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A new look at financial markets efficiency from linear response theory

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ABSTRACT

In this paper we propose a new measure of market efficiency based on the average response of a market price after a market event by using Linear Response Theory. It is shown that the average response to an event in different markets agrees fairly well with this theory's prediction from equilibrium data in absence of external forces or events. In this work it is first found that Linear Response efficiently resolves price dynamics at moderately perturbed financial markets of different types. Namely we study Forex markets, the S&P500 index, Commodities markets and the Bitcoin-US dollar one. Furthermore, we determine a measure of market inefficiency, which can be used to compare the inefficiency between different assets and securities.

1. Introduction

The Efficient Market Hypothesis (EMH) (Fama, 1970) states that in an information efficient market, price changes cannot be forecast if they are properly anticipated, thus, the more efficient the market, the more random the sequence of price changes generated by such market (Malkiel, 1989). However, the trade-off between risk and expected return causes market inefficiencies and sometimes, price changes are not perfectly random, even if markets operate rationally (Farmer and Lo, 1999). Therefore, the EMH by itself is not properly posed and it is an empirically refutable hypothesis.

From the three efficiency classes introduced by Fama (1970), weak, semi-strong, and strong, the most tested one by financial literature is the weak form of efficiency, where market prices are fully and fairly due to information of past ones. After the seminal paper of 1970 from Malkiel and Fama (1970), researchers have used different methodologies for testing such market efficiency. Within this regard, some authors have tested whether technical analysis is able to provide abnormal returns to the investors, see for example the works from Fama and French (1988), Olson (2004) and Shynkevich (2012). Moreover, other ones such as Lo or Matteo, analysed the statistical implication of this hypothesis: that stock returns follow a random walk (Lo and MacKinlay, 1988; Matteo et al., 2005; Dimitrova et al., 2019). In contrast, the strong efficiency class establishes that all current market information, either public or private, is completely reflected at the price of any security or asset. Thus, no investor can gain advantage on the market. However, it is possible to know inner events and future movements in companies and organizations and thus, to accurately estimate market dynamics through Financial Management Theory or insider information (finnerty, 1976; Brigham et al., 2016). To the semi-strong form of efficiency, where a price reflects solely public market information, literature has explored price adjustments

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after market events, see for example research performed by Pettit (1972) Aharony and Swary (1980) and Vidal.Tomás and Ibañez (2018). However, as Farmer and Lo (1999) resolved, most of efficiency tests frequently require several auxiliary hypotheses as well, and a rejection of such joint hypotheses tells us little about which aspect of the joint hypothesis is inconsistent with data. Within this scope, the main contribution of this paper is to resolve evidence of the semi-strong market efficiency by using a well-known model from Statistical Physics, Linear Response Theory (LRT) (Kubo, 1957; Onsager, 1931). This theory depicts the response of a system to an external force in terms of correlation functions of equilibrium quantities, i.e. it links the evolution of the perturbed system to the dynamics of the unperturbed (equilibrium) case.

LRT has been proven efficient in understanding the NASDAQ market response to fluctuations (Puertas et al., 2021), and here it is employed to depict the response to weak and moderate fluctuations in a wide range of financial markets, when such systems are destabilized by weak and moderate market events. Namely, we study a wide variety of financial market data, ranging from Forex markets, Commodities markets, the S&P500 index, and the Bitcoin-US dollar market, where LRT is considered within a twofold scope. It successfully describes market efficiency, as past and current market status and variables are available to market agents, as stated by the EMH. Thus, LRT is set to resolve market relaxation periods, as well as market depth and amplitude response to market fluctuations. Moreover, this approach allows establishing a novel quantitative approach in market efficiency, by establishing the integral of log-return autocorrelations as a measure of market inefficiency in time.

Linear response in colloidal systems

Let us start with a simple example of the application of LRT in a well known physical system, namely, colloids (particles of sub-micron size in a solvent). There, the particles undergo Brownian motion: the single particle mean squared displacement (MSD) grows linearly and the velocity autocorrelation function (VACF) is identically zero for all times $t > 0$. In a dense system, however, the interactions among the particles causes a transient trapping, which is noticed as a shoulder in the MSD between the short and long time diffusion regimes. In any case, if an external force F is exerted onto a given particle, termed tracer, the time evolution of the velocity, $v(t)$ can be calculated within linear response theory as:

$$\langle v(t)\rangle = \beta F \int \langle \mathbf{v}(t') \cdot \mathbf{v}(0)\rangle dt' \tag{1}$$

where $\langle \mathbf{v}(t')\cdot\mathbf{v}(0)\rangle$ is the VACF, calculated in equilibrium (without external force) and $\beta = 1/k_B T$ is the inverse thermal energy. Fig. 1 shows the results from simulations of a system of spherical particles undergoing Langevin dynamics. The upper panel shows the VACF of an isolated particle in equilibrium (black line) and in a bath with a volume fraction of 50% (red line). Whereas the former decays as a single exponential, the VACF of the tracer particle in the dense bath displays a faster decay to a negative minimum, signalling the rebound due to the collision with bath particles, followed by a slow increase toward zero. LRT uses this correlation function to predict the dynamics of a perturbed particle. As indicated by Eq. (1), the integral of this function provides the evolution of a perturbed tracer velocity when an external force is applied (thick lines in the lower panel). This agrees perfectly with the transient velocity of the tracer after the application of the external force (thin lines). The velocity of the isolated tracer (thin lines) increase continuously until a steady velocity is reached, whereas in a dense system, a maximum is observed, occurring when the tracer collides with its neighbours, what reduces its velocity until a steady value is reached.

Linear response in financial markets

The application of linear response theory to a system requires the identification of the variable that is conjugate to the generalized force. In financial markets, this cannot be made in advance, given that it is not a physical system. Nevertheless, we showed previously that in stock markets, the log-return, shows an auto-correlation function with the same time scale and features as the log-price evolution after an event (Puertas et al., 2021). This allowed us to identify the log-return as the conjugate variable and calculate the evolution of the log-price after a dramatic event, assuming that this was caused by an external force that started at the event and kept constant for all positive time. Here, we extend this identification to other financial markets, with different origins, regulations, or practitioners, resulting in different dynamics.

Fig. 2 shows the different markets chosen for this study: FOREX, with a free pair Euro/US dollar (EURUSD) and a pegged pair US dollar/Hong Kong dollar (USDHKD), the Bitcoin price in USD (BTCUSD), commodities (price of Brent crude oil (BCOUSD) and gold (XAUUSD) in USD), and stocks (S&P500 index). In all cases the price with a 1 min resolution has been studied from January 3, 2012 to December 30, 2021[1] (except for the bitcoin, whose price was found only starting in 2015). The figure shows the very different behaviour of these indices. Events in every market are identified as abrupt changes in the price; precisely, when the one-minute log-return is larger in absolute value than 4 times the root mean square deviation of log-returns. For this analysis, only events separated by a time span larger than the typical decay time of the log-return autocorrelation function were selected. The red bars in Fig. 2 shows the number of such events per year in every case. Note that these distributions are also very different from one asset to another, but in all cases the number of events per year is of the order of 10^3.

The average normalized evolution of the price of every asset after an event, $\widetilde{\langle \Delta x(t)\rangle}$ is shown in Fig. 3, what we identify as the response to the event (the normalization is introduced to guarantee that in all cases the security evolves from 0 to 1, allowing the average of different events); this representation also allows to account for positive and negative events. Even more, the separation

[1] Data taken from histdata.com.

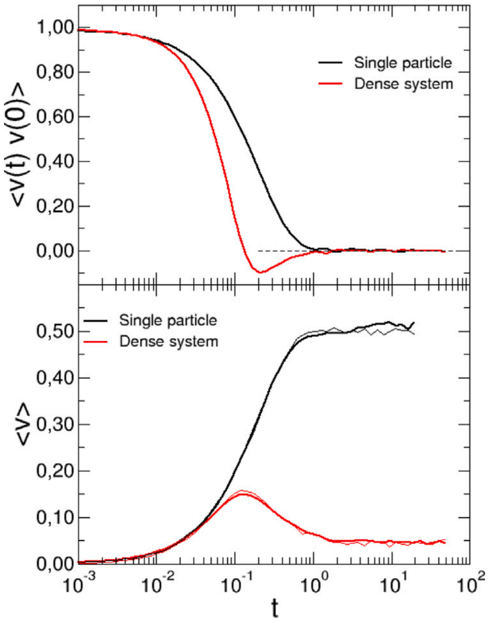

Fig. 1. Tracer velocity autocorrelation function (upper panel) in equilibrium and tracer velocity after the application of the force (lower panel) – the thick line represent the calculation given by Eq. (1), and the thin lines are the results from the simulation.

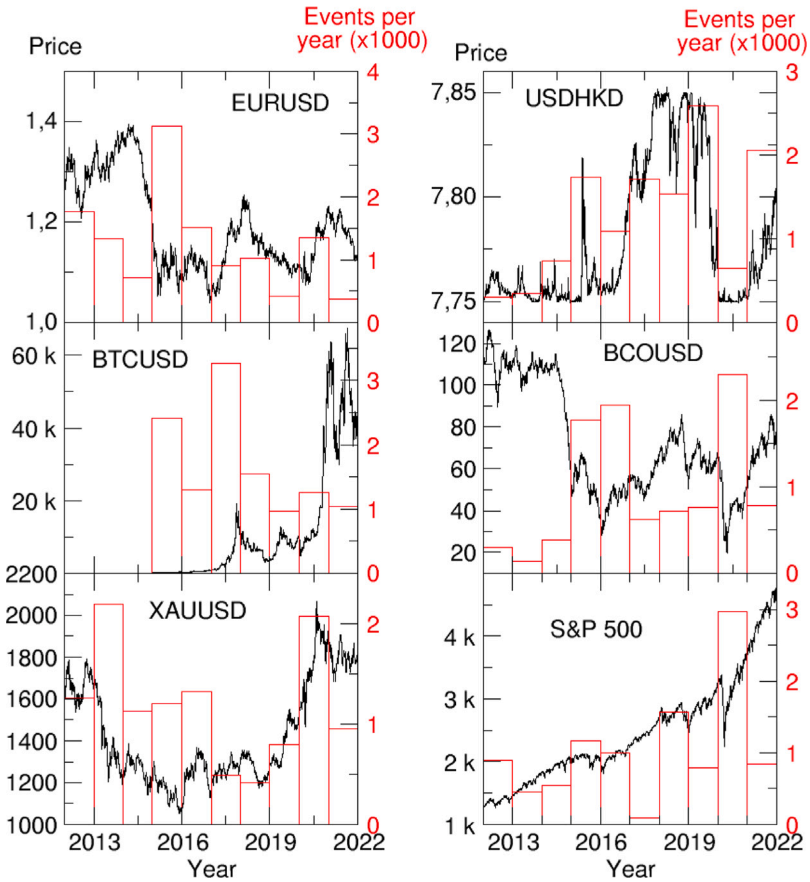

Fig. 2. Evolution of the assets studied in this work, corresponding to the Euro–US Dollar exchange rate, US Dollar–Hong Kong dollar, Bitcoin price in dollars, price of the Brendt crude oil in dollars, price of gold in dollars and S&P500 index.

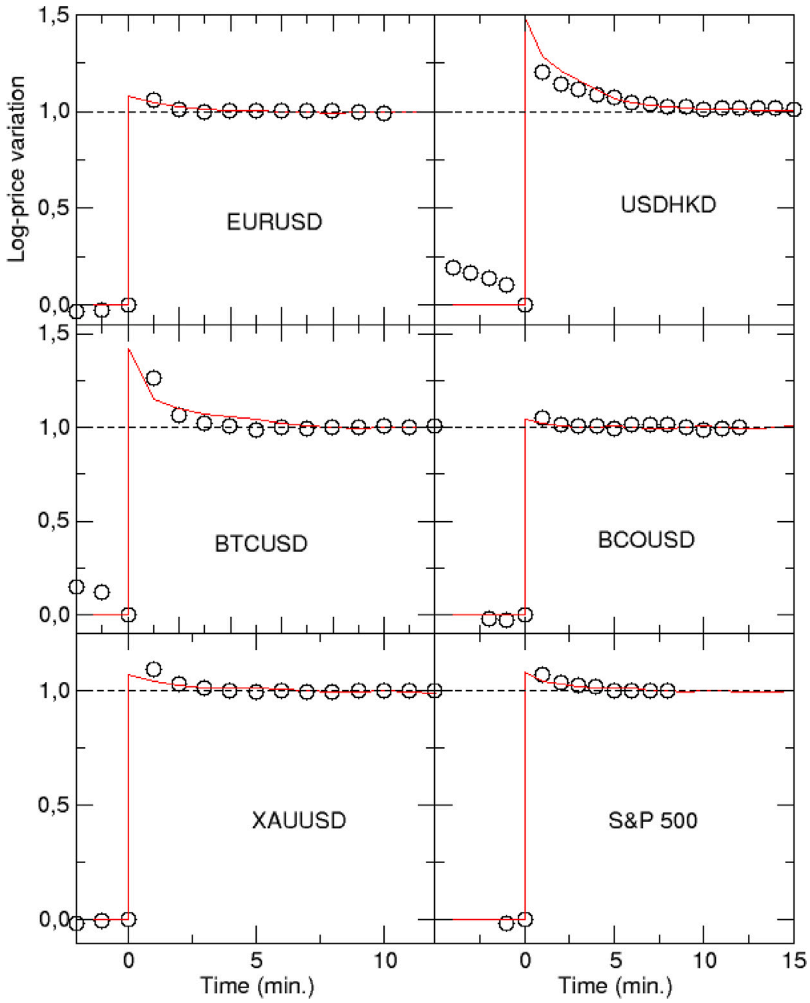

Fig. 3. Average normalized evolution of the log-price for the same assets as Fig. 2. The circles correspond to the observed evolution, and the lines to the calculations from LRT (Eq. (2)).

of events imposed in their selection, guarantees that the responses of different events do no overlap. In all cases, the security grows suddenly, within the first minute, and recovers slightly during the next few minutes, with slight differences from one asset to another. Such slow evolution indicates a transient market inefficiency which is analysed here with LRT.

According to linear response theory, and taking the log-return as the conjugate variable $v(t)$, the response of the log-price, $x(t)$, after the application of the external force is given by:

$$\langle \Delta x(t) \rangle = -\beta \int_0^t F(\tau)\langle \dot{x}(\tau) v(0) \rangle d\tau = -\beta F \int_0^t \langle v(\tau) v(0) \rangle d\tau \tag{2}$$

where $\langle v(\tau)v(0)\rangle$ is the log-return autocorrelation function obtained from the experimental data, and the force follows a Heaviside function, $F(\tau) = F\theta(\tau)$. In order to compare with the normalized evolution, $\langle \Delta x(t)\rangle$ is calculated using the normalized correlation function,

$$C_{AB}(\tau) = \frac{\langle A(\tau + t)B(t)\rangle - \langle A\rangle\langle B\rangle}{\sigma_A \sigma_B} \tag{3}$$

where the brackets imply averages over the time origin, t, and σ_X stands for the standard deviation of the time series of variable $X = A$ or B. The results are shown in Fig. 3 as red lines. In all cases, the agreement is very good; the prediction from Eq. (2) captures not only the shape of the curve but also the right intensity and time scale.

The total change in the log-price upon the application of the external force is measured from the difference between the initial log-price (immediately before the event, and well after it). As given by Eq. (2), this is proportional to the external force, $\langle \Delta x_\infty \rangle = k_x F$,

Table 1
Transport coefficient k_x for the log-price and after-event inefficiency of the market, $\mathcal{I}$.

Asset	k_x [min^{-1}]	$\mathcal{I}$ [min]					
		2012–21	2012–13	2014–15	2016–17	2018–19	2020-21
EURUSD	$8.3 \cdot 10^{-9}$	0.11	0.11	0.11	0.09	0.10	0.14
USDHKD	$5.6 \cdot 10^{-11}$	1.12	0.79	1.16	1.10	0.82	1.84
BTCUSD	$6.1 \cdot 10^{-7}$	0.56	–	0.64[a]	1.96	0.37	0.59
BCOUSD	$1.7 \cdot 10^{-7}$	0.04	0.07	0.06	0.02	0.02	0.02
XAUUSD	$2.8 \cdot 10^{-8}$	0.12	0.08	0.07	0.11	0.02	0.22
S&P500	$3.4 \cdot 10^{-8}$	0.07	0.07	0.12	0.04	0.07	0.06

[a]Only 2015.

Table 2
Comparison of the inefficiency measure $\mathcal{I}$ with other measures from the literature.

Asset	$\mathcal{I}$	TIME	AMIM
EURUSD	0.11	0.976	0.928
USDHKD	1.12	0.993	0.979
BTCUSD	0.56	0.995	0.986
BCOUSD	0.04	0.808	0.43
XAUUSD	0.12	0.976	0.928

and the proportionality constant (or transport coefficient in physical terms) is:

$$k_x = \int_0^\infty \langle v(\tau)v(0)\rangle d\tau \tag{4}$$

Table 1 gives the results of k_x for all securities studied in this work. Because the units of k_x are $\mathrm{p}^2/\mathrm{min}$, where p stands for the price of the asset, all prices are given in USD, to allow comparison among them. k_x can be interpreted as the sensibility of such asset to external forces, and the values in the table indicate that the bitcoin or commodities are more sensitive than currencies. In particular, the bitcoin is affected by external forces around 10^4 times more than the HKD. It is worth recalling that the calculation of k_x uses only equilibrium data, which according to LRT allows the prediction of the average response when an external force is applied.

The slow evolution of the price after the event indicate inefficiencies in the market, as mentioned above, in the picture of the EMH. These can be quantified by calculating the area above the dashed line for positive time in Fig. 3, i.e.

$$\mathcal{I} = \int_0^\infty \left[\widetilde{\langle \Delta x\rangle}(\tau) - 1\right] d\tau \tag{5}$$

where $\widetilde{\langle \Delta x\rangle}(\tau)$ can be taken directly, or calculated using $C_{vv}(t)$. Here, the inefficiency measure $\mathcal{I}$ is intended to account for how long does the market need to incorporate new information into the price. $\mathcal{I} = 0$ depicts that this is instantaneous, as predicted by the EMH, whereas significantly greater values of $\mathcal{I}$ indicate that new information needs some time to be reflected at the market price. The values of $\mathcal{I}$, using $C_{vv}(t)$ for all markets are also included in Table 1, indicating that the USDHKD is the less efficient market, while the Brendt oil, BCOUSD, responds immediately to external events. Even more, $\mathcal{I}$ has been calculated in periods of two years from 2012 to 2021, confirming that the USDHKD is the less efficient market throughout, but also this indicates that most markets have been less efficient in 2020–2021, in coincidence with the COVID19 crisis.

To validate this inefficiency measure, Table 2 compares the results with other methods recently proposed in the literature, TIME and AMIM, which are based on the autocorrelation of returns (Tran and Leirvik, 2019, 2020; Noda, 2016). These also identify the USDHKD and bitcoin as the less efficient markets, although $\mathcal{I}$ is more sensible, as it separates clearly more inefficient markets than almost efficient ones.

Note that validity of inefficiency measure relies on LRT, which can be further tested. The log-return can be also studied with this formalism, as the derivative of expression (2), i.e.

$$\langle \Delta v(t)\rangle = -\beta F \int_0^t \langle \dot{v}(\tau)v(0)\rangle d\tau = -\beta F\langle v(t)v(0)\rangle \tag{6}$$

The normalized correlation function $C_{vv}(\tau)$ is plotted with the normalized experimental evolution of the log-return in Fig. 4. For all securities studied, the log-return peaks when the event occurs ($t = 0$), then it describes a negative minimum whose depth varies from asset to asset, and recovers its base line for long times, i.e. in a few minutes ($\langle \Delta v(t)\rangle$ reaches zero). Therefore, the evolution of the log-return has been normalized differently, evolving from 1 at the event, to 0 at long times. The comparison with the log-return autocorrelation function gives a good agreement in all cases, as predicted by linear response theory.

Once the force and its conjugate are known, the perturbation energy can be calculated as $H' = AF$, where A is the conjugate to the force, and F the force – in a physical system, if F is a conventional force, A is the displacement. In the case of financial markets, the variable A has been identified as the log-return, but the force is unknown. However, using the linear relation between

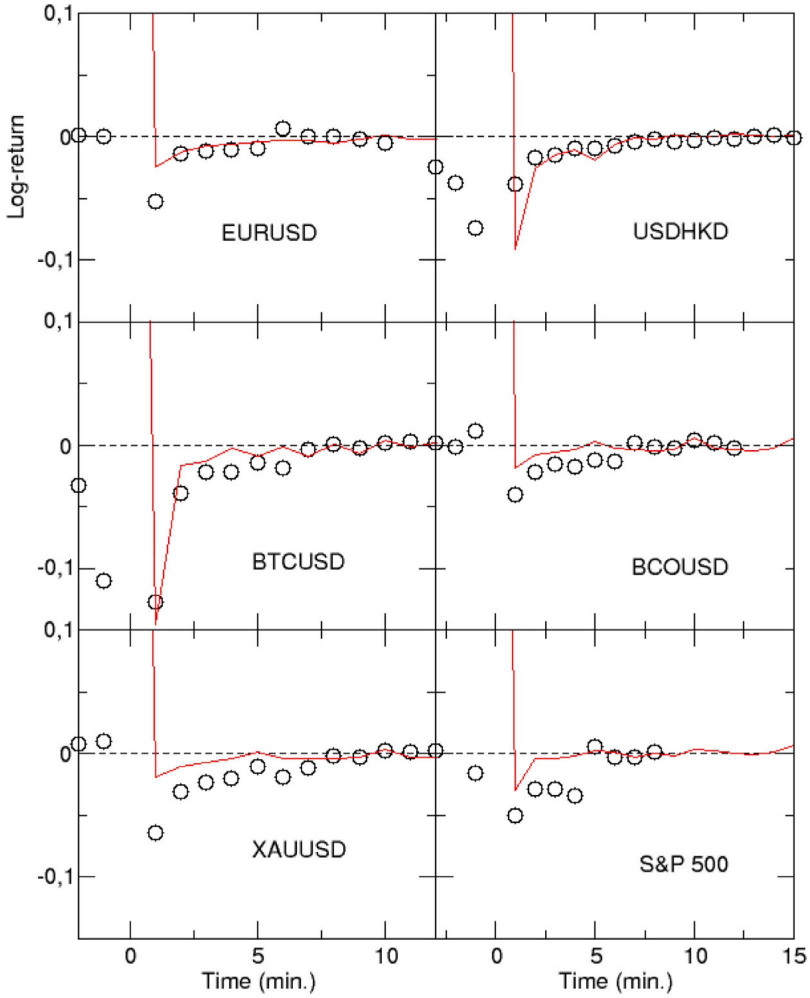

Fig. 4. Average normalized evolution of the log-return for the same indices as Fig. 2. The circles correspond to the observed evolution, and the lines to the calculations from LRT (Eq. (6)).

the force and the total change in the log-price, the perturbation energy can be written as:

$$H' = AF = \frac{1}{k_x}\langle \Delta x_\infty \rangle v(t^*) \quad (7)$$

where $v(t^*)$ is the log-return at the event. Note that since the force is constant, the evolution of this energy after the event is given by the change of the log-return, i.e. it decreases to negative values and reaches a constant base line. In physical terms, this implies that the system relaxes the energy toward the stationary state, which is reached in a few minutes.

Eq. (7) can also be used to calculate the perturbation energy due to all the events accumulated in every day. This is shown in Fig. 5, and allows the identification of the most dramatic dates, in terms of perturbation energy. This evolution can be correlated with the number of events shown in Fig. 2 (red lines), giving the strength of those events. Note that although the number of events is similar in all the cases studied here, the perturbation energy shows more prominent peaks in the currency pairs and the bitcoin, whereas the gold price or the S&P500 index have much less perturbation energy. This implies that the events in currency pairs are more energetic than in the gold or S&P500.

Conclusions

In this paper, we study the semi strong hypothesis of efficiency in different markets by using Lineal Response Theory, which is a well-established physical model. Such model accurately predicts the average price evolution in the next few minutes after a market event. Here, it must be remarked that when implementing LRT to financial markets, the model does not require setting any external parameter or initial condition, as it only depends on the log-return auto-correlation functions. Moreover, LRT performs well

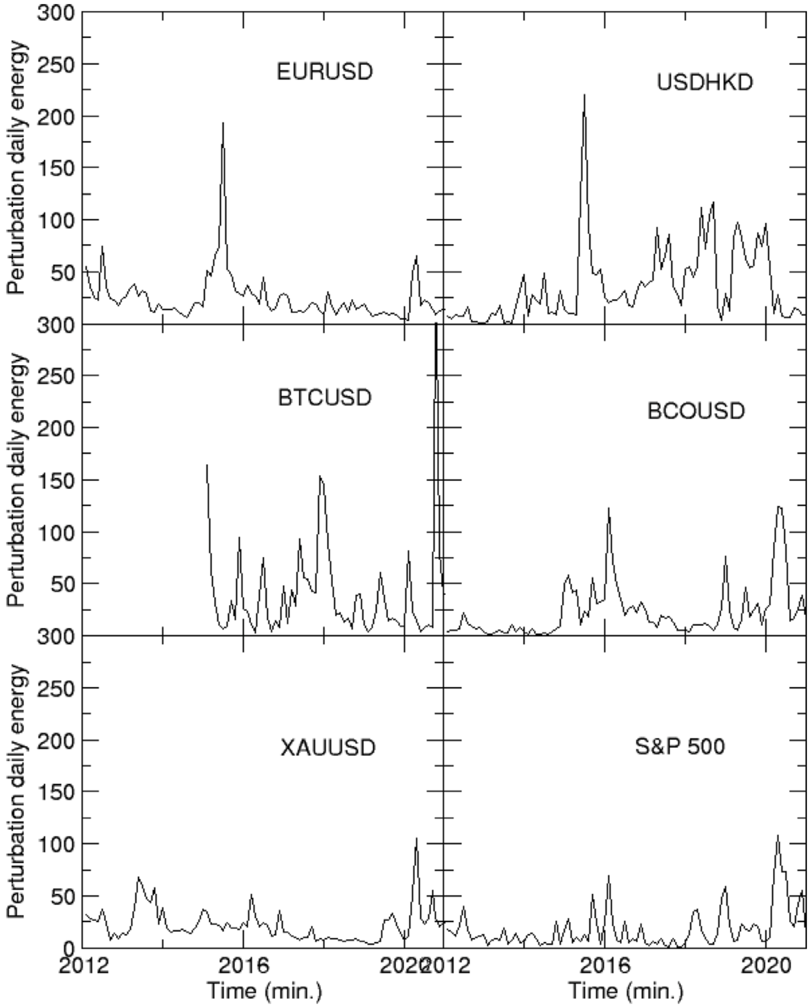

Fig. 5. Evolution of the perturbation energy per day from 2012 to 2021 (except for the bitcoin, which starts in 2015). The panels correspond to the same indices as in Fig. 2.

in depicting market response to moderate fluctuations, indicating an interesting symmetry between natural systems such as colloids or granular systems with an artificial one such as financial markets.

From our findings, it is straightforward to consider a scope of market efficiency with regard to LRT on financial markets. In an efficient market, the response to an event should be instantaneous and then, on average, the log-price should remain unvaried, as the semi-strong class of efficiency establishes. This is the case to most systems studied in this work. However, for the Oil-US Dollar system, we can determine that the response is efficient within statistical noise, but at the Hong Kong-Dollar vs. the US-Dollar market, or at the Bitcoin one, the return to equilibrium takes more time, and hence, there is a temporary loss of efficiency after an event. In the first case, it takes up to 5 to 10 min after the event to return to equilibrium, which is striking as return times are faster in most markets. In the case of the Hong Kong-Dollar, such slower efficiency can be understood due to the different economic policies of its central bank. However, for the Bitcoin case, and although financial literature supports the results obtained, the causes of this inefficiency could be the subject of further research, in order to determine, for example, if it is due to the non-existent regulation of the market, trading volume, or even the timing necessary for authenticating and registering operations at the blockchain. In view of such evidence, we propose the integrated area of the time dependant log-price variation after an event as a measure of efficiency, where an increasing magnitude of such integral indicates stronger inefficiency in each market.

CRediT authorship contribution statement

Antonio M. Puertas: Writing – original draft, Methodology. **Joaquim Clara-Rahola:** Writing – reviewing & editing. **Miguel A. Sánchez-Granero:** Investigation, Writing – original draft. **F. Javier de las Nieves:** Supervision. **Juan E. Trinidad-Segovia:** Conceptualization, Reviewing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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