supply, χ, is calibrated to target a steady state value for labor supply of one third. On the production side the capital share, α, the elasticity of substitution between different varieties, ε, the degree of price indexation, γ_p, the parameter in the functional specification of investment adjustment costs, η_i, and the price rigidity parameter, κ, are within usual ranges. The latter implies an average duration between subsequent price adjustments of more than four quarters. Regarding the functional form for the depreciation of capital, $\delta(u_t)$, the elasticity of marginal depreciation with respect to the utilization rate, ζ, is set to 7.2 and the two remaining parameters, (δ_c, b), are calibrated to match a steady state depreciation rate of 0.025 and a normalized steady state utilization rate of unity, respectively.[15] Following Carlstrom et al. (2016, p. 125), the start-up funds to new firms, X, can be set to an arbitrarily small number.

The government sector is characterized by a Taylor rule, a credit policy rule and a government spending process. The Taylor rule parameters are standard. An inflation coefficient of 1.5 implies that the central bank will respond to an increase of the inflation rate with a more than one-to-one increase in its policy rate, which is referred to as the Taylor principle. The size of the output coefficient is 0.125, because the model runs on a quarterly frequency, and under the baseline calibration, the interest rate smoothing coefficient will be set to zero. The credit policy rule is calibrated such that in the steady state the central bank does not assist as an intermediary, but the central bank will increase its market share in credit markets by ten percent if the spread between the return to capital and the deposit rate increases by one percent. Central bank intermediation is associated with an inefficiency cost, τ, per intermediated unit of ten basis points. The steady state of government spending, $\bar{G}$, is calibrated to match a ratio of government spending over output of 0.2.

The survival rate of banks, θ, is set to 0.94.[16] I calibrate the proportional transfer of start-up funds to new bankers, ω, and the fraction of assets that can be diverted, λ, in order to target an annualized steady state spread between retail return and deposit rate of one hundred basis points and a steady state leverage ratio for banks of twenty—which is a plausible value according to Gertler and Karadi (2011).

Finally, the modeling of intermediate firm's net worth and the costly state verification friction in the borrower lender relationship introduces a survival rate of intermediate firms, γ, monitoring costs, μ, and a standard deviation for the idiosyncratic productivity shock, σ_ω. These parameters are calibrated to match three targets: A leverage ratio of intermediate firms of two, an annualized steady state spread between the return to capital and the retail return of one hundred basis points and an annual bankruptcy rate of intermediate firms of three percent. It should be noted that the choice of relative steady state leverage (banks 20, firms 2) is large compared to previous research papers (banks 4 in Gertler and Karadi (2011), firms 2 in Bernanke et al. (1999)). As I point out in Appendix A.3.1, this choice is more in line with the empirical evidence and additionally a more conservative choice in the sense that it calibrates the model against my conjecture that modeling both types of net worth increases the resilience of the model and leads to smaller gains of unconventional policy.

3.2. Unconditional moments of the business cycle

Before turning to the simulation results, it may be important to compare the model implied unconditional moments across models and with the data. Fig. 1 compares the cross-correlation structure obtained from the equity tandem model (ET), the (Gertler and Karadi, 2011) model (GK) and a version of the model presented in this paper where only the (Bernanke et al., 1999) friction is active (BGG) with the data. Cross-correlations of variables at time t with output at time $t+\tau$ are computed and for the data 95% confidence intervals are estimated using a GMM estimator.[17] For the model simulations I simultaneously turn on the TFP-Shock, the government spending shock, the interest rate shock and the capital quality shock. As can be seen from Fig. 1, all models match the observed

[15] The functional specification of the depreciation rate is $\delta(u_t) = \delta_c + \frac{b}{1+\zeta} u_t^{(1+\zeta)}$. This functional form implies an elasticity of marginal depreciation with respect to the utilization rate of $\frac{\partial \log \delta'(u_t)}{\partial \log u_t} = \zeta$.

[16] I set the survival rate of banks to a smaller number, compared to Gertler and Karadi (2011). Realistically the banking sector operates with a larger leverage ratio. This implies that surviving banks will enjoy a larger gross growth rate of net worth in steady state, provided that limits to arbitrage imply a positive spread. To guaranty that banks do not become fully self-financing, I cut the survival rate of bankers somewhat. This implies an expected lifetime horizon of a banker of more than four years. The large steady state leverage ratio additionally effects the calibration of ω, because more leveraged banks require less start-up funds.

[17] For the implementation of the GMM estimator, I make use of the replication codes from Christiano et al. (2014) available at https://www.openicpsr.org/openicpsr/project/112728/version/V1/view (downloaded 01.05.2023).

Table 1
Calibration.

Parameter	Value	Description
Household		
β	0.9900	Household discount factor
Ξ	0.8150	Habit formation parameter
φ	0.2760	Invers of Frisch elasticity of labor supply
χ	3.3620	Scalling parameter of disutility of labor
Intermediate firms		
α	0.3300	Capital share
b	0.0401	Slope of depreciation rate
δ_c	0.0201	Intercept of depreciation rate
ζ	7.2000	Elasticity of mar. dep. w.r.t. utilization
X	0.0000	Start-up funds of new firms/entrepreneurs
Retail firms		
ϵ	4.1670	Constant elasticity of substitution
κ	0.7790	Calvo price rigidity
γ_p	0.2410	Degree of price indexation
Capital producing firm		
η_i	1.7280	Inverse elasticity of net investment to the price of capital
Banks		
θ	0.9400	Survival rate of banks
ω	0.0002	Proportional transfer to new bankers
λ	0.2337	Risky fraction of loan portfolio
Central bank & Fiscal policy		
κ_π	1.5000	Taylor rule inflation coefficient
κ_y	0.1250	Taylor rule output gap coefficient
κ_i	0.0000	Interest rate smoothing coefficient
υ	10.0000	Responds coefficient of credit policy rule
ψ	0.0000	Steady state fraction of publicly intermediated loans
τ	0.0010	Inefficiency cost per unite intermediated by the central bank
$\bar{G}$	0.1644	Steady state government spending
BGG related parameters		
σ_ω	0.2705	Standard deviation of log of idiosyncratic risk
γ	0.9831	Survival rate of entrepreneurs/intermediate firms
μ	0.0592	Monitoring cost parameter

cross-correlations displayed in the first row reasonably well and tend to have problems in the second row. The Gertler and Karadi (2011) model cannot explain the cross-correlation of total credit and firm leverage with output, respectively, reasonably well. Firm leverage is not properly defined at all. Whereas the BGG mechanism alone cannot explain the correlation between bank leverage and output for the same reason. Also the model presented here (ET) fails to explain this cross-correlation with reasonable accuracy. The counterfactual strong and positive correlation between contemporaneous bank leverage and contemporaneous output stems from the predetermined return to bank loan portfolios introduced by the financial contract from Bernanke et al. (1999). Fig. A.9 in Appendix A.2 compares the model implied autocorrelations with the data. The autocorrelation structure of the equity tandem model tends to resemble the observed pattern in the data, whereas GK again has problems with total credit and firm leverage and BGG cannot explain bank leverage. However, overall, one should note that the equity tandem model is only a small improvement compared to GK and BGG.

3.3. Equity and the resilience of the economy

To begin with, I should note that equity—or similarly leverage—matters for the resilience of the economy. This is illustrated in Fig. 2 which displays the impulse response functions (IRFs) to a contractionary TFP shock of one percent. By assumption, unconventional policy and interest rate smoothing are turned off ($\upsilon = 0, \kappa_i = 0$). The solid black line depicts the response of the baseline GK model where we only have net worth on the supply side (in banks).[18] The GK model assumes a steady state leverage ratio of banks of four. In contrast, my extension of this model is based on the assumption of a steady state leverage ratio of banks of twenty and the dashed and colored IRFs display the dynamics of this model calibrated to different steady state leverage ratios of intermediate firms.[19]

[18] Replication files are available upon request. I make use of the sample code taken from Peter Karadi's homepage to draw the IRFs for the GK model. Relevant snippets from these files are reused and modified to simulate the extended model presented in Section 2.

[19] Large U.S. investment banks tend to have leverage ratios with individual averages across time ranging between 22 to 32 before 2008 (see Table 8.1 in Adrian and Shin, 2008, p. 39). In aggregate, leverage ratios of investment banks can be quite large and volatile compared to commercial banks. For all banks, in aggregate the leverage ratio in the US is quite stable and somewhat below 15. In contrast, for Europe the aggregate bank leverage ratio reached levels somewhat above 25 in 2008 (see Fig. 4 in Kalemli-Ozcan et al., 2012, p. 290). Hence, targeting a steady state leverage ratio of banks of twenty does not seem too implausible.

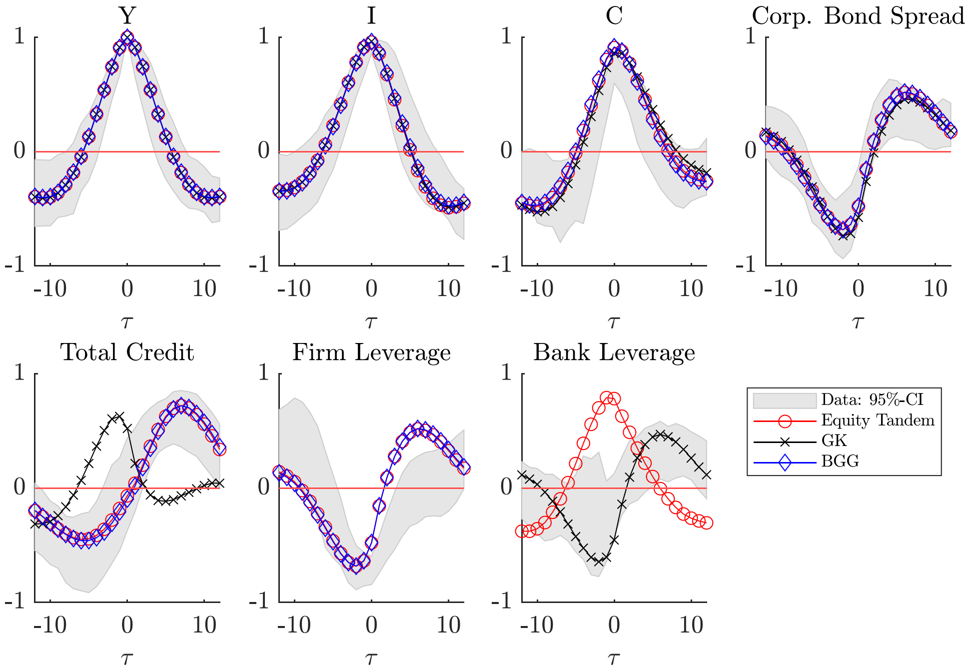

Fig. 1. Selected Cross-Correlations between variable at t and output at $t + \tau$.
Note: The figure compares the model implied cross-correlations with the data. Both data and model simulations are HP-filtered with smoothing parameter set to 1600 for quarterly frequencies. For reference to data sources and transformations see Appendix A.1.

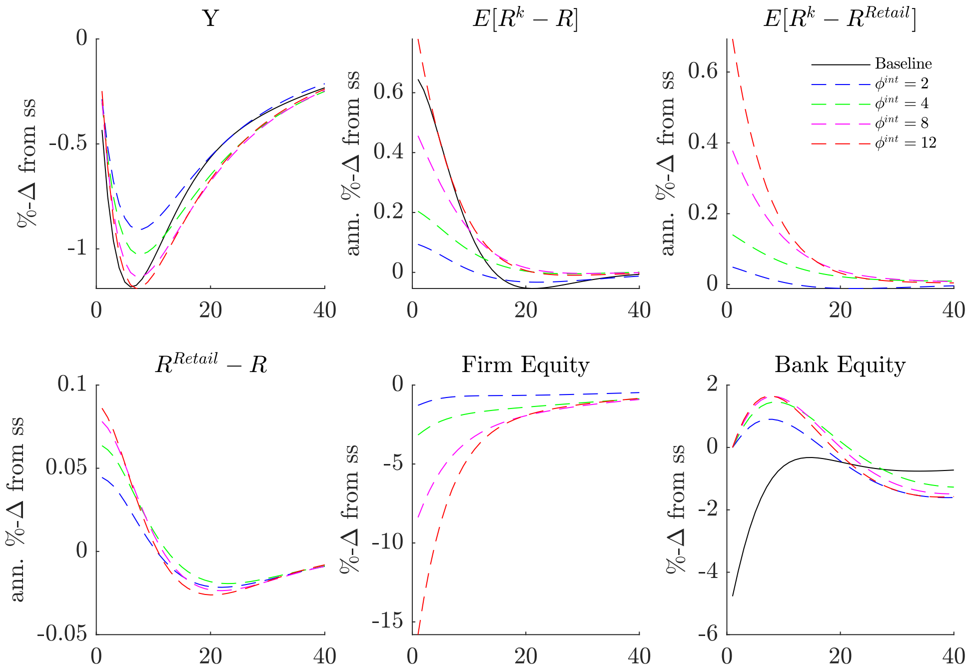

Fig. 2. Firm Leverage and Responsiveness to Shocks.
Note: The figure assumes a decline of TFP by one percent, no unconventional monetary policy, $\upsilon = 0$, and no interest rate smoothing, $\kappa_i = 0$. Impulse response functions are computed for the baseline GK model and the extended model. They are expressed as percentage deviations from steady state (ss). Colored impulse response functions display the dynamics of the extended model calibrated to different steady state leverage ratios of intermediate firms, ϕ^{int}.

It is important to note the differences in financial amplification mechanisms at work. In both models, the unexpected decline in total factor productivity leads to a drop in the ex-post realized return to capital—due to the immediate decline in the marginal productivity of capital. In the baseline model, intermediate firms earn zero profits and due to perfectly state contingent debt, on impact, banks experience a decline of net worth. Because banks are leverage constrained (by depositors), this leads to a shortage of external funds offered to intermediate firms to finance capital investments which leads to an increase in the premium, $E_t[R^k_{t+1} - R_{t+1}]$. Consequently, intermediate firms decide to invest less, which leads to a decline of the price of capital (or asset prices). The latter further depresses the ex-post realized return to capital and bank net worth. As a result, a tighter leverage constraint leads to a higher premium which further depresses investment, output and asset prices. The feedback loop between asset prices and intermediary net worth is the core of the financial amplifier in the baseline GK model.

Once I introduce the equity tandem, the financial amplification mechanism slightly changes. Because firms honor their debt—as long as they do not default on their debt—the unexpected decline in the ex-post return to capital is covered by losses in intermediate firms net worth. In relative terms, the downturn of net worth increases with the steady state leverage of firms—as can be seen from Fig. 2. To cover the higher expected monitoring costs—because firms have less skin in the game—banks will charge a higher spread, $E_t[R^k_{t+1} - R^{Retail}_{t+1}]$. Hence, investment and output will decline as well as asset prices. This leads to a further decline of intermediate firms equity which keeps the contractionary feedback loop going. Because firms are liable for payments of debts and banks charge a premium to cover monitoring costs bank equity does not experience a downturn. Instead, the decline of the policy rate—due to the decline of inflation and output—increases the profitability of banks in the short to medium term. Consequently, the increase in bank equity tends to increase the supply of external finance which somewhat counteracts the BGG-type financial accelerator.

The key message from Fig. 2 is that higher net worth (lower leverage) of intermediate firms improves the economies resilience. Everything else equal, the economic downturn and the financial feedback loops are less pronounced if intermediate firms are less leveraged.[20] Further robustness checks, found in Table 5 within Appendix A.3, show that this result holds for a set of different shocks because a lower steady state leverage of intermediate firms clearly decreases the volatility of key variables. Since non-financial firms are typically less leveraged, compared to financial firms, the propagation and amplification of macroeconomic shocks is muted compared to a *supply side only* approach heavily used in the (theoretical) QE-literature.[21]

3.4. Implications for unconventional policy

I will now turn to the implications of the equity tandem for unconventional monetary policy. Fig. 3 compares the response of both model versions to a capital quality shock (ξ_t) of −5% under a baseline ($\nu = 10$) and an aggressive ($\nu = 100$) credit policy. The key findings are as follows.

As in the previous section, compared to the underlying GK model (labeled as Baseline), the economy responds less strongly to the capital quality shock once non-financial firms are subject to the BGG contract. This implies that the central bank responds with a smaller amount of unconventional monetary policy, since the percentage of the capital stock financed by the CB, ψ, is smaller on impact. However, since the overall spread, $E_t(R^k_{t+1} - R_t)$, is more persistent under the BGG extension (labeled Equity Tandem), this also holds true for the credit policy, ψ. In other words, the unwinding of the unconventional monetary policy is slower once I consider net worth of both financial and non-financial firms.

The most notable difference concerns the evolution of banks and entrepreneurs net worth. Under the BGG contract one can think of the banking sector financing a well diversified debt security which guaranties a predetermined return. Hence, on impact there is no deterioration of bank's net worth. Instead, the impact of the capital quality shock is internalized entirely by non-financial firm's net worth. Because non-financial firms are less leveraged, the overall decline in entrepreneur net worth is about half the magnitude compared to the decline in bank's net worth under the GK model. Following the initial response, financial and non-financial firms share the burden of the capital quality shock in the subsequent adjustment process—due to unconventional policy. This implies that the exit from the unconventional monetary policy occurs at a point in time when both banks and firms are recapitalized.

Another observation is that an aggressive credit policy is shifting net worth from the banking system to the balance sheets of non-financial firms under the equity tandem. On the one hand, the aggressive credit policy ($\nu = 100$) leads to a smaller decline in output and inflation and therefore the rate cut according to the Taylor Rule is smaller. On the other hand, the central bank credit policy increases the supply of external funds and drives down the interest rate R^{Retail}_t. This leads to a smaller spread received by banks. This in turn decreases the opportunity costs of bank's retail branches and leads to a smaller increase in the costs of external finance which ameliorates investment and leads to a smaller downturn in asset prices (price of capital Q). Overall, one can see a small improvement of non-financial firms net worth at the cost of lower net worth of banks compared to the baseline credit policy.

Another lesson taken from Fig. 3 is that the relative effectiveness of unconventional monetary policy is smaller under the equity tandem. Switching from the baseline to the aggressive credit policy improves output only a bit. In the GK model this relative gain is much larger. Because non-financial firms are less leveraged—which increases the resilience of the economy—neglecting entrepreneur's net worth leads to an overestimate of the effectiveness of unconventional monetary policies. The same picture is confirmed when considering short-run and long-run effects displayed in Table 2. Without the extra equity in the economy, replacing a non-aggressive with an aggressive credit policy leads to higher absolute gains for output and investment, both on impact and accumulated over forty quarters. With the extra equity, also spreads are more stable in the long-run even under a non-aggressive policy. These differences tend to be of an economically relevant order. Finally, Table 2 shows that under the equity tandem replacing a non-aggressive with an aggressive credit policy leads to a considerable accumulated loss in bank equity and a comparably small improvement of firm equity.

[20] For the US, between 2004 and 2009 the average leverage ratio over all non-financial firms was around 2.4–2.5 (see Fig. 9 in Kalemli-Ozcan et al., 2012, p. 293).

[21] For other macroeconomic shocks I obtain similar IRFs compared to Fig. 2. Results are available upon request.

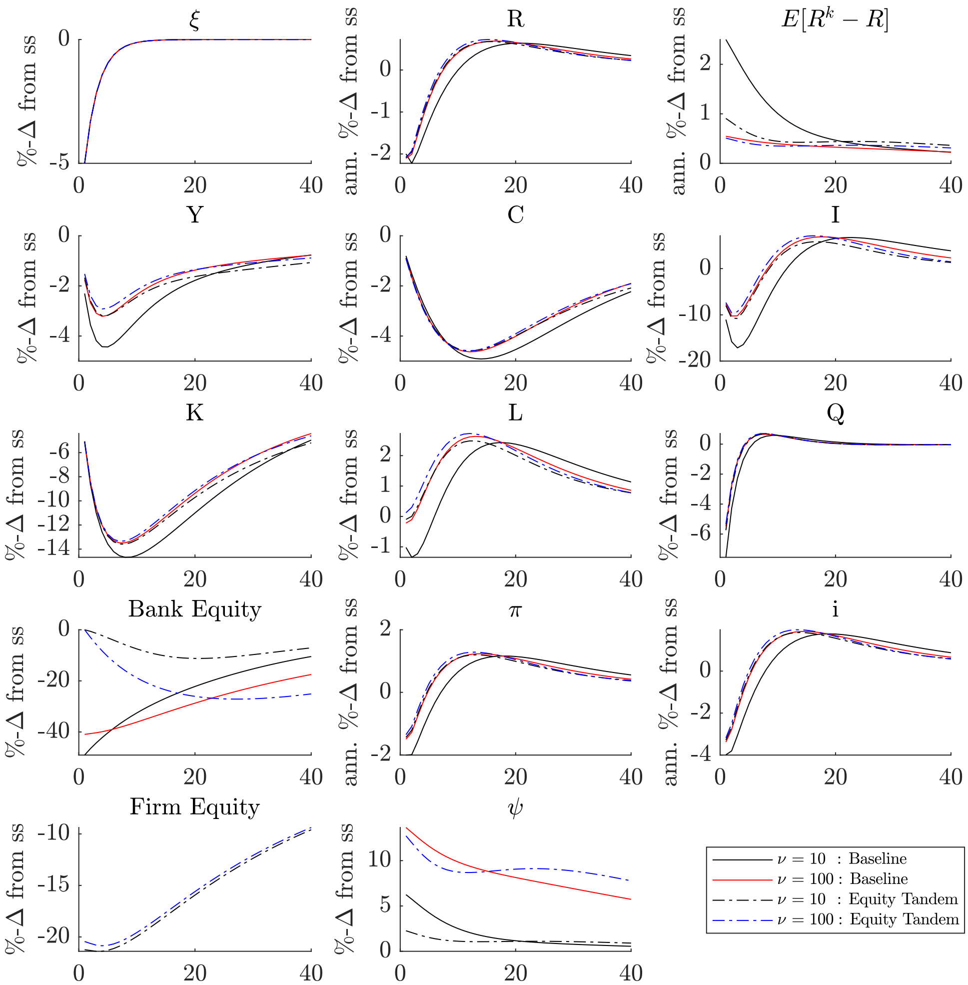

Fig. 3. Responses to a Capital Quality Shock with Credit Policy.
Note: The figure replicates the crisis experiment of Gertler and Karadi (2011, p. 29). An unexpected decline of capital quality of five percent and no interest rate smoothing by the central bank are assumed. Impulse response functions are computed for the baseline GK model and the extended model (Equity Tandem) from Section 2. They are expressed as percentage deviations from steady state (ss).

Table 2
Short-run and long-run effects of a capital quality shock with credit policy.

Shock/	Y		I		$E[R^k - R]$		Firm equity		Bank equity	
Model	IR	CE	IR	CE	IR	CE	IR	CE	IR	CE
Capital quality shock										
GK: $\nu = 10$	−2.30	−84.96	−11.02	37.31	2.50	29.48	–	–	−49.04	−971.12
GK: $\nu = 100$	−1.76	−64.76	−7.71	100.42	0.55	13.62	–	–	−40.95	−1156.65
ET: $\nu = 10$	−1.67	−72.80	−8.03	59.85	0.91	18.82	−21.24	−634.06	0.00	−327.10
ET: $\nu = 100$	−1.52	−62.72	−7.34	99.88	0.51	14.55	−20.45	−619.79	−0.00	−872.92

Note: The table displays the impact response (IR) and cumulative effect (CE) of an unexpected decline of capital quality of five percent on selected variables for the Gertler & Karadi (2011) model and the equity tandem under a non-aggressive, $\upsilon = 10$, and an aggressive credit policy, $\upsilon = 100$. No interest rate smoothing, $\kappa_i = 0$, is assumed. Cumulative Effects are computed over 40 quarters and units are in terms of percentage deviations from steady state.

Table 6 in Appendix A.3 explores the robustness of these results for the model implied standard deviations of selected variables across a set of different shocks. Overall, a more aggressive unconventional monetary policy leads to a larger relative decline of standard deviations under the GK model for most of the scenarios. This supports the observation that neglecting net worth on one side of the capital market leads to an overestimate of the effectiveness of unconventional policy.

A discrepancy between the models concerns the dynamics of bank's leverage ratio. Fig. 4 displays the bank's leverage ratio, ϕ_t, as well as the coefficients of the value function. In the GK model the leverage ratio and both the marginal value of net worth, η_t, (holding the asset holdings constant) and the marginal value of increasing asset holdings, ν_t, (holding net worth constant) increase

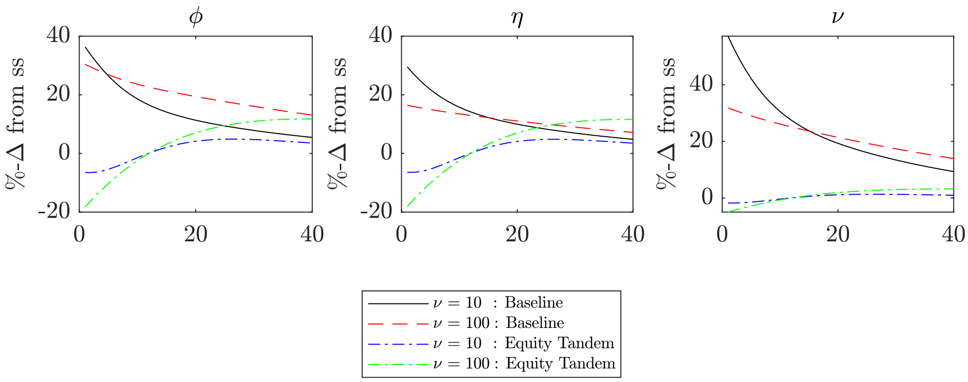

Fig. 4. Banks Leverage Ratio and Partial Derivatives of the Value function.
Note: An unexpected decline of capital quality of five percent and no interest rate smoothing by the central bank are assumed. Impulse response functions are computed for the baseline GK model and the extended model (Equity Tandem) from Section 2. They are expressed as percentage deviations from steady state (ss).

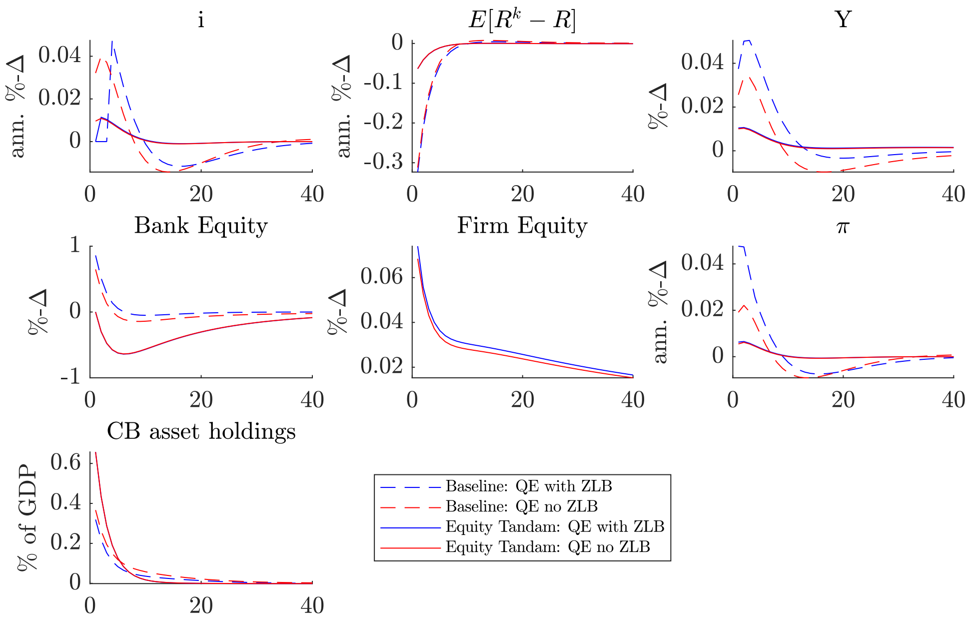

Fig. 5. Expansionary Credit Policy, Zero Lower Bound and Equity.
Note: An unexpected decline of capital quality of 7.5% in the first two quarters, an unexpected increase in the central banks (credit) market share, ψ_t, by 1% in the third quarter, a baseline credit policy with $\upsilon = 10$, and no interest rate smoothing by the central bank are assumed. Impulse responses are displayed as the difference between the trajectory based on all shocks minus the trajectory based on both capital quality shocks. The first two shock periods are discarded from the displayed IRFs.

in the presence of the capital quality shock. This is because in the GK model the overall spread, $E_t(R^k_{t+1} - R_{t+1})$, increases and is entirely internalized by banks. In the extended model, banks earn the spread, $R^{Retail}_t - R_t$, which is below trend. Hence, the banking system is modestly deleveraging on impact and increases leverage over the course of the crisis under the equity tandem. In contrast to Gertler and Karadi (2011), where the deleveraging of banks slows down the recovery, here it is the deleveraging of non-financial firms.

3.5. Implications of the ZLB

In order to generate a binding zero lower bound, I assume a sequence of unforeseen innovations which hold the capital quality shock (ξ_t) at −7.5% in the first two quarters. To account for the kink in the decision rules, which is generated by the zero lower bound, I consider the ZLB as an occasionally-binding constraint and use the OccBin toolkit of Guerrieri and Iacoviello (2015) which is part of Dynare (see Adjemian et al., 2022).

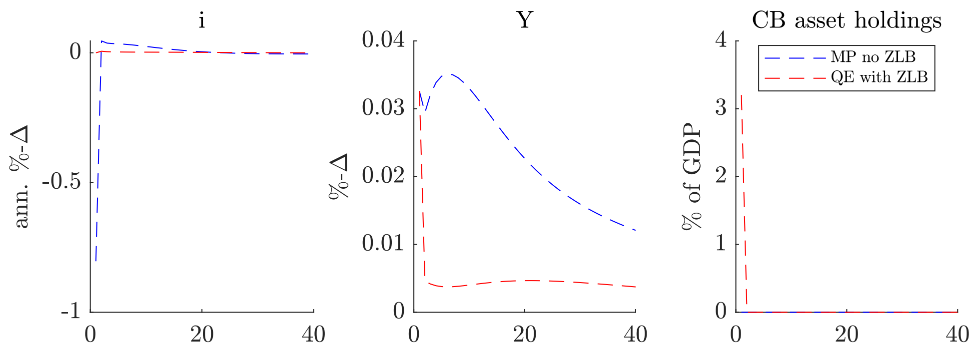

Fig. 6. Comparison of conventional and unconventional policy.
Note: The figure uses the extended model from Section 2 and compares a conventional rate cut in normal times with an unconventional credit policy in times of a binding ZLB. For the credit policy an unexpected decline of capital quality of 7.5% in the first two quarters is supposed to generate a binding ZLB. The increase in the central banks (credit) market share, ψ_t, in the third period is calibrated to match the initial output response from the conventional policy. Further, no endogenous response of credit policy $\upsilon = 0$, one-off shocks $\rho_i = \rho_\psi = 0$ and no interest rate smoothing by the central bank are assumed. Impulse responses are displayed as the difference between the trajectory based on all shocks minus the trajectory base on both capital quality shocks. For the conventional policy a rate cut by 1% annualized is assumed to take place in period three. The first two shock periods are discarded from the displayed IRFs.

Fig. 5 compares the effectiveness of an expansionary credit policy or QE under both model versions with and without a binding ZLB. Impulse responses for each model version are constructed as the difference between two trajectories. Both trajectories assume a capital quality shock (ξ_t) of −7.5% in the first two quarters to push the shadow interest rate into negative territories. Additionally, one trajectory is constructed by taking into account an expansionary credit policy shock of 1% in the third quarter. The first two shock periods are discarded from the displayed impulse response functions. The nominal interest rate, the overall spread and the inflation rate are expressed as annualized percentage differences between both simulations. Output, bank and firm equity are expressed as percentage difference and asset holdings of the central bank are expressed in percentages of annualized steady state output. Impulse responses with ZLB are constructed with the piecewise-linear solution of Guerrieri and Iacoviello (2015) and impulse responses without ZLB are based on a first-order perturbation.

In all configurations, the expansionary credit policy leads to a compression of the overall spread by about 6 to 32 annualized basis points. After taking the endogenous response of the feedback rule into account, the central bank's asset holdings increase, initially, between 0.3% and 0.65% of annualized steady state output. The increase in credit intermediation by the central bank stimulates investments and consequently increases output and inflation. The additional demand for capital increases asset prices which in turn increases the ex-post realized return to capital (and leads to a decline in the expected return to capital). In the baseline GK model this leads to an increase in bank equity, meanwhile in the extended model firms internalize the unexpected increase in the ex-post realized return to capital. In Fig. 5 the influence of the ZLB is clearly visible in the baseline case as it takes one year to leave the lower bound constraint. Since there is no interference with conventional interest rate policy during the ZLB episode, unconventional monetary policy is more effective at the interest rate lower bound.[22] The peak response of output is about 50% stronger and inflation increases more than twice as much under the ZLB. Once I consider the equity tandem, the impact of unconventional monetary policy on output and inflation is more muted and the influence of the ZLB vanishes almost completely. After the expansive credit policy shock, it takes only one additional quarter for the economy to leave the lower bound and the influence of unconventional monetary policy on output and inflation is only marginally stronger at the ZLB. Hence, there are two implications from the simultaneous modeling of borrowers and lenders net worth. First, gains in terms of output and inflation from QE policies might be smaller. Second, it might be less relevant for the effectiveness of QE policies whether they are applied within a zero interest rate environment.

An important question concerns whether and, if so, under which conditions unconventional monetary policies like QE are a viable substitute for conventional monetary policy.[23] Provided QE is a substitute, a key question is how much QE is necessary to replace a given rate cut, if the latter is unattainable due to the ZLB. Fig. 6 addresses this question for the extended model derived in Section 2. It compares a decline in the nominal interest rate by one percentage point at an annualized rate with an expansion of the central bank's balance sheet when the interest rate is constrained from below. For comparability, both shocks feature no persistence ($\rho_i = \rho_\psi = 0$). Displayed IRFs are generated along the same line as in Fig. 5. If there is no endogenous response of credit policy to the decline in the overall spread, an increase in the central bank's balance sheet by about 3.2% of annual steady state output is

[22] This is a well known result in the literature (e.g., Gertler and Karadi, 2013, p. 41).

[23] I do not provide further insides into this discussion. Sims and Wu (2020) argue that they are substitutes. Sims et al. (2023) also argue that QE policies might be useful at any time, because optimal policy adjusts the interest rate in response to natural rate shocks and QE in response to credit market shocks. Furthermore, they argue that QE seems to be an imperfect substitute within a ZLB-environment. Similarly, Karadi and Nakov (2021) find that the nature of the shock matters since QE can achieve the best outcome if the origin of the shock is in the financial sector. However, they warn that QE might not necessarily be a useful substitute for conventional interest rate policy when the latter is constrained, because in their model the effectiveness of QE depends on the state of the financial sector (whether the occasionally-binding balance sheet constraints are binding or not).

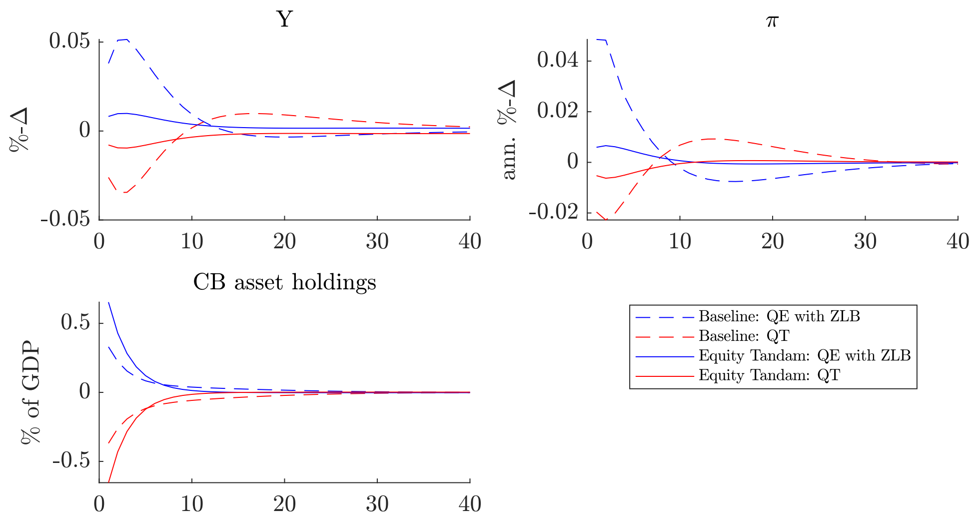

Fig. 7. Exogenous QE and QT.
Note: Only for QE an unexpected decline of capital quality of 7.5% in the first two quarters is assumed. Additionally, in the third quarter, an unexpected increase in the central banks (credit) market share, ψ_t, by 1% takes place, a baseline credit policy with $\upsilon = 10$, and no interest rate smoothing by the central bank are assumed. Impulse responses are displayed as the difference between the trajectory based on all shocks minus the trajectory based on both capital quality shocks. For QT the sign of the policy shock in the third quarter is changed and conventional IRFs are displayed. The first two shock periods are discarded from the IRFs.

sufficient to proxy for the initial output response of a conventional decline in the policy rate.[24] However, since the expansionary credit policy leads to a decline in bank equity and the assumed persistence parameter, ρ_ψ, implies an immediate unwinding of the central bank's balance sheet, the initial output increase is very short-lived compared to the temporary rate cut.[25] Since also the inflationary effects are comparably small, unconventional asset purchases seem to be rather an imperfect substitute.

3.6. Quantitative tightening

Because the size of central banks balance sheets like the Fed or the ECB reached unprecedented levels, especially after the recent use of QE to overcome the COVID-19 crisis and its lockdown measures, the question arises of how to normalize monetary policy. I now consider the reverse operation of QE referred to as Quantitative Tightening (QT). First, I will compare an exogenous increase in central bank credit intermediation in an environment featuring a binding ZLB with an exogenous decrease in central bank credit intermediation in normal times. This exercise addresses the question whether the working of QE and QT is symmetric, assuming that both policies are native to different environments. Second, I shall consider an experiment, similar to Sims and Wu (2021, p. 156), where endogenous QE is followed by a credibly communicated and anticipated QT policy once the ZLB lifts. This allows to study the implications of QT as well as the interaction between QE and QT.

Fig. 7 compares an exogenous QE shock under a binding ZLB with an exogenous QT shock in normal times, assuming both policies are native to different environments. To generate a ZLB environment, similar to Fig. 5, an unexpected decline of capital quality of 7.5% in the first two quarters is assumed. To allow for exogenous QT in normal times I recalibrate the steady state fraction of intermediated loans by the central bank, ψ, targeting steady state central bank asset holdings relative to GDP of 14%.[26] As indicated by Fig. 7, exogenous QE and QT have somewhat asymmetric effects under the GK model (labeled as Baseline). In absolute terms, the impact of the QE shock on output and inflation is about twice as large. In contrast, under the equity tandem both policies behave rather symmetric. The main reason for the asymmetry under the GK model is the larger impact of the ZLB due to a stronger financial amplification mechanism. Compared with the equity tandem, the counteracting conventional policy is delayed for additional two quarters in the GK model. As a consequence, the expansionary impact of the QE shock is larger than the contractionary impact of the QT shock in absolute value. Since under the equity tandem financial amplifiers are weaker, the counteracting conventional policy is delayed by the ZLB only for one quarter and both policies have rather similar impacts on output and inflation, though with opposite sign.

[24] In terms of 2007-US-GDP, this implies an increase of the central bank's balance sheet by about 460 billions of US-Dollar. This corresponds to over three-quarters of the initial asset purchases by the Fed during the Great Financial Crisis, where the Fed bought agency debt and mortgage backed securities for $600 billion.

[25] Note that Sims and Wu (2021, p. 146) also find that the effect of a QE shock on output is less persistent and the inflation response is comparably small. For euro area data, Ouerk et al. (2020) also find smaller and less persistent effects of unconventional monetary policy shocks.

[26] This roughly matches the central bank assets to GDP for the United States six years after the Great Financial Crisis. For instance, see the FRED data series (DDDI06USA156NWDB).

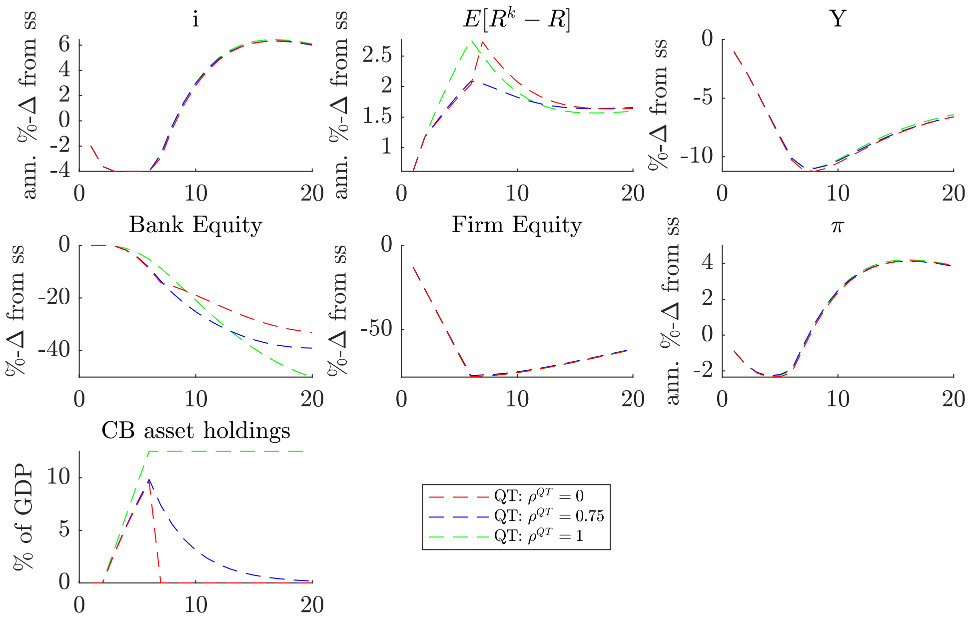

Fig. 8. Different paces of anticipated QT
Note: A sequence of capital quality innovations of −3% in the first six quarters, a baseline credit policy with $\upsilon = 10$, a persistence parameter of credit policy of $\rho^{QE} = 0.75$ and no interest rate smoothing by the central bank are assumed. Impulse response functions are displayed as percentage deviations from steady state.

In Fig. 8 I consider a different setup to compare various speeds of unwinding the central bank balance sheet. Following Sims and Wu (2021), I assume that endogenous QE is only active when the ZLB binds. Once the economy overcomes the ZLB, the subsequent unwinding of the central bank balance sheet is characterized by an AR(1) process. Additionally, in order to allow for the accumulation of a large balance sheet, as it is the case in practice for many central banks, I introduce persistence into Eq. (18). Hence, once the ZLB binds unconventional monetary policy is characterized by

$$\psi_t = (1 - \rho^{QE})\psi + \rho^{QE}\psi_{t-1} + \upsilon E_t \left(\log R^k_{t+1} - \log R_{t+1} - \log \frac{R^k}{R} \right) + \varepsilon^{\psi}_t. \tag{19}$$

In contrast, there is no endogenous QE type policy in normal times ($\upsilon = 0$) and because I allow for various speeds of unwinding the balance sheet, the subsequent QT process is given by

$$\psi_t = (1 - \rho^{QT})\psi + \rho^{QT}\psi_{t-1}. \tag{20}$$

I assume a sequence of innovations to capital quality of minus three percent in the first one and a half year in Fig. 8. This drives the economy into the ZLB environment between the third and sixth quarter. During this episode the central bank accumulates asset holdings between ten and twelve and a half percent relative to annualized steady state output. Fig. 8 compares three different QT policies after the ZLB phase using the equity tandem model. An immediate unwinding ($\rho^{QT} = 0$), a moderate unwinding over three and a half years ($\rho^{QT} = 0.75$) and no unwinding ($\rho^{QT} = 1$) of the central bank balance sheet are considered. The results suggest that the different anticipated QT policies have a very small impact on the performance of output and inflation. In contrast, Sims and Wu (2021) find that no QT leads to a somewhat worse economic downturn during the ZLB period. However, similar to Sims and Wu (2021), no balance sheet normalization requires more QE during the ZLB phase. This points to an interesting interaction between QE and QT. If agents anticipate that there will be no unwinding of the balance sheet after the ZLB lifts, QE is less effective in stabilizing spreads at the ZLB which requires more QE. Additionally, the larger amount of QE is detrimental for the profitability of banks in the medium term. On the other hand, both unwinding policies lead to a better stabilization of the overall spread at the ZLB, a smaller decline in bank equity in the medium term and a shorter central bank balance sheet. This comes at the cost of a slightly larger decline in output in the medium term because spreads tend to be slightly higher due to the contractionary monetary policy. Since in my simulation trajectories of output and inflation are very similar it might be difficult to suggest a certain QT policy to a central banker. In line with my previous result that different QE policies have more muted effects in my model this result also holds for QT. Because an immediate unwinding ($\rho^{QT} = 0$) leads to a sudden rise in the spread after the ZLB and the largest downturn in output, this seems to suggest that a credible commitment to a smooth future QT policy might be desirable if financial variables are important.[27] The argument against maintaining a large balance sheet could be further strengthened if the political cost of a large

[27] Also note that in the above experiment the smallest downturn in welfare, measured by the household's utility function (1), is obtained by the smooth QT path ($\rho^{QT} = 0.75$). Additionally, though differences are numerically small, the smooth QT path yields to the smallest contraction in output during and immediately after the ZLB. This result is available upon request.

central bank balance sheet are relatively large. This conclusion, in favor of a smooth QT path, aligns with the conclusion of Karadi and Nakov (2021) that an optimal exit from a large central bank balance sheet is gradual.

4. Conclusion

A key feature of many studies using the DSGE framework to analyze quantitative easing or unconventional monetary policy is that net worth only plays a relevant role on bank's balance sheets—if it is considered at all. In reality, however, net worth of borrowers *and* lenders plays a relevant role in financing investment projects. The present paper argues that this *equity tandem* has important implications. Net worth of non-financial firms acts as a *first line of defense*, since non-financial firm's balance sheets are hit in the first place by macroeconomic shocks originating in the real sector. Modeling the equity tandem increases the resilience of the model—because non-financial firms are typically less leveraged—and therefore implies smaller gains of unconventional monetary policy.[28] Hence, neglecting the dual role of net worth has relevant implications. Inter alia, it might lead to an overestimate of the effectiveness of unconventional monetary policy within a DSGE model when only bank's net worth is taken into account.

A novel insight from the simultaneous modeling of borrowers and lenders net worth is that a credit or QE policy is redistributing net worth between banks and non-financial firms. In this way, unconventional policies can influence the division of burdens between banks and non-financial firms in the aftermath of an economic downturn. Central bankers concerned about the health of banks should take this redistribution into account when using QE. Additionally, under the equity tandem the influence of the ZLB on the efficacy of credit policy decreases compared to the (baseline) supply side approach. This might suggest that the unconventional policy is a substitute for conventional interest rate policy under the model that I have developed in this paper. However, compared with conventional policy, large scale asset purchases lead to a very short-lived increase in output and a smaller inflation response. Even though it seems reasonable to conduct unconventional asset purchases when conventional measures are unavailable they are rather an imperfect substitute for conventional interest rate policy. Also previous papers offered arguments against a perfect substitutability. E.g. Karadi and Nakov (2021) show that under slack balance sheet constraints of banks QE might be entirely ineffective even at the ZLB. Within an overlapping generations model Sheedy (2017) argues that unconventional policies may be a poor substitute for conventional interest rate policy because of distributional effects and intertemporal trade-offs.

Finally, considering normalization policies under the model presented here exogenous QE and QT shocks tend to have rather symmetric effects on the macroeconomy, though both policies are assumed to be native to different environments—whether or not the ZLB binds. Further, different anticipated QT paths are shown to have little consequences for output and inflation but influence the build up of central bank asset holdings and the ability to stabilize spreads at the ZLB. In line with the literature, QE tends to drag on the profitability of banks and consequently a smooth QT path should be preferred (e.g. Karadi and Nakov, 2021; Sims and Wu, 2021).

Since the underlying financial structure of the macroeconomy is important for monetary transmission future research could further elaborate on the equity tandem model and its implications for unconventional monetary policy. E.g. from the literature about corporate finance and monetary policy transmission it is known that whether firms in aggregate rely more on bank loans or bond financing is relevant for the strength of the monetary transmission mechanism (e.g. Holm-Hadulla and Thürwächter, 2021). Hence, relaxing the assumption of perfect substitutability between both sources of funding and analyzing different corporate debt structures might be an interesting extension of the model.

Declaration of competing interest

I have no conflicts of interest to disclose and I am the only responsible author.

Appendix A

A.1. Data and transformations

All data series used in Sections 3.2 and A.2 are downloaded from the FRED database. The only exception is the GZ-Spread, which is taken from the Federal Reserves homepage https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv (downloaded 20.06.2023). The estimates of 95% confidence intervals are based on quarterly data ranging from Q1:1990 to Q4:2022. Table 3 summarizes the data downloaded from FRED. Table 4 documents the construction of observables that match with the model counterparts in real per capita terms.

A.2. Autocorrelation over the business cycle

See Fig. A.9.

[28] In principle, the increased resilience of the model might also limit the space for other stabilization policies. Because the model analyzed here belongs to the class of monetary DSGE models and inherits features from the literature about unconventional monetary policy, I restricted my attention to credit or QE type policies and leave the consideration of other stabilization policies for future research.

Table 3
FRED data used for observed unconditional business cycle moments.

Short name	FRED mnemonics	Description
GDP	GDP	Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
GDP deflator	GDPDEF	Gross Domestic Product: Implicit Price Deflator , Index 2012=100, Quarterly, Seasonally Adjusted
Labor force	CLF16OV	Civilian Labor Force Level, Thousands of Persons, Quarterly, Seasonally Adjusted
Investment	GPDI	Gross Private Domestic Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
PCE durable goods	PCEDG	Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
PCE	PCEC	Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
FFR	FEDFUNDS	Federal Funds Effective Rate, Percent, Quarterly, Not Seasonally Adjusted
Total credit	QUSNAMUSDA	Total Credit to Non-Financial Corporations, Adjusted for Breaks, for United States, Billions of US Dollars, Quarterly, Not Seasonally Adjusted
Firm total assets	TABSNNCB	Nonfinancial Corporate Business; Total Assets, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted
Firm net worth	TNWMVBSNNCB	Nonfinancial Corporate Business; Net Worth, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted
Bank total assets	QBPBSTAS	Balance Sheet: Total Assets, Millions of U.S. Dollars, Quarterly, Not Seasonally Adjusted
Bank equity	QBPBSTLKTEQKTBKEQK	Balance Sheet: Total Liabilities and Capital: Total Equity Capital: Total Bank Equity Capital, Millions of U.S. Dollars, Quarterly, Not Seasonally Adjusted

Table 4
Construction of observables.

Variable name	Transformation
Y	log(GDP/GDPDEF/CLF16OV $*$ $1e+08$)
I	log((GPDI+PCEDG)/GDPDEF/CLF16OV $*$ $1e+08$)
C	log((PCEC-PCEDG)/GDPDEF/CLF16OV $*$ $1e+08$)
Corp. bond spread	GZ-Spread/4
Total credit	log(QUSNAMUSDA/GDPDEF/CLF16OV $*$ $1e+08$)
Firm leverage	log(TABSNNCB/TNWMVBSNNCB)
Bank leverage	log(QBPBSTAS/QBPBSTLKTEQKTBKEQK)

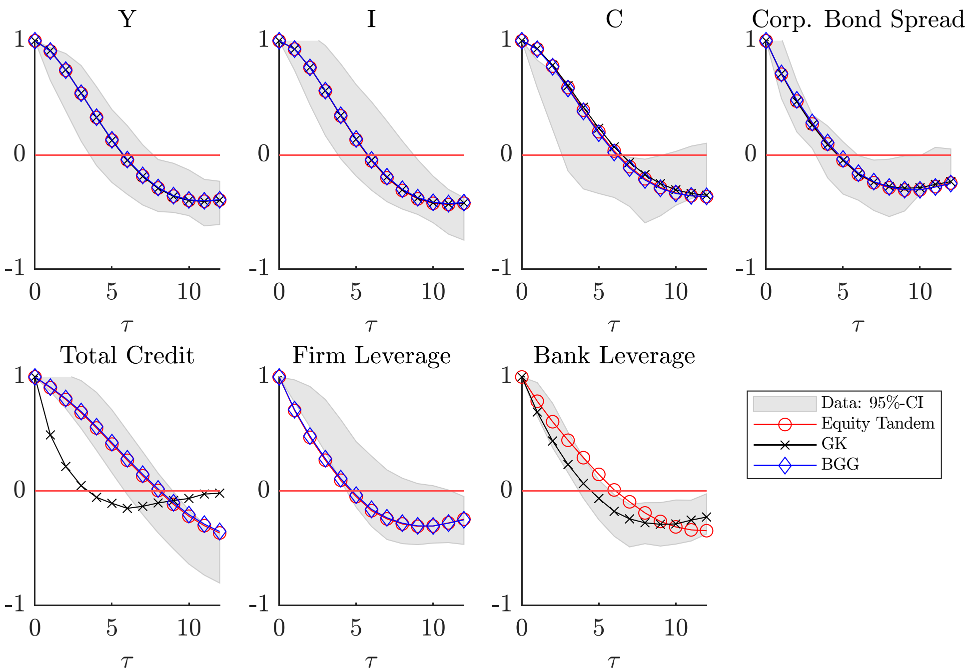

Fig. A.9. Selected Autocorrelations of order τ.
Note: The figure compares the model implied autocorrelations with the data. Both data and model simulations are HP-filtered with smoothing parameter set to 1600 for quarterly frequencies. For reference to data sources and transformations see Appendix A.1.

Table 5
Firm leverage and responsiveness to shocks.

Shock/Model	Y	I	$E[R^k - R]$	Firm equity	Bank equity
All shocks					
Baseline (GK)	0.18192	0.72707	0.05048	–	1.68383
ET: $\phi^{int} = 2$	0.13930	0.33021	0.00930	1.12455	0.47865
ET: $\phi^{int} = 10$	0.20307	1.05795	0.04574	4.26418	1.12360
TFP-Shock					
Baseline (GK)	0.04458	0.14259	0.00346	–	0.09385
ET: $\phi^{int} = 2$	0.03779	0.10446	0.00058	0.04888	0.09844
ET: $\phi^{int} = 10$	0.04668	0.16721	0.00309	0.26360	0.10151
Capital quality shock					
Baseline (GK)	0.17465	0.69718	0.03320	–	1.65444
ET: $\phi^{int} = 2$	0.13394	0.31297	0.00911	1.12330	0.42474
ET: $\phi^{int} = 10$	0.19749	1.04375	0.04544	4.25238	1.08880
Government spending shock					
Baseline (GK)	0.00472	0.00302	0.00019	–	0.00398
ET: $\phi^{int} = 2$	0.00527	0.00319	0.00007	0.00438	0.00194
ET: $\phi^{int} = 10$	0.00478	0.00151	0.00005	0.00263	0.00482
Bank equity shock					
Baseline (GK)	0.00415	0.02393	0.00123	–	0.03972
ET: $\phi^{int} = 2$	0.00022	0.00131	0.00005	0.00054	0.02661
ET: $\phi^{int} = 10$	0.00048	0.00316	0.00013	0.00578	0.02249
Monetary policy shock					
Baseline (GK)	0.00271	0.01532	0.00075	–	0.02494
ET: $\phi^{int} = 2$	0.00164	0.00851	0.00021	0.01288	0.01738
ET: $\phi^{int} = 10$	0.00330	0.02406	0.00103	0.08496	0.01598
QE-Shock					
Baseline (GK)	0.02358	0.14640	0.03783	–	0.29508
ET: $\phi^{int} = 2$	0.00208	0.00947	0.00178	0.01593	0.19492
ET: $\phi^{int} = 10$	0.00459	0.03598	0.00413	0.15422	0.25671

Note: The table displays the standard deviations for the baseline GK model and the Equity Tandem under different steady state firm leverage ratios of selected variables considering each shock separately. The first rows display the standard deviations considering all shocks jointly. No unconventional monetary policy, $\upsilon = 0$, and no interest rate smoothing, $\kappa_i = 0$, are assumed.

A.3. Robustness checks

A.3.1. On the choice of relative leverage

As mentioned in the main text, the choice of relative steady state leverage (banks 20, firms 2) is large compared to other research papers (banks 4 in Gertler and Karadi, 2011, firms 2 in Bernanke et al., 1999). However, this choice is appropriate for two reasons.

First, it is a more conservative choice because it allows for more shock amplification stemming from the bank lending channel and the associated financial accelerator. In other words, I calibrate the model against my conjecture that modeling both types of net worth increases the resilience of the model and leads to smaller gains of unconventional policy. The choice of steady state bank leverage of 20 is rather inconvenient to support this conjecture. However, the results presented in the main text are insensitive to smaller steady state bank leverage ratios. These results are available upon request.

Second, this choice is closer to the empirical evidence. Large U.S. investment banks tend to have leverage ratios with individual averages across time ranging between 22 to 32 before 2008 (see Table 8.1 in Adrian and Shin, 2008, p. 39). In aggregate, leverage ratios of investment banks can be quite large and volatile compared to commercial banks. For all banks, in aggregate the leverage ratio in the US is quite stable and somewhat below 15. In contrast, for Europe the aggregate bank leverage ratio reached levels somewhat above 25 in 2008 (see Fig. 4 in Kalemli-Ozcan et al., 2012, p. 290). Hence, targeting a steady state leverage ratio of banks of twenty does not seem too implausible. Note that this choice is also within the plausible ranges mentioned by Gertler and Karadi (2011, p. 26). On the other hand, steady state firm leverage is motivated by the choice of Bernanke et al. (1999) and therefore in line with the literature. Looking at the most recent data confirms that this is a reasonable choice.

A.3.2. Firm leverage and shock amplification

Table 5 shows that firm leverage has a clear influence on the responsiveness of the economy to various exogenous shocks. For most of the scenarios an empirically realistic value of steady state firm leverage, $\phi^{int} = 2$, decreases the standard deviations of key model variables. Matching the standard deviations of the GK model with the equity tandem model requires implausibly large values of firm leverage. Hence, a realistic calibration of firm leverage, in conjunction with the modeling of bank net worth, limits the space for additional policies (like QE) to further stabilize the model presented here. The only notable exception is the scenario with government spending shocks. However, the relative change of standard deviations in this case is small compared to other shocks and the equity tandem roughly aligns with the volatility under GK.

A.3.3. Effectiveness of aggressive credit policy

See Table 6.

Table 6
Effectiveness of aggressive credit policy.

Shock/Model	Y	C	$E[R^k - R]$	Firm equity	Bank equity
All shocks					
GK:	−39.86	−15.00	−86.56	–	40.89
ET:	−21.85	−5.05	−39.14	−4.38	846.42
TFP-Shock					
GK:	−6.63	−4.71	−95.18	–	32.80
ET:	2.23	0.37	−71.38	−9.58	−58.89
Capital quality shock					
GK:	−42.43	−15.28	−83.79	–	41.09
ET:	−23.87	−5.20	−37.73	−4.36	936.84
Government spending shock					
GK:	0.53	−1.72	−94.70	–	18.89
ET:	−3.85	0.63	−27.28	0.14	5054.09
Bank equity shock					
GK:	−62.76	−48.37	−87.19	–	60.08
ET:	−61.23	−51.47	−72.08	−74.14	28.78
Monetary policy shock					
GK:	−61.13	−47.73	−87.27	–	55.15
ET:	−43.94	−31.62	−56.88	−12.93	132.62
QE-Shock					
GK:	−97.41	−96.99	−98.41	–	−97.31
ET:	−80.95	−81.29	−82.72	−82.13	−81.81

Note: The table displays the percentage change of standard deviations due to a change from an non-aggressive, $\upsilon = 10$, to an aggressive credit policy, $\upsilon = 100$, considering one shock at a time. The first two rows display the percentage change considering all shocks jointly. No interest rate smoothing, $\kappa_i = 0$, is assumed.

A.4. Model details

The model is a synthesis of the frameworks from Gertler and Karadi (2011) and Bernanke et al. (1999). The economic environment is characterized by households, banks, non-banks, intermediate firms, retailers, final goods producers, capital goods producers and a government sector. The decision problems of these agents are outlined within this appendix.

A.4.1. Households

Households consume, save and supply labor to the general labor market. By assumption, there are infinitely many identical households in the economy. Each of them is indexed by $h \in [0,1]$. Within each household there exists a continuum of workers with measure $1 - f$ and a continuum of bankers with measure f.[29] Between household members there is, by assumption, perfect consumption insurance. As their names suggests, workers supply labor and return their labor income to their household. Each banker runs a financial intermediary and transfers a stochastic dividend payment to the household he or she belongs to. To avoid that bankers accumulate sufficient net worth to become fully self financing, which would imply a non-binding leverage constraint, bankers and workers randomly switch their states. That is, each period a banker faces the event of interrupting its business, returning its accumulated net worth to its household and becoming a worker with probability $1-\theta$. Exiting bankers are replaced with an equal number of randomly drawn workers becoming bankers, in order to keep the relative sizes of both populations fixed across time. Since the survival probability, θ, of a bank each period is iid across individuals and time the average (or expected) lifetime of a banker is $\frac{1}{1-\theta}$.

Overall the household derives utility from consumption C_t, real holdings of liquid assets B_{t+1} and disutility from labor supply, L_t. The expected lifetime utility function of a household is described by

$$E_t \sum_{i=0}^{\infty} \beta^i \left(\log(C_{t+i} - \Xi C_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} + \frac{s_{t+i}}{1-\sigma_b} B_{t+i}^{1-\sigma_b} \right) \tag{21}$$

where $\beta \in (0,1)$ is the household discount factor, $\Xi \in (0,1)$ determines the degree of internal habit formation, φ is the inverse of the Frisch elasticity of labor supply, χ is a scaling parameter of disutility from labor supply and s_t is a preference shock for holding liquid assets. Each household can save by holding one period real deposits on accounts with banks, not belonging to their own household, or by holding riskless one period real government bonds. By construction of the equilibrium, both saving vehicles are perfect substitutes and pay the predetermined gross real return R_t from $t-1$ to t. Overall, the total amount of the household's real holding of one period real bonds and real deposits at banks, acquired at $t-1$ and due in t, is given by B_t. Additionally, the household can save by holding real deposits at non-bank intermediaries D_t^{NB} that pay $R_t^{d,\mathrm{NB}}$. The household faces a sequence of budget constraints given by

$$C_t + B_{t+1} + D_{t+1}^{\mathrm{NB}} = W_t L_t + R_t B_t + R_t^{d,\mathrm{NB}} D_t^{\mathrm{NB}} + \Pi_t^{\mathrm{Prof.}} - T_t \tag{22}$$

[29] Christiano et al. (2014) refer to this assumption of Gertler and Karadi (2011) as "large family" assumption.

where W_t, $\Pi_t^{\text{Prof.}}$ and T_t denote the real wage, real profits from the ownership of financial and non-financial firms net of start-up funds for new bankers and lump sum taxes at time t, respectively. Since households display price taking behavior, because they operate only in competitive markets, the first order conditions for the household's maximization problem of its lifetime utility function subject to the sequence of budget constraints are well known. The dynamic optimization problem of the household is summarized by the Lagrangian

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \Bigg(\log(C_{t+i} - \Xi C_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} + \frac{s_{t+i}}{1-\sigma_b} B_{t+i}^{1-\sigma_b} + \varrho_{t+i} \left[W_{t+i} L_{t+i} + \Pi_{t+i}^{\text{Prof.}} - T_{t+i} + R_{t+i} B_{t+i} + R_{t+i}^{d,\text{NB}} D_{t+i}^{\text{NB}} - B_{t+i+1} - D_{t+i+1}^{\text{NB}} - C_{t+i} \right] \Bigg)$$

and the corresponding FOCs are given by

$$\varrho_t = (C_t - \Xi C_{t-1})^{-1} - \beta \Xi E_t (C_{t+1} - hC_t)^{-1} \tag{23}$$

$$1 = s_t \frac{B_{t+1}^{-\sigma_b}}{\varrho_t} + E_t \Lambda_{t,t+1} R_{t+1} \tag{24}$$

$$1 = E_t \Lambda_{t,t+1} R_{t+1}^{d,\text{NB}} \tag{25}$$

$$\chi L_t^{\varphi} = \varrho_t W_t \tag{26}$$

where ϱ_t defines the Lagrange multiplier on the budget constraint. For later use, the household's real stochastic discount factor (SDF) is defined by

$$\Lambda_{t,t+1} = \beta \frac{\varrho_{t+1}}{\varrho_t}. \tag{27}$$

A.4.2. Banks

Banks combine net worth and deposits, obtained from households, to finance lending to intermediate firms. Intermediate firms in turn also use internal and external funds to finance the capital stock used in production. Hence, net worth becomes a relevant state variable on both sides of the financial market. The modeling of banks is similar to Gerali et al. (2010).[30] Each bank j can be thought of as consisting out of three competitive branches. The deposit branch issues deposits, $D_{t+1}(j)$ to households which are remunerated at the real rate R_{t+1} and pass-through the funds to the management branch without making any profits.[31] The management branch combines net worth and deposits to finance the lending activities of the retail branch. At the stage of the management branch, I introduce the incentive compatibility constraint from Gertler and Karadi (2011) to allow for supply side frictions in the intermediation process (see below). The retail branch works like the intermediary in Bernanke et al. (1999). It obtains funds from the management branch and uses this funds to finance a well diversified portfolio of loans to intermediate firms. At the stage of the retail branch, I introduce the costly state verification problem in order to allow intermediate firm's net worth to become a relevant state variable.[32]

A.4.3. Management branch

The core of each bank j is the management branch. On the liability side management branch j has net worth $N_t(j)$ at the end of period t and deposits $D_{t+1}(j)$, due in $t+1$, received from the deposit branch j. On the asset side it holds bonds, $B_t^{Retail,MB}(j)$, issued by retail branch j which can be thought of as a claim on a perfectly diversified bond portfolio. Retail bankers, further described below, guaranty a predetermined return on each bond they issue at time t, which is given by R_{t+1}^{Retail}. The balance sheet of management branch j is given by

$$B_t^{Retail,MB}(j) = N_t(j) + D_{t+1}(j). \tag{28}$$

As we will see later, due to limits to arbitrage, imposed by the incentive constraint, the management branch will accumulate net worth out of retained profits until exiting the industry. That is, the difference between interest earnings on assets and payments on liabilities. Hence, net worth of management branch j at the end of period $t+1$ accumulates as

$$N_{t+1}(j) = R_{t+1}^{Retail} B_t^{Retail,MB}(j) - R_{t+1} D_{t+1}(j). \tag{29}$$

Combining the latter equation with the balance sheet condition (28) yields

$$N_{t+1}(j) = \left(R_{t+1}^{Retail} - R_{t+1}\right) B_t^{Retail,MB}(j) + R_{t+1} N_t(j). \tag{30}$$

[30] The reader should note, however, that there are some conceptual differences. E.g., in Gerali et al. (2010) the deposit branch and retail branch both act under monopolistic competition and apply a markdown and markup, respectively.

[31] Since the deposit branch is like a friction-less pass-through, I could also omit this branch and let the management branch issue deposits.

[32] Instead of modeling a retail branch, a similar model representation could be obtained by modeling mutual funds as in Christiano et al. (2014). In this case the banks would buy bonds issued by mutual funds.

Recall that each bank is confronted with the uncertainty of survival. A bank that operates today (from t to $t+1$) will discontinue its operations and make a stochastic dividend payment to the household it belongs to in period $t+1+k$ with probability $(1-\theta)\theta^k$ and $k \geq 0$. Since the household sector is the owner of the banking sector, each bank applies the household's SDF to value future earnings. Overall, the objective of each management branch is to maximize the franchise value of the bank

$$V_t(j) = \max E_t \sum_{k=0}^{\infty} (1-\theta)\theta^k \Lambda_{t,t+1+k} \left[(R^{Retail}_{t+1+k} - R_{t+1+k}) B^{Retail,MB}_{t+k}(j) + R_{t+1+k} N_{t+k}(j) \right]. \tag{31}$$

Because the model assumes imperfect capital markets with limits to arbitrage, the risk adjusted spread $E_t \Lambda_{t,t+1+k}(R^{Retail}_{t+1+k} - R_{t+1+k})$ can be positive.[33] Hence it is optimal for the bank to retain profits/accumulate net worth until receiving the exogenous signal to exit the industry. Additionally, each bank would like to borrow an unbounded amount of funds from households in order to take advantage of the arbitrage opportunities which in turn would make the excess return vanish in equilibrium. However, the following agency problem implies a capital market imperfection and thus limits arbitrage.

Each period the management board of a bank can decide to divert a fraction λ of assets. If it does so, this comes at the cost of being forced into bankruptcy by depositors that recover the remaining fraction $1-\lambda$. Only if there is no incentive to divert funds from the balance sheet households are willing to lend to banks. This implies that the following incentive compatibility constraint

$$V_t(j) \geq \lambda B^{Retail,MB}_t(j) \tag{32}$$

must be satisfied at any time. The left hand side indicates that the bank would lose its entire franchise value in the event of diverting funds. The right hand side indicates that the bank would gain the amount $\lambda B^{Retail,MB}_t(j)$ by cheating depositors. As long as this gains do not exceed losses there is no incentive for fraud and depositors are willing to lend to banks.

The solution for the value function (31) can be obtained by a standard guess and verify approach. The guess for the solution is given by

$$V_t(j) = \nu_t B^{Retail,MB}_t(j) + \eta_t N_t(j)$$

where ν_t and η_t are undetermined coefficients. To demonstrate the solution for the value function, note that the incentive constraint for the banker

$$V_t(j) = \nu_t B^{Retail,MB}_t(j) + \eta_t N_t(j) \geq \lambda B^{Retail,MB}_t(j) \tag{33}$$

implies a trivial solution for the maximization problem. As long as $0 < \nu_t < \lambda$ and $E_t \Lambda_{t,t+1+k}(R^{Retail}_{t+1+k} - R_{t+1+k}) \geq 0$, it is optimal, in any period, to hold the highest possible amount of assets implied by a binding incentive constraint

$$B^{Retail,MB}_t(j) = \frac{\eta_t}{\lambda - \nu_t} N_t(j). \tag{34}$$

In other words, the bank will borrow from households until the point is reached where depositors are not willing to hold additional deposits with this bank. By rewriting the value function and plugging in the guess I obtain the solution

$$\nu_t = E_t \left[(1-\theta)\Lambda_{t,t+1}(R^{Retail}_{t+1} - R_{t+1}) + \Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1} \right] \tag{35}$$

$$\eta_t = E_t \left[(1-\theta) + \Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1} \right] \tag{36}$$

for the coefficients where $x_{t,t+1} = \frac{B^{Retail,MB}_{t+1}(j)}{B^{Retail,MB}_t(j)}$ and $z_{t,t+1} = \frac{N_{t+1}(j)}{N_t(j)}$ are the gross growth rates of asset holdings and net worth, respectively.[34] Eq. (35) determines the marginal value for a bank of an additional unit of financing its retail branch holding net worth fixed. Eq. (36) determines the marginal value for a bank of having an additional unit of net worth holding its balance sheet size fixed. In the friction-less case arbitrage would imply that ν_t is zero at any point in time.

Note that Eq. (34) implies that the lending capacity of the management branch is tied to its equity stock. Hence, the incentive constraint implies an upper bound $\bar{\phi}_t = \frac{\eta_t}{\lambda - \nu_t}$ for the leverage ratio $\phi_t = \frac{B^{Retail,MB}_t(j)}{N_t(j)}$, e.g. the asset to net worth ratio. The calibration used will satisfy a binding incentive constraint, e.g. $0 < \nu_t < \lambda$ holds, in the neighborhood of the steady state. Therefore, the leverage ratio will increase in ν_t. Since a higher spread $E_t \Lambda_{t,t+1}(R^{Retail}_{t+1} - R_{t+1})$ implies a larger ν_t, this also increases the opportunity cost of the management branch and therefore makes depositors tolerate a larger leverage ratio.

An additional way for expressing the law of motion of bank j's net worth is given by

$$N_{t+1}(j) = \left[(R^{Retail}_{t+1} - R_{t+1})\phi_t + R_{t+1} \right] N_t(j) \tag{37}$$

which demonstrates that the sensitivity of net worth with respect to the spread, $R^{Retail}_{t+1} - R_{t+1}$, increases with the leverage ratio. It is important to note that all leverage ratios of all banks will be the same in equilibrium. To see this first recall that each management branch will love to expand its balance sheet to the point where the maximum possible leverage ratio is reached, e.g. $\phi_t = \bar{\phi}_t$. The

[33] This spread cannot be negative, as in this case the bank management would stop funding retail loans. The resulting credit scarcity would immediately increase this spread.

[34] See Appendix A.6 for further derivations.

maximum possible leverage ratio in turn is independent from individual specific characteristics since Eqs. (35) and (36) and the gross growth rate of each bank's net worth and asset holdings

$$z_{t,t+1} = \frac{N_{t+1}(j)}{N_t(j)} = \left[(R^{Retail}_{t+1} - R_{t+1})\phi_t + R_{t+1}\right] \tag{38}$$

$$x_{t,t+1} = \frac{B^{Retail,MB}_{t+1}(j)}{B^{Retail,MB}_{t}(j)} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1} \tag{39}$$

are independent from individual specific characteristics. Hence, in equilibrium, all banks are scaled versions of each other. This feature facilitates aggregation across banks.

Since each management branch finances the loans of its retail branch, $B^{Retail,P}_t(j)$, to intermediate firms and retail branches simply hand over funds, the equality $B^{Retail,P}_t(j) = B^{Retail,MB}_t(j)$ holds. In aggregate, the total amount of *privately* intermediated credit offered to non-financial firms, $B^{Retail,P}_t$, is given by

$$B^{Retail,P}_t = \int_0^1 B^{Retail,P}_t(j)dj = \bar{\phi}_t \int_0^1 N_t(j)dj = \bar{\phi}_t N_t. \tag{40}$$

Eq. (40) shows that the private credit supply in the economy is linked to the aggregate net worth stock in the banking sector.

Aggregate net worth, N_t, at the end of period t is given by the sum over all banks operating from t to $t+1$. This is equivalent to the sum over all surviving banks plus the sum over all new bankers. Existing banks accumulate net worth from $t-1$ to t and at the end of period t only a randomly chosen fraction θ will survive. Shifting time subscripts on (37), integrating and applying the rules of the lottery, net worth of surviving banks is given by

$$N^S_t = \theta\left[(R^{Retail}_t - R_t)\phi_{t-1} + R_t\right] N_{t-1}. \tag{41}$$

New bankers need start-up funds from their household at the beginning of their operations. I follow Gertler and Karadi (2011) and assume that start-up funds are proportional to the intermediated funds in the final operation period of exiting bankers. Accordingly, in the aggregate, new bankers at time t receive a transfer

$$N^{new}_t = \omega B^{Retail,P}_{t-1} \tag{42}$$

from households. Aggregate net worth can now be written as

$$N_t = N^S_t + N^{new}_t \tag{43}$$

$$= \theta\left[(R^{Retail}_t - R_t)\phi_{t-1} + R_t\right] N_{t-1} \exp(\mathcal{E}^{NS}_t) + \omega B^{Retail,P}_{t-1} \tag{44}$$

where I augment surviving bank's net worth by the factor $\exp(\varepsilon^{NS}_t)$ as a shorthand to introduce exogenous shocks to the banking sector's aggregate net worth.[35] During the calibration exercise the parameter ω will turn out to be useful for targeting the steady state leverage ratio of banks.

A.4.4. Retail branch

As mentioned in the previous subsection, the retail branch j obtains funds from the management branch of bank j and intermediates these funds to intermediate firms. Intermediate firms use external funds from retail branches and their own net worth to finance capital used in production next period. The relationship between the retail branch and intermediate firms follows along the lines of Bernanke et al. (1999). Retail branches take the gross real interest rate R^{Retail}_{t+1}, which they return to management branches, as given and lend to a continuum of firms subject to idiosyncratic firm-level shocks. The loan contract between each retail branch and intermediate firm, signed at time t, will guaranty that the loan portfolio of each retail branch earns the gross real interest rate R^{Retail}_{t+1} from t to $t+1$ in any state of nature.[36] In order to obtain funding from the management branch, a retail branch issues claims, $B^{Retail,MB}_t(j)$, on its portfolio. These claims can be thought of as well diversified composites of a continuum of corporate bonds, which gives a return equal to $R^{Retail}_{t+1} B^{Retail,MB}_t(j)$ at $t+1$. Due to perfect competition between retail branches revenues from holding the portfolio will always equal the cost of funds from the management branches. By assumption, there are no frictions within retail branches that might introduce a wedge between the overall return on the loan portfolio and the return demanded by management branches. Hence, retail branches simply pass-through the funds from management branches to intermediate firms. The details of the contractual arrangement between retail branches and intermediate firms are outlined in the following subsection.

35 The replication files from Gertler and Karadi (2011) show that they proceed the same way. For comparability reasons, I follow this approach.

36 As noted by Carlstrom et al. (2016, p. 128) the key assumption in the BGG framework is that the lender's return on the loan portfolio is predetermined because the borrower is risk neutral and bears all the aggregate risk and the remaining idiosyncratic risk is perfectly diversified within the loan portfolio by lending to a large number of firms.

A.4.5. Non-bank intermediaries

In addition to banks, non-banks are introduces as a second type of intermediary in order to allow intermediate firms to obtain funds from the corporate bond market as a second source of external finance. Banks and non-banks offer intermediate firms the exact same financial contract (see below). Hence, intermediate firms regard the external funds raised via the banking channel and the market channel as perfect substitutes. A representative non-bank intermediary creates a loan

$$B_t(i)^{\mathrm{NB}} = \left(D_{t+1}^{\mathrm{NB}}(i)\right)^{\alpha_{NB}}, \quad \alpha_{NB} < 1 \tag{45}$$

subject to a decreasing returns to scale technology. Under perfect competition the returns earned by the non-bank intermediary R_{t+1}^{NB} equal R_{t+1}^{Retail}. Since a non-bank remunerates household deposits, D_{t+1}^{NB}, with $R_{t+1}^{d,\mathrm{NB}}$ and due to profits maximization, in a symmetric equilibrium, the total supply via the market channel is given by

$$B_t^{\mathrm{NB,P}} = \int_0^1 B_t(i)^{\mathrm{NB}} di = \left(\frac{R_{t+1}^{Retail}}{R_{t+1}^{d,\mathrm{NB}}}\right)^{\frac{\alpha_{NB}}{1-\alpha_{NB}}}. \tag{46}$$

A.4.6. Intermediate firms

In order to allow for a dual role of net worth intermediate firms play an important role in the model.[37] By assumption, there exists a continuum of infinitely many competitive intermediate firms in the economy. Similar to banks, each intermediate firm f faces a constant survival probability γ. This implies a finite horizon and an expected lifetime equal to $\frac{1}{1-\gamma}$. Again, births and deaths are a modeling device to prevent intermediate firms from accumulating sufficient net worth to become fully self-financing. In the event of death an intermediate firm distributes its accumulated net worth to the representative household. Dead intermediate firms are replaced by new firms which receive real start-up funds, X, from the household. Another important assumption is that intermediate firms are risk-neutral. As shown below, this implies that these firms are willing to hold the entire aggregate risk involved in the capital investment.

Each intermediate firm produces intermediate output, $Y_t^m(f)$, according to a constant returns to scale technology

$$Y_t^m(f) = A_t K_t^{\mathrm{S}}(f)^{\alpha} L_t(f)^{1-\alpha} \tag{47}$$

using capital services $K_t^{\mathrm{S}}(f)$ and labor $L_t(f)$ inputs acquired in competitive factor markets, where A_t denotes an exogenous total factor productivity process and α is the capital share of production. The capital stock, $K_t(f)$, purchased in period t, is used in production in $t+1$. Capital acquisitions are financed by internal funds (e.g., net worth) and external funds, by borrowing from a bank's retail branch and/or a non-bank intermediary. Net worth of intermediate firms, beyond the start-up funds, is accumulated out of retained profits from previous capital investments. On the demand side of the financial contract net worth is important, because it determines the costs of external financing for a firm.

As in Bernanke et al. (1999) there is sufficient anonymity in financial or capital markets. This assumption implies that only one-period contracts are made between financial and non-financial firms. Additionally, in the event that an intermediate firm endogenously defaults on its loan, it will receive a new loan in the next period if it does not receive the exogenous signal to leave. In order to make the agency problem located on the demand side apply to the entire capital stock and not only to the flow of current investments, it is assumed that intermediate firms repurchase (and refinance) the capital stock each period.

At the end of period t, intermediate firm f purchases the amount of physical capital $K_t(f)$ from a competitive capital goods producer at the relative price Q_t. The capital expenditures are financed from net worth $N_t^{int}(f)$ and borrowing $B_t^{int}(f)$ from a retail branch or a non-bank intermediary. Hence, the balance sheet identity applies

$$Q_t K_t(f) = N_t^{int}(f) + B_t^{int}(f). \tag{48}$$

Following the exposition of Christiano et al. (2014) and Carlstrom et al. (2016), after purchasing capital, each intermediate firm receives an idiosyncratic shock ω_{t+1}^f before production in $t+1$ takes place. This shock converts the acquired physical capital, $K_t(f)$, into $\omega_{t+1}^f K_t(f)$ units of capital. The idiosyncratic shock is drawn from a log-normal distribution with mean $E\{\omega_{t+1}^f\} = 1$ and is iid across both, individual firms and time.[38] The idea behind the idiosyncratic shock is to allow for individual firm specific risk in a simplified way. Following Gertler and Karadi (2011), on top of this firm specific shock there is an aggregate capital quality shock, ξ_{t+1}, which is identical across firms such that the effective capital of firm f is $K_{t+1}^{\mathrm{eff}}(f) = \xi_{t+1}\omega^f K_t(f)$. After observing the aggregate state at $t+1$, the intermediate firm chooses the utilization rate u_{t+1} of effective capital, such that $K_{t+1}^{\mathrm{S}}(f) = u_{t+1} K_{t+1}^{\mathrm{eff}}(f)$ units of capital services enter production.

At the end of period $t+1$, after production, the intermediate firm f sells its used capital to a capital producing firm at the relative price Q_{t+1}. Following Gertler and Karadi (2011), the replacement cost for depreciated capital is fixed at unity. Hence, the value of capital at the end of period $t+1$, after production, is $[Q_{t+1} - \delta(U_{t+1})]\xi_{t+1}\omega^f K_t(f)$, where $\delta(U_{t+1})$ denotes the depreciation rate. After paying wages the intermediate firm f has left $P_{t+1}^m \alpha Y_{t+1}^m(f)$ from production, where P_t^m is the competitive relative price

[37] I will use the terms entrepreneurs and intermediate firms interchangeably.

[38] Note that I do not introduce risk shocks via the standard deviation of the normally distributed variable $\log \omega^f$, contrary to Christiano et al. (2014).

for intermediate output. Since the amount invested into capital at time t was $Q_t K_t(f)$, the ex-post realized return to capital that intermediate firm f receives at $t+1$ is

$$\omega^f_{t+1} R^k_{t+1} = \frac{P^m_{t+1} \alpha \frac{Y^m_{t+1}(f)}{\xi_{t+1}\omega^f_{t+1} K_t(f)} \xi_{t+1}\omega^f_{t+1} K_t(f) + (Q_{t+1} - \delta(U_{t+1}))\xi_{t+1}\omega^f_{t+1} K_t(f)}{Q_t K_t(f)} \tag{49}$$

$$= \omega^f_{t+1} \cdot \frac{\left[P^m_{t+1} \alpha \frac{Y^m_{t+1}}{\xi_{t+1} K_t} + (Q_{t+1} - \delta(U_{t+1})) \right] \xi_{t+1}}{Q_t} \tag{50}$$

where R^k_{t+1} is the ex-post average or aggregate real gross return to capital.[39] As can be seen, the real gross return to the individual investment project can be decomposed into a firm specific and an aggregate component, which are two independent random variables. The overall payoff from firm f's investment project is $\omega^f_{t+1} R^k_{t+1} Q_t K_t(f)$. Because the intermediate firm relies on external finance, this payoff is not entirely contributing to the accumulation of firm's net worth. Instead, the financial contract will decompose this payoff into a fraction going to the borrower and a fraction going to the lender.

Following Bernanke et al. (1999), the relationship between borrowers and lenders is characterized by an agency problem that arises from a situation of asymmetric information. That is, only intermediate firms can observe the realization of the firm specific shock ω^f_{t+1}. If a retail branch from a bank wishes to observe the actual return of a certain project it has to pay a monitoring cost, $\mu\omega^f R^k_{t+1} Q_t K_t(f)$, which is proportional to the realized project outcome with the proportionality factor μ determining the degree of the friction. This is basically the costly state verification problem in the spirit of Townsend (1979).

The loan contract can be characterized as follows. Given the current net worth, $N^{int}_t(f)$, intermediate firm f would like to finance a capital investment of $Q_t K_t(f)$ at time t which implies the level of debt $B^{int}_t(f)$. The intermediate firm promises to pay a gross non-default loan rate Z_{t+1}, but the loan is risky due to the possibility of default. This gross non-default loan rate can equivalently be mapped into a cutoff value for the idiosyncratic shock, $\bar{\omega}^f{}_{t+1}$, defined by

$$\bar{\omega}^f{}_{t+1} R^k_{t+1} Q_t K_t(f) = Z_{t+1} B^{int}_t(f). \tag{51}$$

For realizations of the firm specific shock $\omega^f_{t+1} \leq \bar{\omega}^f{}_{t+1}$ below this cutoff, the intermediate firm is not able to repay the loan because the overall payoff from firm f's investment project, $\omega^f_{t+1} R^k_{t+1} Q_t K_t(f)$, is too small. In this case firm f defaults and the bank will pay the monitoring cost to recover the rest of the project $(1-\mu)\omega^f_{t+1} R^k_{t+1} Q_t K_t(f)$. For realizations of $\omega^f_{t+1} \geq \bar{\omega}^f{}_{t+1}$ above the cutoff the intermediate firm repays $Z_{t+1} B^{int}_t(f)$ and keeps $(\omega^f_{t+1} - \bar{\omega}^f{}_{t+1}) R^k_{t+1} Q_t K_t(f)$. Denote the pdf and cdf of ω^f_{t+1} by $f(\omega^f_{t+1})$ and $F(\omega^f_{t+1})$, respectively, the cutoff value determines the division of the project outcome between the intermediate firm and the retail branch. The expected share of the project going to the intermediate firm is

$$\zeta^{\text{int}}(\bar{\omega}^f_{t+1}) = \int_{\bar{\omega}^f_{t+1}}^{\infty} \omega^f_{t+1} f(\omega^f_{t+1}) d\omega^f_{t+1} - (1 - F(\bar{\omega}^f_{t+1}))\bar{\omega}^f_{t+1} \tag{52}$$

and the expected share net of monitoring cost going to the bank's retail branch is

$$\zeta^{\text{bank}}(\bar{\omega}^f_{t+1}) = (1 - F(\bar{\omega}^f_{t+1}))\bar{\omega}^f_{t+1} + (1-\mu) \int_0^{\bar{\omega}^f_{t+1}} \omega^f_{t+1} f(\omega^f_{t+1}) d\omega^f_{t+1}. \tag{53}$$

When signing a debt contract the objective of an intermediate firm is to maximize its expected net worth next period given by

$$E_t \zeta^{\text{int}}(\bar{\omega}^f_{t+1}) R^k_{t+1} Q_t K_t(f). \tag{54}$$

Using the definition of the intermediate firm's leverage ratio, $\phi^{\text{int,f}}_t = \frac{Q_t K_t(f)}{N^{int}_t(f)}$, this can be rewritten as

$$E_t \zeta^{\text{int}}(\bar{\omega}^f_{t+1}) R^k_{t+1} \phi^{\text{int,f}}_t N^{int}_t(f). \tag{55}$$

Conditional on the aggregate return to capital at $t+1$ the expected payoff earned by the retail branch is

$$\zeta^{\text{bank}}(\bar{\omega}^f_{t+1}) R^k_{t+1} \phi^{\text{int,f}}_t N^{int}_t(f). \tag{56}$$

Due to perfect competition and free market entry in lending markets retail branches earn zero profits in equilibrium. Hence the expected payoff must equal the cost of funds, $R^{Retail}_{t+1} B^{\text{int}}_t(f)$. That is, in any state of the world at time $t+1$ the participation constraint

$$\zeta^{\text{bank}}(\bar{\omega}^f_{t+1}) R^k_{t+1} \phi^{\text{int,f}}_t N^{int}_t(f) = R^{Retail}_{t+1}(Q_t K_t(f) - N^{int}_t(f)) \tag{57}$$

must hold, where I make use of Eq. (48). Since the cost of funds are predetermined, as noted earlier, the financial contract is now characterized by a leverage ratio, $\phi^{\text{int,f}}_t$, and a schedule for $\bar{\omega}^f_{t+1}$ which links the cutoff value to each possible realization of R^k_{t+1}, such that the participation constraint is satisfied. More conveniently, the participation constraint can be rewritten as

$$\zeta^{\text{bank}}(\bar{\omega}^f_{t+1}) R^k_{t+1} \phi^{\text{int,f}}_t = R^{Retail}_{t+1}(\phi^{\text{int,f}}_t - 1). \tag{58}$$

[39] Note that in the second step I use the fact that due to perfect competition and constant returns to scale all intermediate firms produce along the same output expansion path. This implies that the output capital ratio is constant across intermediate firms.

As pointed out by Christiano et al. (2014, p. 37), the combinations of $(\phi_t^{\text{int,f}}, \bar{\omega}_{t+1}^f)$ which satisfy (58) constitute a menu of state-contingent standard debt contracts from which the firm can choose. The optimal financial contract maximizes the expected net worth of the borrower subject to the participation constraint of the lender and this problem can be summarized by the Lagrangian

$$\mathcal{L} = max_{\bar{\omega}_{t+1}^f, \phi_t^{\text{int,f}}} E_t \left\{ \zeta^{\text{int}}(\bar{\omega}_{t+1}^f) R_{t+1}^k \phi_t^{\text{int,f}} + \Psi_{t+1} \left[\zeta^{\text{bank}}(\bar{\omega}_{t+1}^f) R_{t+1}^k \phi_t^{\text{int,f}} - R_{t+1}^{Retail}(\phi_t^{\text{int,f}} - 1) \right] \right\} \tag{59}$$

where Ψ_{t+1} denotes the ex post value of the Lagrange multiplier on the participation constraint, after the $t+1$ aggregate state is known.[40] The corresponding first order conditions are[41]

$$\frac{\partial \mathcal{L}}{\partial \bar{\omega}_{t+1}^f} : \zeta^{\text{int}\,\prime}(\bar{\omega}_{t+1}^f) + \Psi_{t+1} \zeta^{\text{bank}\prime}(\bar{\omega}_{t+1}^f) = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t^{\text{int,f}}} : E_t \left\{ \zeta^{\text{int}}(\bar{\omega}_{t+1}^f) R_{t+1}^k + \Psi_{t+1} \left[\zeta^{\text{bank}}(\bar{\omega}_{t+1}^f) R_{t+1}^k - R_{t+1}^{Retail} \right] \right\} = 0 \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial \Psi_{t+1}} : \zeta^{\text{bank}}(\bar{\omega}_{t+1}^f) R_{t+1}^k \phi_t^{\text{int,f}} = R_{t+1}^{Retail}(\phi_t^{\text{int,f}} - 1). \tag{62}$$

As pointed out by Carlstrom et al. (2016, p. 129), to guaranty the lender a predetermined return, $R_{t+1}^{Retail} = \zeta^{\text{bank}}(\bar{\omega}_{t+1}^f) R_{t+1}^k \frac{\phi_t^{\text{int,f}}}{\phi_t^{\text{int,f}} - 1}$, the cutoff value, $\bar{\omega}_{t+1}^f$, will move oppositely with realizations of R_{t+1}^k according to (62).[42] As can be seen from the FOCs, there are no individual specific variables beyond the choice variables $(\phi_t^{\text{int,f}}, \bar{\omega}_{t+1}^f)$ and therefore all intermediate firms will choose the same cutoff and leverage ratio $(\phi_t^{\text{int}}, \bar{\omega}_{t+1})$. This facilitates aggregation.

Operating intermediate firms, in aggregate, accumulate $\zeta^{\text{int}}(\bar{\omega}_{t+1}) R_{t+1}^k \phi_t^{\text{int}} N_t^{int}$ from t to $t+1$. Since only a portion, γ of them is allowed to continue, existing (or surviving) firms net worth is $N_{t+1}^{\text{int,ex}} = \gamma \zeta^{\text{int}}(\bar{\omega}_{t+1}) R_{t+1}^k \phi_t^{\text{int}} N_t^{int}$. Firms that discontinue their operations are replaced with new firms that in aggregate start with $N_t^{\text{int,new}} = X$. Aggregate net worth of intermediate firms is therefore given by

$$N_{t+1}^{\text{int}} = \gamma \zeta^{\text{int}}(\bar{\omega}_{t+1}) R_{t+1}^k \phi_t^{\text{int}} N_t^{int} + X. \tag{63}$$

The remaining first order conditions of the intermediate firm are obtained by considering its profit maximization problem at time t. Period profits of intermediate goods producers are given by

$$\text{Prof}_t(f) = P_t^m Y_t^m(f) - W_t L_t(f) - Z_t B_{t-1}^{\text{int}}(f) + (Q_t - \delta(u_t))\xi_t \omega_t^f K_{t-1}(f).$$

From the FOC of the competitive intermediate firm w.r.t. $L_t(f)$ and u_t we obtain

$$W_t = P_t^m (1-\alpha) \frac{Y_t^m}{L_t} \tag{64}$$

$$P_t^m \alpha \frac{Y_t^m}{u_t \xi_t K_{t-1}} = \delta'(u_t) \tag{65}$$

which are the same across the population of firms.

A.4.7. Retailers and final goods firm

Aggregate output, Y_t is a composite of a continuum of differentiated retail outputs and produced by a representative final goods producer with a CES technology

$$Y_t = \left(\int_0^1 Y_t(r)^{\frac{\epsilon-1}{\epsilon}} dr \right)^{\frac{\epsilon}{\epsilon-1}} \tag{66}$$

where $Y_t(r)$ denotes the output from retailer r. From profit maximization under perfect competition one obtains the standard formulas for the demand for variety r and the definition of the aggregate price level

$$Y_t(r) = \left(\frac{P_t(r)}{P_t} \right)^{-\epsilon} Y_t \tag{67}$$

$$P_t = \left(\int_0^1 P_t(r)^{1-\epsilon} dr \right)^{\frac{1}{1-\epsilon}} \tag{68}$$

where $P_t(r)$ is the price for retail output r.

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[40] Note that net worth only scales the objective function and can be dropped.

[41] Recall that the conditional expectation operator degenerates when computing a partial derivative w.r.t. a variable indexed by a future state.

[42] Applying the Leibniz rule, the derivative of (53) is $\zeta^{\text{bank}\,\prime}(\bar{\omega}_{t+1}^f) = (1 - F(\bar{\omega}_{t+1}^f)) - \mu \bar{\omega}_{t+1}^f f(\bar{\omega}_{t+1}^f)$. Bernanke et al. (1999, p. 1380–1382) show that there exists a range of values for $\bar{\omega}_{t+1}^f < \bar{\omega}^*$ such that $\zeta^{\text{bank}\,\prime}(\bar{\omega}_{t+1}^f) > 0$.

Each monopolistically competitive retailer purchases intermediate output as the sole input and produces its variety according to a linear production function.

$$Y_t(r) = Y_t^{\mathrm{m}}(r) \tag{69}$$

Therefore, the real marginal cost of each retailer are given by the relative price for intermediate goods P_t^m. The primary reason for modeling retail firms on the supply side of the economy is to introduce nominal price stickiness in a convenient way. Nominal rigidities are introduced by assuming that the price setting process follows the rules of a Calvo lottery. Each period a retailer has the opportunity to reoptimize his/her price with probability $1-\kappa$. Firms that are not allowed to reoptimize in the current period index their price to lagged inflation, where γ_p denotes the degree of indexation to past inflation. Hence, a retailer with the opportunity to adjust in period t faces the uncertainty that he or she might be stuck in period $t+i$ with the price $P_{t+i|t}(r) = P_t(r)\prod_{k=1}^{i} \Pi_{t+k-1}^{\gamma_p}$ with probability κ^i. The real profit at time $t+i$, given that the price was last reoptimized at time t, is

$$\mathrm{Prof}_{t+i|t}^{R} = \left(\frac{P_{t+i|t}(r)}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - P_{t+i}^{m}\left(\frac{P_{t+i|t}(r)}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i} \tag{70}$$

and the relevant part for maximization of the discounted value of current and future real profits is

$$\max_{P_t(r)} E_t \sum_{i=0}^{\infty} \kappa^i \Lambda_{t,t+i}\left[\left(\frac{P_{t+i|t}(r)}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - P_{t+i}^{m}\left(\frac{P_{t+i|t}(r)}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i}\right] \tag{71}$$

From the retailers price setting problem we obtain the FOC

$$E_t \sum_{i=0}^{\infty} \kappa^i \Lambda_{t,t+i}\left(\frac{P_t^*}{P_{t+i}}\prod_{k=1}^{i}\Pi_{t+k-1}^{\gamma_p} - \mu P_{t+i}^{m}\right) Y_{t+i|t}(r) = 0$$

where P_t^* is the optimal reset price and $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the desired gross markup.

Due to the properties of the Calvo lottery the law of motion for the aggregate price level is

$$P_t^{1-\varepsilon} = (1-\kappa)P_t^{*\,1-\varepsilon} + \kappa\left(\Pi_{t-1}^{\gamma_p} P_{t-1}\right)^{1-\varepsilon}. \tag{72}$$

A.4.8. Capital producer

Following Gertler and Karadi (2011), and for the sake of comparability, investment adjustment costs are implemented such that they only apply to the net rather than gross investment at the stage of the capital producing firm. By assumption the real replacement costs of depreciated capital are fixed at unity. That is, a representative capital producer purchases capital from intermediate firms after production at the relative price Q_t, refurbishes depreciated capital $\delta(u_t)\xi_t K_{t-1}$ and produces new capital (net investments). Since this firm hands over the costs (one for one) of refurbished capital $\delta(u_t)\xi_t K_{t-1}$, it can earn profits only by producing new capital subject to adjustment costs on net investment. At time t revenues are given by $Q_t I_{n,t}$, where $I_{n,t}$ denotes net investments, and real costs associated with the production of new capital are equal to $I_{n,t} + f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss})$. Since households are the ultimate owners of this firm, it applies the household's stochastic discount factor to value the stream of current and future profits. Its objective is to maximize discounted profits by adjusting the quantity of new capital, taking prices as given.

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}\left\{(Q_\tau - 1)I_{n,\tau} - f\left(\frac{I_{n,\tau}+I_{ss}}{I_{n,\tau-1}+I_{ss}}\right)(I_{n,\tau}+I_{ss})\right\} \tag{73}$$

The FOC w.r.t. net investments $I_{n,t}$ yields

$$\begin{aligned} Q_t = {} & 1 + f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right) + f'\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}} \\ & - E_t\left[\Lambda_{t,t+1} f'\left(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\right)\left(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\right)^2\right] \end{aligned} \tag{74}$$

with the functional forms specified as

$$\begin{aligned} f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right) &= \frac{\eta_i}{2}\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}} - 1\right)^2 \\ f'\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right) &= \eta_i\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}} - 1\right). \end{aligned}$$

In aggregate the capital stock evolves according to the law of motion

$$K_t = \xi_t K_{t-1} + I_{n,t} \tag{75}$$

where gross investments are given by $I_t = I_{n,t} + \delta(u_t)\xi_t K_{t-1}$.

A.4.9. Conventional and unconventional monetary policy

In the tradition of the literature, conventional monetary policy is characterized by an instrument rule of the Taylor-type for the short term nominal interest rate

$$\frac{R_t^{\text{shad}}}{R^{\text{shad}}} = \left(\frac{R_{t-1}^{\text{shad}}}{R^{\text{shad}}}\right)^{\kappa_i} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\pi} \left(\frac{\chi_t}{\frac{\varepsilon}{\varepsilon-1}}\right)^{-\kappa_y}\right)^{1-\kappa_i} e^{\mathcal{E}_t^i} \tag{76}$$

where the markup gap over its steady state, $\frac{\chi_t}{\frac{\varepsilon}{\varepsilon-1}}$, is used to proxy for the output gap.[43] R_t^{shad} denotes the notional or shadow rate, which is the unconstrained gross nominal interest rate. To take into account the zero lower bound, the gross nominal interest rate is given by

$$R_t^{\text{nom.}} = \max(1, R_t^{\text{shad}}) \tag{77}$$

where the gross nominal interest rate is related to the safe gross real interest rate via the Fisher equation $R_t^{\text{nom.}} = R_{t+1} E_t \Pi_{t+1}$.

To allow for unconventional monetary policy, the central bank assists in the process of creating loans. That is, it purchases composite bonds, $\tilde{B}_t^{CB}$, from the continuum of bank retail branches and non-bank intermediaries.[44] This purchases are financed by issuing government debt, $B_t^{CB} = \tilde{B}_t^{CB}$, to households which pay the riskless return and are perfect substitutes for deposits. Note that there is no agency problem between households and government. Therefore, in contrast to banks, the government is not constrained by obtaining funds. Note that due to its intermediation activities the central bank makes a profit of $(R_t^{Retail} - R_t)B_{t-1}^{CB}$ in any period t. These profits are handed over to the fiscal policy maker and therefore serve to finance government expenditures. Let the total amount of loans created by banks and non-banks and demanded by intermediate firms be $B_t^{Total} = \int_0^1 B_t^{int}(f)df = \int_0^1 B_t^{Retail}(j)dj + \int_0^1 B_t^{\text{NB}}(i)di$. By aggregating over all bank retail branches and non-banks one obtains

$$B_t^{Total} = B_t^{Retail,P} + B_t^{\text{NB},P} + B_t^{CB}. \tag{78}$$

The total amount of retail loans is due to private lending and lending under government assistance, where the former is constrained by the aggregate stock of banks net worth and decreasing returns to scale of non-banks. Unconventional monetary policy is implemented by assuming that the central bank finances a fraction, ψ_t, of the total value of intermediated loans, B_t^{Total}.

$$B_t^{CB} = \psi_t B_t^{Total} \tag{79}$$

Because the central bank is a less efficient intermediary, government intermediation causes a deadweight loss which is by assumption proportional to the amount of credit intermediated under central bank assistance, $\tau\psi_t B_t^{Retail}$. By combining Eqs. (78), (79), (46) and (40) I obtain

$$B_t^{Total} = \phi_t^{\text{total}} N_t + \frac{1}{1-\psi_t}\left(\frac{R_{t+1}^{Retail}}{R_{t+1}^{d,NB}}\right)^{\frac{\alpha_{NB}}{1-\alpha_{NB}}} \tag{80}$$

where $\phi_t^{\text{total}} = \bar{\phi}_t/(1-\psi_t)$ is the total leverage applied to bank's net worth taking account of central bank intermediation.

The fraction, ψ_t, of the total value of intermediated loans, intermediated under central bank assistance, follows a feedback rule of the form

$$\psi_t = \psi + \upsilon E_t\left(\log R_{t+1}^k - \log R_{t+1} - \log\frac{R^k}{R}\right) + \mathcal{E}_t^{\psi}. \tag{81}$$

According to (81), the central bank credit policy is characterized by a steady state fraction ψ of loans intermediated under public assistance, a non-negative response coefficient υ and an exogenous credit policy shock ϵ_t^{ψ}. The idea is that the central bank uses the overall spread, $\log R_{t+1}^k - \log R_{t+1}$, in deviation from its steady state, $\log\frac{R^k}{R}$, in order to proxy for the degree of credit scarcity in the economy. In an economic downturn external finance becomes more expensive for non-financial firms as their net worth deteriorates. To stabilize aggregate demand the central bank expands its credit policy as the spread increases. This will increase the leverage ratio, ϕ_t^{total}, and therefore the total amount of loans.

A.4.10. Fiscal policy, aggregate resource constraint and exogenous processes

The fiscal policy maker's budget constraint is given by

$$G_t + \tau\psi_t B_t^{Total} = T_t + (R_t^{Retail} - R_t)B_{t-1}^{CB}. \tag{82}$$

Accordingly, government expenditures and inefficiency costs of the central bank are financed by lump-sum taxes and net earnings from central bank intermediation. Government expenditures follow an exogenous AR(1) process in logs:

$$\log G_t = (1-\rho_G)\log\bar{G} + \rho_G \log G_{t-1} + \varepsilon_t^G. \tag{83}$$

[43] Without this simplification additional model equations are necessary to explain the natural (or flexible price) equilibrium of the model. Gertler and Karadi (2011) also proxy for the output gap in this way.

[44] At this point my modeling differs from Gertler and Karadi (2011). Their model considers central bank's direct lending to non-financial firms financed via government debt issued to households or equivalently interest bearing reserves issued to financial intermediaries. In my framework the central bank channels household's savings to the retail branches in order to bypass the agency friction within management boards. In contrast to direct lending, the costly state verification friction between non-financial firms and retail branches still applies.

Table 7
Calibration of exogenous processes.

Parameter	Value	Description
AR parameters		
ρ_a	0.950	AR parameter of TFP
ρ_ξ	0.660	AR parameter of capital quality shock
ρ_{N^S}	0.000	AR parameter bank equity shock
ρ_g	0.950	AR parameter of government spending process
ρ_ψ	0.660	AR parameter of credit policy shock
ρ_i	0.000	AR parameter monetary policy shock
Shock standard deviations		
σ_ξ	0.050	Standard deviation of capital quality innovation
σ_a	0.010	Standard deviation of TFP innovation
σ_g	0.010	Standard deviation of government spending innovation
σ_{N^S}	0.010	Standard deviation of banks net worth innovation
σ_i	0.010	Monetary policy shock
σ_ψ	0.072	Standard deviation of credit policy innovation

The aggregate resource constraint for the closed economy is given by

$$Y_t = C_t + I_t + G_t + f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) + \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t R_t^k Q_{t-1} K_{t-1} + \tau \psi_t B_t^{Retail} \tag{84}$$

As usual, output is used for private and government consumption as well as gross investment into the capital stock. The last three terms indicate that part of the aggregate output is lost in the production of capital, due to adjustment costs, because banks need to pay monitoring costs and the central bank intermediation of funds involves inefficiency costs.[45]

In addition to the government spending process, the model dynamics are driven by five exogenous shock processes. These are given by

$$\log A_t = \rho_a \log A_{t-1} + \varepsilon_t^A \tag{85}$$

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_t^\xi \tag{86}$$

$$\mathcal{E}_t^\psi = \rho_\psi \mathcal{E}_{t-1}^\psi + \varepsilon_t^\psi \tag{87}$$

$$\mathcal{E}_t^i = \rho_i \mathcal{E}_{t-1}^i + \varepsilon_t^i \tag{88}$$

$$\mathcal{E}_t^{N^S} = \rho_{N^S} \mathcal{E}_{t-1}^{N^S} + \varepsilon_t^{N^S} \tag{89}$$

where $\varepsilon_t^n \sim \mathcal{N}(0, \sigma_n)$ and $n \in \{G, A, \xi, \psi, i, N^S\}$. That is, the natural logarithm of the total factor productivity process and the capital quality process as well as the shocks to the conventional Taylor rule, the credit policy rule and surviving bank's net worth follow zero-mean AR(1) processes.

A.5. Calibration of exogenous processes

See Table 7.

A.6. Coefficients of value function

By rewriting the value function and plugging in the guess one obtains

$$V_t = E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i} \left[(R_{t+1+i}^{Retail} - R_{t+1+i}) B_{t+i}^{\text{Retail,MB}}(j) + R_{t+1+i} N_{j,t+i} \right] \tag{90}$$

$$= E_t \Big((1-\theta)\Lambda_{t,t+1} \left[(R_{t+1}^{Retail} - R_{t+1}) B_t^{\text{Retail,MB}}(j) + R_{t+1} N_{j,t} \right] + \theta \Lambda_{t,t+1} V_{t+1} \Big) \tag{91}$$

$$= E_t \Big(\; (1-\theta)\Lambda_{t,t+1} \left[(R_{t+1}^{Retail} - R_{t+1}) B_t^{\text{Retail,MB}}(j) + R_{t+1} N_{j,t} \right] \tag{92}$$

$$+ \; \theta \Lambda_{t,t+1} \left(\nu_{t+1} B_t^{\text{Retail,MB}}(j) + \eta_{t+1} N_{j,t+1} \right) \Big) \tag{93}$$

$$= E_t \Bigg(\left[(1-\theta)\Lambda_{t,t+1} (R_{t+1}^{Retail} - R_{t+1}) + \theta \Lambda_{t,t+1} x_{t,t+1} \nu_{t+1} \right] B_t^{\text{Retail,MB}}(j) \tag{94}$$

[45] Since I use the OccBin Toolkit, which relies on a first order approximation, quadratic adjustment costs will vanish at first order. Also the monitoring costs and inefficiency terms are numerically small.

$$+\left[(1-\theta)\underbrace{\Lambda_{t,t+1}R_{t+1}}_{E_t(\cdot)=1}+\theta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}\right]N_{j,t}\Bigg) \tag{95}$$

$$= E_t\Big[(1-\theta)\Lambda_{t,t+1}(R^{Retail}_{t+1}-R_{t+1})+\theta\Lambda_{t,t+1}x_{t,t+1}v_{t+1}\Big]B_t^{\text{Retail,MB}}(j) \tag{96}$$

$$+\ E_t\Big[(1-\theta)+\theta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}\Big]N_{j,t} \tag{97}$$

with $x_{t,t+1}=\frac{B_{t+1}^{Retail,MB}(j)}{B_t^{Retail,MB}(j)}$ and $z_{t,t+1}=\frac{N_{t+1}(j)}{N_t(j)}$. The leading expectation terms determine the coefficients

$$v_t = E_t\Big[(1-\theta)\Lambda_{t,t+1}(R^{Retail}_{t+1}-R_{t+1})+\Lambda_{t,t+1}\theta x_{t,t+1}v_{t+1}\Big] \tag{98}$$

$$\eta_t = E_t\Big[(1-\theta)+\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\Big]. \tag{99}$$

A.7. Aggregate resource constraint

The aggregate resource constraint is obtained by consolidating the household and government budget constraint and applying some convenient aggregation results. Recall that the household obtains all profits and dividend payments from financial and non-financial firms and pays start-up funds to new bankers.

$$\Pi_t^{\text{Prof.}} = \text{Profit}^{\text{Retailers}}+\text{Profit}^{\text{Intermediate firms}}+\text{Profit}^{\text{Capital firms}}+\text{Profit}^{\text{Banks}} - \text{Start-up funds} \tag{100}$$

$$\text{Profit}^{\text{Retailers}} = \int_0^1\left[\left(\frac{P_t(r)}{P_t}\right)Y_t(r)-P_t^mY_t(r)\right]dr = Y_t - P_t^m\int_0^1 Y_t^m(f)df = Y_t-P_t^mY_t^m \tag{101}$$

$$\text{Profit}^{\text{Intermediate firms}} = \int_0^1 \text{Prof}_t(f)df \tag{102}$$

$$= P_t^mY_t^m - W_tL_t - \left[(1-F(\bar\omega_t))\bar\omega_t+\int_0^{\bar\omega_t}\omega_tf(\omega_t)d\omega_t\right]R_t^kQ_{t-1}K_{t-1} + (Q_t-\delta(u_t))\xi_tK_{t-1} \tag{103}$$

$$\text{Profit}^{\text{Capital firms}} = (Q_t-1)I_{n,t}-f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss}) = (Q_t-1)\left(I_t-\delta(u_t)\xi_tK_{t-1}\right)-f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss}) \tag{104}$$

$$\text{Profit}^{\text{Banks}} - \text{Start-up funds} = (1-\theta)\left[(R_t^{Retail}-R_t)\phi_{t-1}+R_t\right]N_{t-1}-\omega B_{t-1}^{Retail,P} = \left[(R_t^{Retail}-R_t)\phi_{t-1}+R_t\right]N_{t-1}-N_t \tag{105}$$

This implies that overall profits distributed to the household are

$$\Pi_t^{\text{Prof.}} = Y_t - W_tL_t - \left[(1-F(\bar\omega_t))\bar\omega_t+\int_0^{\bar\omega_t}\omega_tf(\omega_t)d\omega_t\right]R_t^kQ_{t-1}K_{t-1} \tag{106}$$

$$+\underbrace{(Q_t-\delta(u_t))\xi_tK_{t-1}+(Q_t-1)\left(I_t-\delta(u_t)\xi_tK_{t-1}\right)}_{=Q_tK_t-I_t} - f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss})+\left[(R_t^{Retail}-R_t)\phi_{t-1}+R_t\right]N_{t-1}-N_t \tag{107}$$

$$= Y_t - W_tL_t - \left[(1-F(\bar\omega_t))\bar\omega_t+\int_0^{\bar\omega_t}\omega_tf(\omega_t)d\omega_t\right]R_t^kQ_{t-1}K_{t-1} \tag{108}$$

$$+Q_tK_t-I_t-f\left(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\right)(I_{n,t}+I_{ss}) + \underbrace{\left[(R_t^{Retail}-R_t)\phi_{t-1}+R_t\right]N_{t-1}}_{(R_t^{Retail}-R_t)B_{t-1}^{Retail,P}+R_tN_{t-1}}-N_t \tag{109}$$

Since

$$\left[(1-F(\bar\omega_t))\bar\omega_t+(1-\mu)\int_0^{\bar\omega_t}\omega_tf(\omega_t)d\omega_t\right]R_t^kQ_{t-1}K_{t-1} = R_t^{Retail}\left(B_{t-1}^{Retail,P}+B_{t-1}^{Retail,CB}\right) \tag{110}$$

this simplifies the expression for overall profits

$$\Pi_t^{\text{Prof.}} = Y_t - W_t L_t - R_t^{Retail} B_{t-1}^{Retail,CB} - \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t R_t^k Q_{t-1} K_{t-1} + Q_t K_t - I_t - f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) - R_t B_{t-1}^{Retail,P} + R_t N_{t-1} - N_t. \tag{111}$$

The household's budget constraint is

$$C_t + B_{t+1} = W_t L_t + R_t B_t + \Pi_t^{\text{Prof.}} - T_t \tag{112}$$

and the corresponding budget constraint of the government sector is

$$G_t + \tau\psi_t B_t^{Retail} = T_t + (R_t^{Retail} - R_t) B_{t-1}^{Retail,CB}. \tag{113}$$

By consolidating Eqs. (111), (112) and (113), I obtain

$$\begin{aligned} C_t + B_{t+1} &= W_t L_t + R_t B_t + \Pi_t^{\text{Prof.}} - G_t - \tau\psi_t B_t^{Retail,CB} + (R_t^{Retail} - R_t) B_{t-1}^{Retail,CB} \\ &= Y_t - \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t R_t^k Q_{t-1} K_{t-1} + Q_t K_t - I_t - f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) \\ &- N_t - G_t - \tau\psi_t B_t^{Retail} \end{aligned} \tag{114}$$

$$C_t = Y_t - \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t R_t^k Q_{t-1} K_{t-1} - I_t - f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) - G_t - \tau\psi_t B_t^{Retail} \tag{115}$$

$$Y_t = C_t + I_t + G_t + f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) + \mu \int_0^{\bar{\omega}_t} \omega_t f(\omega_t) d\omega_t R_t^k Q_{t-1} K_{t-1} + \tau\psi_t B_t^{Retail} \tag{116}$$

where I use the aggregate relationships

$$D_{t+1} = B_t^{Retail,P} - N_t \tag{117}$$

$$B_{t+1} = D_{t+1} + B_t^{Retail,CB} = B_t^{Retail,P} + B_t^{Retail,CB} - N_t \tag{118}$$

$$B_t^{Retail} = B_t^{Retail,P} + B_t^{Retail,CB} \tag{119}$$

$$Q_t K_t = N_t + B_{t+1}. \tag{120}$$

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