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Central bank intervention and financial bubbles

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ABSTRACT

This paper develops a model to study the impact on asset prices arising from central bank intervention during bubble bursts. In particular, we explore how investors react to a policy whereby the central bank decreases interest rates to alleviate market crashes or significant price reversals. We show that the possibility of central bank intervention creates incentives for investors to inflate asset prices since larger bubbles lead to (i) higher capital gains in case of no bust and (ii) higher probability of intervention (i.e, losses being absorbed by central bank's intervention) upon bubble burst episodes. Our model predicts that (i) bubbles should be smaller in more fragile economies (i.e., economies where bubbles are more likely to burst) and (ii) larger in those scenarios where it is less costly for the central bank to cut rates and/or assets are more liquid. Finally, we show that central bank policies of the type studied in the paper are welfare enhancing if the cost imposed on the economy from a decrease in rates is sufficiently low (e.g. during non-inflationary periods) which, in turn, provides scope for forward guidance.

1. Introduction

Central bank policy in times of crises is typically a hot topic among academics, policy makers, and other market participants. Views on policies tend to be reassessed after major crises as observed in the aftermath of the global financial crisis (GFC) of 2007–2009 (Dell'Ariccia, Habermeier, Haksar, & Mancini-Griffoli, 2015; Klein, 2016; The Economist, 2013) and, most recently, the Covid-19 pandemic (Menon, 2021; Restoy, 2020). In both, central bank interventions (e.g. quantitative easing and lowering of interest rates) presumably helped economies to recover faster. In so doing, however, central banks arguably reinforced their image of lenders of last resort, i.e., institutions standing by and ready to extend a helping hand that financial markets are so often in need. As the story goes, such actions create scope for moral hazard among investors (Blinder & Reis, 2005; Buiter, Cecchetti, Dominguez, & Sanchez Serrano, 2023; Miller, Weller, & Zhang, 2002) and hence potentially leading central banks to sow the seeds of the very same crises they wish to avoid in the first place. In this context, the goal of this paper is to embed the mechanism above in an environment where central banks' reaction function is responsive to large price drops in asset prices. Our objective is to study how such a policy can affect the behavior of investors and, as a result, the corresponding impact on the inflation of bubbles.

The motivation behind this paper follows from the debate around the role of central banks in safeguarding financial stability (Boyarchenko, Favara, & Schularik, 2022; Carstens, 2019). There is significant evidence that price reversals (in the form of bubble bursts or change in fundamentals) can severely impact economic activity for prolonged periods of time (Bordo & Jeanne, 2002) which then begs the question of whether financial stability should be factored in by central banks in the design and implementation of monetary policy. This issue has been extensively studied with one strand of the literature ('clean' side) arguing that central banks should react to asset prices only to the extent that they impact inflation (Bernanke & Gertler, 2000, 2001; Evans, 2014; Gilchrist & Leahy, 2002; Svensson, 2019; Vickers, 2000) and another ('lean') advocating for central bank intervention in scenarios of financial imbalances, i.e., rapid credit growth and asset price booms (Borio & Lowe, 2005; Cecchetti, Genberg, Lipsky, & Wadhwani, 2000; Filardo &

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Rungcharoenkitkul, 2016). The 'lean' versus 'clean' debate (Book, 2021) is mostly driven by considerations about the costs and benefits of monetary policy deviating from central banks' traditional dual mandate (i.e., price stability and full employment) in order to address financial stability concerns (Adrian & Liang, 2016; Gourio, Kashyap, & Sim, 2017; Smets, 2014; Svensson, 2017). The trade-off entailed by the debate, i.e. easing of financial conditions vis a vis an increase in financial vulnerabilities, got reinvigorated after the GFC (Bernanke, 2015; Kocherlakota, 2014; Mishkin, 2011; Stein, 2014; White, 2009) and in the context of the impact of financial cycles – driven by credit growth, house and equity prices – on the macroeconomy and likelihood of recessions (Borio, 2012; Borio, Drehmann, & Xia, 2018; Filardo, Hubert, & Rungcharoenkitkul, 2022). It is to this discussion that the present paper contributes to.

The analysis presented in the sequence assumes that central banks act only upon the burst of asset price bubbles however not in preventing their build up (the so called benign neglect). In other words, the paper takes a stance that central banks engage with the 'clean' but not the 'lean' side of the debate or, in other words, that there is an asymmetric policy response over the business cycle (Hoffmann, 2013; Ravn, 2012). The bubble burst episodes warranting central bank intervention (and focus of the paper) are those vindicated from a reduced-form cost–benefit analysis (with respect to reacting or not to the bust in asset prices). The reaction function considered entails the easing of monetary policy (i.e., decrease of the short-term interest rate) similarly to what was observed in 1987 (stock market crash), 1990–91 (Saving & Loans crisis), 1998 (Long Term Capital Management collapse), 2001–04 (dotcom bubble), 2007–08 (GFC) (Brunnermeier, 2009; Greenlaw, Hatzius, Kashyap, & Shin, 2008; White, 2009), and during Covid-19 most recently (Cantú, Cavallino, De Fiore, & Yetman, 2021). In the presence of a central bank policy represented by such reaction function, the paper studies the impact on investors' behavior and the consequences for the magnitude of bubbles. The importance of such an approach is well summarized in Filardo et al. (2022, p. 2):

> *A connection between the systematic behavior of monetary policy and the financial cycle could arise because market participants' expected returns depend on how the central bank would react to scenarios of financial market frothiness. The conditional paths of the policy rate shape beliefs about the prospective returns to investment, including under the scenarios of speculative bubbles, thus influence investors' incentives to take on risks and financial variables today.*

The bubble bursts considered in the paper are not specific to an asset class although we interpret them as a collapse in equity prices. In this context, Mishkin and White (2003a, 2003b) point out that the stock market should play a role in the intervention decision of a central bank only to the extent that it puts stress on the financial system. Cieslak and Vissing-Jorgensen (2021, p. 4046) show that "*since the mid-1990s, the Fed has engaged in a sequence of policy easings following large stock market declines in the intermeeting period"*. Similar evidence of policy accommodation upon poor stock returns is found in Kurov, Olson, and Zaynutdinova (2022) and Rigobon and Sack (2003) where a 5% fall in the S&P500 is found to lead to a decrease in the federal funds rate of about 14 basis points. There is evidence suggesting that central banks' policy (regardless of whether conventional or unconventional) indeed affect asset prices (Putniņš, 2022; Thorbecke, 1997).

To study the impact on investors' behavior from the possibility of central bank intervention following market crash episodes that encapsulates the discussion above, we present a model where investors and central banks play a game. In this game, investors decide whether to buy an asset and, the more of them doing so, the more a bubble is inflated. Conditional on the bubble bursting, the central bank chooses to keep the interest rate unchanged or to decrease it, the former option entailing a cost proportional to the magnitude of the bust and, the later, representing the cost of deviating from an interest rate that otherwise would be dictated solely by the goals of price stability and full employment (e.g., Taylor rule). Our modeling choice is motivated by the seminal currency attack model developed in Morris and Shin (1998) and, in particular, the global games methodology behind it. The global games approach rely on assumptions that better capture the dynamics of financial markets and the information differential of central banks relative to investors. Moreover, it provides for a unique equilibrium according to which a comparative statics analysis can be performed in relation to the decision of investors and central bank based on changes in the primitives of the model: cost of intervention, transaction costs, and likelihood of bubble bursting.

In order to shed light on how the availability of information about the economy affects investors' decisions (since it indicates the likelihood of bubble bursts and intervention itself), we study settings with complete and incomplete information. In both, we show that a decrease in interest rates as a response to bubble bursts incentivizes investors to magnify bubbles. The driver of this result is investors acting strategically in light of central bank's policy: if they buy the asset and the bubble does not burst, large capital gains will be made; if the bubble bursts, the larger the asset price collapse is and, therefore, the more likely the central bank is to intervene (and hence alleviate losses). Similarly to Morris and Shin (1998), we show that the equilibrium strategy of investors is characterized by a unique threshold against which they compare their private signals when deciding to invest. This is a very appealing outcome since it matches simple trading rules used by investors (Brock, Lakonishok, & LeBaron, 1992). The unique equilibrium derived allows us to conduct a comparative statics leading to the following conclusions: (i) in economies with less developed financial markets (i.e., with an inherently higher susceptibility to bubble bursts), asset prices are less inflated since investors are more cautious to participate in the market; (ii) in scenarios where the cost of decreasing the interest rate is prohibitively high (for example, during periods of high inflation), investors are less certain of central bank intervention which leads to lower buying activity and smaller bubbles as well; and (iii) bubbles are smaller in illiquid assets/markets: by virtue of investors paying higher transaction costs, there are less incentives to buy which then makes it harder to inflate bubbles to a magnitude that would warrant central bank intervention in case of a bust. These results also relate to the literature exploring the nexus between financial development and crises (Ben Naceur, Candelon, & Lajaunie, 2019; Gourinchas, Valdes, & Landerretche, 2001; Loayza & Rancière, 2005; Rajan, 2005).

The optimality of the intervention policy discussed in the paper is studied from both an ex-ante and ex-post perspective. In particular, we consider whether a decrease in interest rates by the central bank in response to a collapse in asset prices is a desirable policy from a social welfare perspective. We show that the answer to this question depends primarily on the costs entailed by a reduction in interest rates (e.g., potential misallocation of capital or compromising the central bank price stability/full employment dual mandate). Ex-post, intervention is welfare enhancing if the cost it imposes on the economy is low whereas, ex-ante, whenever the *expected* cost is low. Given that the cost of intervention depends on variables that are difficult for a central bank to observe and forecast, these results suggest that intervention ex-post might be easier to be warranted than ex-ante, therefore landing support for discretionary intervention policies rather than pre-committed rules (Kocherlakota, 2016). In this context, central bank communication regarding its economic outlook and policy plans (which in our case refers to the likelihood of lowering interest rates), i.e., forward guidance, turns out to be very important as well (Baker, 2013; Bernanke, 2020).

The main contribution of the paper is to show that a policy whereby a central bank reacts to asset prices collapses but not their build up (only cleaning but not leaning) inflates asset price bubbles and, if they burst, a decrease in interest rates alone might not be sufficient to absorb losses that otherwise could spillover to the real economy. Hence, our message is twofold, namely that (i) leaning against the wind is critical since it prevents the inflation of asset prices, and (ii) unconventional monetary policies should be part of a central bank's tool box since changes in interest rates alone might not be able to do the clean up after a crisis (and particularly so in scenarios of liquidity trap, i.e., interest rates close to the zero lower bound). This is in contrast to the existing 'lean' vs. 'clean' literature that puts emphasis predominantly on the side effects of leaning against the wind to curb asset price inflation however without discussing the consequences on investors' behavior from the cleaning policy adopted. Another contribution is to show that asset price bubbles can be rationalized based on investors incorporating the impact of the central bank's put into asset prices, i.e., the guarantee that potential losses will be absorbed by accommodative policies creates incentives for investors to buy an asset at a larger price relative to its fair value.

1.1. Related literature

The paper contributes to the literature around the debate of the 'lean' vs. 'clean' approach to monetary policy. In particular, we focus on the impact of accommodative policies (i.e., lowering of interest rates) in episodes of price reversals (characterized as bubble bursts) on investors' behavior. Although with a different modeling approach, our paper obtain similar results to Drechsler, Savov, and Schnabl (2018) showing that lower nominal rates lead to higher asset prices and volatility. The mechanism driving this result is risk-shifting in the presence of financial intermediaries that results in bubbles as pointed out earlier in Allen and Gale (2000). Evidence of this mechanism in the U.S. is provided in Dell'Ariccia, Laeven, and Suarez (2017). There is also a growing literature on laboratory experiments studying the impact of monetary policy on asset price bubbles (Bao & Zong, 2019; Fischbacher, Hens, & Zeisberger, 2013; Galí, Giusti, & Nousssair, 2021). Although we do not attempt to characterize the best response of a central bank to a collapse in prices that might compromise financial stability nor discuss alternative intervention policies (Allen & Gale, 1998, 1999), our welfare analysis also shows that a central bank intervention absorbing losses (and hence decreasing the size of a price collapse) might be warranted.

The model developed in the paper aims to formalize the idea that central bank policies in periods of financial distress can sow the seeds of future crises. Our focus on the impact from the 'cleaning' intervention from central banks is due to a large literature already studying the consequences associated with credit booms induced by monetary policy, for instance the leverage cycle theory (Geanakoplos, 1997, 2003; Mendoza, 2010) driving financial cycles (Claessens, Kose, & Terrones, 2011). Moreover, it has been well documented empirically the role played by credit and lending booms in creating financial instability (Dell'Ariccia, Igan, Laeven, & Tong, 2016; Jordà, Schularik, & Taylor, 2013), to the point of if being considered a leading indicator of future crises (Greenwood, Hanson, Shleifer, & Sørensen, 2022; Schularik & Taylor, 2012). The literature also points out that not every episode of credit boom leads to busts in asset prices (Gorton & Ordoñez, 2020); bubbles fueled by credit (Galí, 2014; Grimm, Jordà, Schularick, & Taylor, 2023), though, tend to bring more pernicious effects to the real economy once they burst (Jordà, Schularik, & Taylor, 2015), and this is the scenario that we envisage in the model discussed in the sequence. It can be said that it was the GFC that re-sparked the discussion around monetary policy as a crisis preventing mechanism with the last two decades landing extra support to the financial instability hypothesis discussed in Minsky (1986).

The game that we rely on to study the impact on investors' behavior and asset prices from central bank's intervention policy during a crisis is reminiscent of the model in Morris and Shin (1998). In particular, we modify their payoff function studying scenarios of perfect and imperfect information about the state fundamentals in order to highlight how beliefs about bubble bursts and likelihood of intervention impact asset prices. In the scenario with perfect information, a multiplicity of equilibria in the same fashion of Diamond and Dybvig (1983) ensues. Breaking up with the assumptions of (i) common knowledge of fundamentals and (ii) agents knowing each others behavior in equilibrium, i.e., using a global games approach (Carlsson & van Damme, 1993; Morris & Shin, 2000, 2003), leads to a unique equilibrium that, in turn, allows us to perform a comparative static analysis and discuss policy implications. Among these, we shed light on how the magnitude of asset price bubbles are related to the characteristics of the economy and financial markets where they develop including financial development (Dell'Ariccia & Marquez, 2006; Loayza, Ouazad, & Rancière, 2018), liquidity (Cecchetti, 2004), and intervention costs faced by central banks operating under the dual mandate of price stability and full employment (Cao & Chollete, 2017; Ikeda, 2022).

Although it has been argued in the literature that a 'leaning' policy of increasing interest rates and hence tightening financial conditions during booms might be counterproductive to fight bubbles since they are difficult to identify, our results suggest two reasons for the opposite. Firstly, one reason for increasing rates might be to prevent bubbles getting too large in which case the consequences of sharp price collapses can be much harder to contain. Secondly, it is possible that interest rates remain low for too long in which case there might not be enough room for maneuver once a crises triggered by the burst of a bubble happens, i.e., monetary policy is not effective closer to the zero lower bound (Kontonikas, MacDonald, & Saggu, 2013). Whereas the second reason has been studied in the literature, it seems that we are the first paper formally arguing in favor of a 'leaning' policy based on the first.

The structure of the paper is as follows: the next section details the model, first analyzing the benchmark case of perfect information about the state of fundamentals. Subsequently we assume that investors only receive a noisy signal about the state of the economy allowing for a unique equilibrium of the game between investors and the central bank to be derived. Under the unique equilibrium, a comparative static analysis is performed and the impact of changes in the parameters of the model is discussed. Finally, the last section explores the optimality of intervention (ex-ante and ex-post) establishing the conditions for the central bank policy of decreasing interest rates in bubble burst episodes to be welfare improving.

2. Model

The model consists of a 1-asset, 3-date economy ($t = 0, 1, 2$) populated by a unit mass of agents (investors) represented by the set $I = [0, 1]$. There is also a central bank that sets the interest rate (cash rate) in the economy. At the initial date ($t = 0$) investors must decide whether to buy one unit of the asset, i.e., each $i \in I$ chooses either $X_0^i = 1$ (buy) or $X_0^i = 0$ (not buy). The asset is available in fixed unitary supply and if an investor decides to buy it, a fixed price p_0 must be paid at $t = 0$. The asset must be sold at date $t = 2$ (and only then) with a transaction cost of t_c. After investors make their decisions, at $t = 1$ (interim stage) the price of the asset adjusts from p_0 to $p_1 := g(\theta, r_0, \alpha)$, where θ is the state of fundamentals of the economy, r_0 is the current interest rate, and α is the fraction of investors who have decided to buy the asset. The function g is interpreted as the observed price of the asset satisfying[1] $g_\theta > 0$, $g_{r_0} < 0$, and $g_\alpha > 0$ in order to account for, respectively, (i) asset prices increasing with the state of fundamentals as a result of the procyclicality of credit and leverage (Adrian & Shin, 2014), (ii) the inverse relationship between interest rates and asset prices (time-value of money), and (iii) positive impact on price from excess demand. In a partial equilibrium framework (as in our setup), the function g can be interpreted as the stochastic process driving the price behavior of the asset with θ, r_0, and α being parameters of the chosen specification.

We postulate there is a wedge causing the observed price to differ from the asset's fundamental value which is given by $f(\theta, r)$ with $f_\theta > 0$ and $f_r < 0$. The wedge between the two prices is based on the premise that the fundamental value should reflect only discounted future cash flows[2] whereas the observed price is impacted by the mass of investors buying the asset at the initial date.[3] Given the wedge between the observed price and the asset's fundamental value, a bubble is defined as[4] $b(\theta, r_0, r_1, \alpha) := g(\theta, r_0, \alpha) - f(\theta, r_1)$. To simplify the analysis, we focus on positive bubbles, i.e., $b(\theta, r_0, r_1, \alpha) > 0$. It is also assumed that $b_\theta > 0$ (once again to account for cyclicality of credit supply and the corresponding impact on bubbles) and that $b_r > 0$ (the central bank can decrease the magnitude of a bubble burst by decreasing interest rates).

A bubble bursts in our framework as a function of two factors, namely (i) intrinsic characteristics of the economy captured by parameter L, and (ii) the state of fundamentals θ. The parameter L represents institutional arrangements that better support and justify an increase in prices; in our framework, it is the lowest state of fundamentals according to which a bubble can persist. For example, if an increase in asset prices is being fueled by availability of credit, an economy with high (low) L would be one with low (high) lending standards and, accordingly, the likelihood of a bubble burst would be high (low). In this sense, we classify an economy as strong if $\theta > L$ and weak otherwise, $\theta \le L$. In a strong economy, bubbles do not burst and the final price of the asset ends up being equal to the observed one, $p_2 = g(\theta, r_1, \alpha)$. Conversely, in a weak economy the bubble collapses causing the final price of the asset to be equal to its fundamental value, $p_2 = f(\theta, r_1)$. The final price of the asset, p_2, is dependent on the interest rate r_1 set by the central bank at the interim stage once it has been realized whether the bubble has burst (and its size).

The decision of the central bank to change the interest rate from r_0 to r_1 is considered as follows: if the bubble bursts (weak economy) and there is no change in the interest rate ($r_1 = r_0$), the cost imposed on the economy is assumed to be proportional to the magnitude of the price collapse and equal to $b(\theta, r_0, r_0, \alpha)$. With central bank intervention, the interest rate is set at $r_1 = r^* < r_0$ such that the cost for the economy of a bubble burst is reduced to[5] c. Since the fundamental value given by f is decreasing in the interest rate, investors who bought the asset always benefit when the central bank lowers the interest rate which happens whenever the bubble bursts and $b(\theta, r_0, r_0, \alpha) > c$. On the other hand, if the bubble does not burst (strong economy) the central bank does not change the interest rate.[6]

The above specification can be thought in the following way. When the economy is strong, bubbles do not burst and hence do not require central bank intervention which is then vindicated by the fact that the economy is "healthy" to begin with. On the

[1] The subscript represents the partial derivative of the function with respect to the corresponding argument.

[2] We abstract from modeling them in order to keep the framework simple.

[3] Hence the reason for $g(\cdot)$ to be a function of α however not $f(\cdot)$.

[4] The observed price is based on the initial interest rate r_0 whereas the fundamental value will depend on the interest rate r_1 subsequently set by the central bank — the decision of the central bank to change the interest rate is discussed in the sequence.

[5] In other words, r^* is such that $b(\theta, r_0, r^*, \alpha) = g(\theta, r_0, \alpha) - f(\theta, r^*) = c$.

[6] We justify this assumption on the basis that it might be too risky for the central bank to try to cool down the "market". Although not embedded in our model, there could be uncertainty in relation to the true fundamental value of the asset leading the central bank to be cautious and hence to avoid leaning against the bubble.

other hand, bubbles cannot persist in weak economies and, if the price collapse is sufficiently large, the economy is better-off with a decrease in interest rates by the central bank. The parameter c can be viewed as the side effects brought about by the adoption of a laxer monetary policy in the context of a bubble burst that otherwise can compromise financial stability. Among these side effects we could cite: (i) inflation in case a significant price reversal takes place whilst the economy is operating at full employment; (ii) moral hazard and the associated cost of future crises[7] triggered by excessive risk taking once it is perceived that the central bank will act as either lender (or market maker) of last resort; (iii) induced financial fragility due to excessive liquidity (Lopomo Beteto Wegner, 2020). It is out of the scope of the paper (given the static nature of the model studied and the parameters it is based on) to provide a micro-foundation of the trade-off and costs represented by c; the reduced-form analysis that it encapsulates, however, applies without loss of generality and shows how factors compromising the implementation of an accommodative policy can serve as a disciplining device for central banks to tame bubbles (if it can be credibly communicated to market participants and in the scenario where financial stability is considered as a goal itself).

Conditional on the decision of the central bank to change the interest rate (intervention for short), the payoff to an arbitrary investor at $t = 2$ can be described as follows:

$$X_0^i\left(p_2 - p_0 - t_c\right) = \begin{cases} X_0^i\left[g\left(\theta, r_0, r_0, \alpha\right) - p_0 - t_c\right] & \text{if } \theta > L \text{ (no burst, no intervention)} \\ X_0^i\left[f\left(\theta, r_0\right) - p_0 - t_c\right] & \text{if } \theta \le L \text{ (burst, no intervention)} \\ X_0^i\left[f\left(\theta, r^*\right) - p_0 - t_c\right] & \text{if } \theta \le L \text{ (burst, intervention)} \end{cases} \tag{1}$$

The model so far is silent in relation to the information available to investors at the initial date when they must decide whether to buy the asset. As it turns out, information about the state of fundamentals will play a pivotal role in determining the way that investors behave. Next turn our focus to the analysis of two settings with investors having perfect and imperfect information about θ at $t = 0$, respectively.

2.1. Perfect information about the state of fundamentals

In the scenario with perfect information, the central bank knows the state of fundamentals of the economy, θ, which is assumed common knowledge among investors too. The parameter L that determines the threshold for bubble bursts as well as the observed price and fundamental value of the asset are also considered common knowledge. Investors' decision and the central bank's reaction to it unfold in the following way:

1. Nature draws θ from a uniform distribution $U[0, 1]$;
2. The central bank and investors observe θ;
3. Investors decide whether to buy the asset;
4. The central bank observes the fraction of investors who bought the asset, α, and decides whether to intervene;
5. Game ends (investors sell the asset).

The equilibrium of the game is a profile of strategies followed by investors and the central bank with no incentives to deviate from. For investors, the strategy specifies a course of action for each possible realization of θ whereas for the central bank it will be the decision to lower the interest rate conditional on the size of the bubble if it bursts (recall that intervention is warranted if and only if $b\left(\theta, r_0, r_0, \alpha\right) > c$). To structure investors' decision, we impose that $b\left(L, r_0, r_0, 0\right) < c$ (the central bank never intervenes if the fraction of investors who purchased the asset is negligible), $b\left(0, r_0, r_0, 1\right) < c$ (intervention by the central bank is never vindicated if the economy if too weak), and $b\left(L, r_0, r_0, 1\right) > c$ (else investors would not entertain the possibility of central bank intervention in case of a bubble burst). Moreover, with $\underline{\theta}$ being the state of fundamentals such that the central bank is indifferent to intervention when all investors buy the asset (i.e., $b\left(\underline{\theta}, r_0, r_0, 1\right) = c$), we also assume that $f\left(\underline{\theta}, r_0\right) = p_0 + t_c$ (which implies that investors are not guaranteed a profit from buying the asset in case the bubble bursts and there is no intervention; if there is intervention, investors would at least breakeven since $f\left(\theta, r^*\right) > p_0 + t_c$ for $\theta \in \left(\underline{\theta}, L\right]$ and $r^* < r_0$). In case of no burst, we consider $g\left(\theta, r_0, \alpha\right) > p_0 + t_c$ (investors make a profit). Under these "boundary conditions", the possible scenarios for θ and the corresponding outcomes are:

1. $\theta \in \left[0, \underline{\theta}\right)$: since $\underline{\theta}$ is given by $b\left(\underline{\theta}, r_0, r_0, 1\right) = c$ and the bubble is increasing in the state of fundamentals, it follows that the cost for the central bank to intervene is larger than the cost of letting the bubble burst and hence no intervention ensues. This implies that it is not profitable for investors to buy the asset as $f\left(\theta, r_0\right) < p_0 + t_c$. Therefore, "not intervene" (central bank) and "not buy" (investors) is an equilibrium strategy profile.
2. $\theta \in \left[\underline{\theta}, L\right]$: from the definition of $\underline{\theta}$, there exists a critical mass of investors, $\underline{\alpha}$, such that $b\left(\theta, r_0, r_0, \underline{\alpha}\right) = c$. The assumption that the bubble is increasing in α implies that the central bank will lower the interest rate if and only if $\alpha \ge \underline{\alpha}$. Since investors are better-off not buying the asset when the central bank does not intervene, it follows that "not intervene" (central bank) and "not buy" (investors) is an equilibrium. On the other hand, since $f\left(\underline{\theta}, r_0\right) = p_0 + t_c$ and the fundamental price is decreasing in the state of fundamentals, if the central bank intervenes lowering the interest rate then investors prefer to buy the asset and hence leading to a second equilibrium of the form "intervene" (central bank) and "buy" (investors).

[7] Albeit difficult to measure.

3. $\theta \in (L, 1]$: in this scenario, the economy is strong allowing bubbles to persist and, therefore, central bank intervention does not take place. Since $g(\theta, r_0, \alpha) > p_0 + t_c$, it is then a dominant strategy for investors to buy the asset making the profile "not intervene" (central bank) and "buy" (investors) an equilibrium.

Hence, if the state of fundamentals is either too low, $\theta \in [0, \underline{\theta})$, or too high, $\theta \in (L, 1]$, the central bank does not intervene which causes "not buy" and "buy" to be the best response by investors, respectively. For intermediate levels of the state of fundamentals, $\theta \in [\underline{\theta}, L]$, investors having perfect information about θ gives rise to multiple equilibria as in Diamond and Dybvig (1983). Notice that, in case they could coordinate their decisions, investors would eliminate the multiplicity of equilibria by always buying the asset if $\theta > \underline{\theta}$ as it would lead to a payoff of $f(\theta, r^*) - (p_0 - t_c) > f(\underline{\theta}, r_0) - (p_0 - t_c) = 0$ (the payoff from not buying the asset).

The multiplicity of equilibria in the perfect information setting prevents us from knowing how the model's parameters affect the equilibrium outcome since we do not know which particular equilibrium will prevail in the first place. Moreover, the assumption of perfect information puts investors and the central bank on the same footing in the sense that both have access to the same information regarding the state of fundamentals. We find this troublesome since the decision of the central bank to change the interest rate will have a concomitant effect on investors' payoffs and hence can be classified as an endogenous component of the state of fundamentals itself. In this way, a setting where the central bank has "better" information relative to investors would be a more compelling alternative to represent the dynamics observed in the real world. As shown in the next section, both issues are addressed by adding uncertainty to the information obtained by investors before they make their decision — that prevents them from discerning whether the bubble will burst (and from eliciting central bank's reaction as a result).

2.2. Imperfect information about the state of fundamentals

We now relax the assumption of investors having perfect information about the state of fundamentals, θ. In particular, we are interested in a scenario where the central bank has an informational advantage over investors that in turn forces the latter to infer what action the former will take. This is a relevant situation for us to analyze since in the real world investors "price" or try to anticipate the conduct of monetary policy in the economy. In this context, the timeline of events is modified as follows:

1. Nature draws θ according to a uniform distribution, $U[0, 1]$;
2. The central bank observes θ and each investor receives a private signal x, $x \sim U[\theta - \varepsilon, \theta + \varepsilon]$, with $\varepsilon > 0$;
3. Investors decide whether to buy the asset;
4. The central bank observes the fraction of investors who bought the asset, α, and decides whether to intervene;
5. Game ends (investors sell the asset).

As in the previous section, the equilibrium of the game is found by determining the best response of the central bank to the realization of θ and α and, conditional on that, the reaction function of investors is derived.[8] The reasoning presented in the sequence (and the proofs in the Appendix) closely follow Morris and Shin (1998). First, investors know that the central bank lowers the interest rate in a bubble burst episode if and only if $b(\theta, r_0, r_0, \alpha) > c$. Let $a(\theta)$ denote the critical mass of investors buying the asset that vindicates central bank intervention when the state of fundamentals is θ. In other words, $a(\theta)$ is given by $b(\theta, r_0, r_0, a(\theta)) = c$. Denote by $\pi(x)$ the probability of an investor to buy the asset[9] upon receiving signal x and by $s(\theta, \pi)$ the total fraction of investors purchasing the asset as a result. Since $x \sim U[\theta - \varepsilon, \theta + \varepsilon]$, it follows that:

$$s(\theta, \pi) = \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x)\, dx. \tag{2}$$

Define by $A(\pi)$ the event where central bank intervention ensues conditional on investors following strategy π:

$$\begin{aligned} A(\pi) &:= \{\theta \mid s(\theta, \pi) \geq a(\theta)\} \\ &= \left\{ \theta \mid \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x)\, dx \geq a(\theta) \right\}. \end{aligned} \tag{3}$$

The payoff from buying the asset conditional on θ and investors following strategy π is:

$$h(\theta, \pi) := \begin{cases} g(\theta, r_0, \alpha) - p_0 - t_c & \text{if } \theta > L \text{ (no burst, no intervention)} \\ f(\theta, r_0) - p_0 - t_c & \text{if } \theta \leq L \text{ (burst, no intervention)} \\ f(\theta, r^*) - p_0 - t_c & \text{if } \theta \leq L \text{ (burst, intervention)} \end{cases} \tag{4}$$

As investors do not observe the state of fundamentals, the investment decision must be based on the expected payoff from buying conditional on signal x and given by:

$$\begin{aligned} u(x, \pi) &:= \frac{1}{2\varepsilon} \int_{x-\varepsilon}^{x+\varepsilon} h(\theta, \pi)\, d\theta \\ &= \frac{1}{2\varepsilon} \left[\int_{[L, x+\varepsilon]} h(\theta, \pi)\, d\theta + \int_{[x-\varepsilon, L] \cap A(\pi)^c} h(\theta, \pi)\, d\theta + \int_{[x-\varepsilon, L] \cap A(\pi)} h(\theta, \pi)\, d\theta \right] \end{aligned} \tag{5}$$

[8] We continue to assume the boundary conditions discussed in Section 2.1.

[9] In other words, π is the investment strategy.

which follows from splitting the domain of integration into three mutually exclusive sets (the first where a bubble does not burst, the second where there is a bubble burst but no intervention, and the third with both a bubble burst and intervention by the central bank). Using the definition of $h(\theta, \pi)$ allows us to write:

$$u(x,\pi) = \frac{1}{2\varepsilon}\left[\int_{[L,x+\varepsilon]} g\left(\theta, r_0, s(\theta,\pi)\right) d\theta + \int_{[x-\varepsilon,L]\cap A(\pi)^c} f\left(\theta, r_0\right) d\theta + \int_{[x-\varepsilon,L]\cap A(\pi)} f\left(\theta, r^*\right) d\theta\right] - p_0 - t_c. \tag{6}$$

Since an investor can guarantee a payoff of zero by not buying the asset, the decision of whether or not to buy the asset is tantamount to $u(x,\pi)$ being positive or negative, respectively. Thus, if the central bank follows its intervention policy, π is an equilibrium of the first period game if $\pi(x) = 1$ for $u(x,\pi) > 0$ and $\pi(x) = 0$ otherwise. We are now in the position to enunciate the following proposition:

Proposition 1. *There is a unique equilibrium in the imperfect information game played by investors and the central bank. In particular, there are unique θ^* and x^*such that the central bank lowers the interest rate if and only if $\theta^* \leq \theta \leq L$ and investors buy the asset if and only if their signal x is such that $x > x^*$.*

The proof of the proposition can be found in the Appendix. One of the crucial steps to prove this result is to show that investors' equilibrium strategy satisfies the property of strategic complementarity. In other words, investors are better-off buying the asset when others are also doing so. This is a nice feature as it captures a type of behavior often observed in stock markets, namely that of investors clustering their decisions (buying when everybody is buying and selling when everybody is selling). The driver of this result is the positive externality that investors impose on each other when they buy the asset: the capital gain is larger if the bubble being inflated does not burst whereas if it does, the likelihood of central bank to intervene and absorb losses is higher the more inflated the price of the asset is.

Another important feature of the equilibrium in Proposition 1 is that investors follow a cut-off rule to make their decisions — that also agrees with heuristics followed by participants in stock markets,[10] i.e., buying the asset if the signal is above a particular threshold and not buying/selling it otherwise. Indeed, this type of investment strategy can explain patterns of booms and busts observed in real markets: if all investors follow the same rule and signals are not too noisy, the majority of them would be either buying or selling the asset leading asset prices to behave accordingly.

The most beneficial outcome from the introduction of uncertainty into investors' decision making process is the uniqueness of equilibrium that it entails. In particular, it allows us to study how the cut-off levels θ^* and x^* that the central bank and investors base their decisions on are impacted by changes in the parameters of the model. This is the subject of the analysis presented next.

2.2.1. Comparative statics

As discussed in the previous section, the assumption of imperfect information leads to a unique equilibrium with investors buying the asset only if the signal received is above a threshold x^* and the central bank intervening only if the state of fundamentals is such that $\theta \in [\theta^*, L]$. The objective of this section is to analyze how these thresholds are impacted by changes in the parameters of the model so that we can ultimately contrast alternative central bank policies.

Changes in the Threshold Point of Bubble Bursts, L

The parameter L determines whether bubble bursts: this is the outcome whenever $\theta \in [0, L]$. In this way, L can be associated with the economy's fragility from the perspective of exposure to bubble bursts, i.e., more (less) fragile economies would command a higher (lower) L. Hence, we are implicitly taking a standpoint such that the fragility of the economy is not the same as its state of fundamentals, the former being related to L and the later, to θ. As a testable implication, if bubbles could be identified we would claim that they should last longer in economies featuring characteristics that actually make it harder for bubbles to start in the first place. One example relates to access to mortgages and housing bubbles: in spite of bubbles existing both in jurisdictions with low and high lending standards (e.g., United States and Australia), they are obviously less likely to burst in those jurisdictions where the screening process has been more effective (i.e., we would argue that $L_{Australia} < L_{USA}$).

By virtue of having a high L, fragile economies are more susceptible to bubble bursts and, therefore, investors would feel less compelled to buy the asset. Less investors buying the asset leads to a lower α in the model: bubbles become less inflated and, in case they burst, it is less likely that the central bank will intervene lowering the interest rate. In this way, even though L does not explicitly appear in the characterization of the equilibrium threshold point (see Proposition 7 in the Appendix), in more fragile economies both θ^* and thus x^* should be larger (and the opposite for less fragile economies).

Even though not formally done here, one could hypothesize about what would happen if investors had imperfect information about L (with beliefs being interpreted as market sentiment). For instance, if investors were to believe the switching point of bubble bursts to be L' rather than L with $L' < L$, then they would be more likely to buy the asset due to underestimating the likelihood of bubble bursts. Investors in this scenario could be classified as overconfident which is yet another factor leading to bubbles.[11]

[10] One example being decision making driven by technical analysis although in this case the signal is the price of the asset itself.

[11] Shiller (2005) discusses the effects of market sentiment on the inflation of bubbles. According to his argument, bubbles come to be at times associated with the beginning of the so-called new eras, often depicted in a rosy way with people believing that prosperous times came for good and to the benefit of everyone.

Changes in the Cost of Central Bank Intervention, c

The model defines central bank intervention as a decrease in the interest rate following a bubble burst episode. The cost of this intervention is meant to capture the side effects of the adoption of a more expansionary monetary policy (e.g., inflation and misallocation of capital, to cite a few). The more significant these side effects are, the less likely the central bank is to intervene. Accordingly, investors in those scenarios will be more reluctant to buy the asset, i.e., they will adopt a higher threshold x^* to assess whether they should invest. This in turn translates in lower market participation (the variable α in the model) dampening the magnitude of bubbles.

The case where intervention becomes prohibitively costly can be thought of as one where $c \to \infty$. In this case, the interest rate is kept constant, $r_1 = r_0$, regardless of the magnitude of bubble bursts. From the proof of Proposition 7 in the Appendix, the threshold point of investors' equilibrium strategy, x^*, converges to θ^* as the signal received becomes less noisy, i.e., $\varepsilon \to 0$. Moreover, expression (41) in the same proposition implies that $r^* = r_0$ leads to θ^* being determined by:

$$f\left(\theta^*, r_0\right) = p_0 + t_c. \tag{7}$$

Therefore, upon acknowledging the impossibility of central bank intervention and receiving clear information about the state of fundamentals, investors adopt a strategy according to which they will at least break-even in case of a bubble burst — this is an intuitive result given that the possibility of the interest rate being decreased that alleviates losses is now precluded from the outset. As shown in Section 4, the threshold θ^* implied by (7) when central bank intervention is precluded due to being too costly is higher than the analogous one obtained from (41) when a lowering of the interest rate is a possibility. Hence, this confirms that the mere possibility of central bank intervention during bursts increases the magnitude of bubbles.

Just as a thought exercise and without entering the discussion about the merits of the policy to be pursued by the central bank, one way of credibly committing to a non-intervention policy would be by signaling to investors that either the cost of lowering the interest rate is substantially high[12] or that the central bank has no flexibility of doing so. Indeed, this was the scenario faced by the Federal Reserve: the S&P500 has collapsed by more than 20% in the first semester of 2022 however due to inflation going beyond the 2% target, the possibility of a decrease in interest rates has been heavily discounted (Jones & Altunbas, 2022).

Changes in Transaction Costs, t_c

The transaction cost parameter t_c is meant to represent the easiness at which investors can sell the asset. As such, it can be viewed as a measure of asset liquidity: the higher t_c, the lower asset liquidity is. Intuitively, the costlier to sell the asset, the less prone investors will be to buy it, particularly so when there is uncertainty in regard to the state of fundamentals. If investors are more cautious in their buying decision, bubbles will be smaller with burst episodes vindicating intervention by the central bank only at state of fundamentals beyond higher levels of θ^* (since it is only when that the magnitude of bursts will be significant). Formally, the effect of changes in t_c on the equilibrium threshold point can be analyzed based on expression (41) in Proposition 7 (see Appendix). In particular, using the fact that:

$$\underbrace{\phi_0'\left(-\theta^*\right)}_{<0}\underbrace{\left[f\left(\phi_0\left(-\theta^*\right), r_0\right) - f\left(\phi_0\left(-\theta^*\right), r^*\right)\right]}_{<0} > 0, \tag{8}$$

expression (41) can be re-written as:

$$K\left(\theta^*\right) + f\left(\theta^*, r_0\right) + f\left(\theta^*, r^*\right) = 2\left(p_0 + t_c\right) \tag{9}$$

where $K\left(\theta^*\right) := \phi_0'\left(-\theta^*\right)\left[f\left(\phi_0\left(-\theta^*\right), r\right) - f\left(\phi_0\left(-\theta^*\right), r^*\right)\right]$. The question is then how θ^* on the left-hand side of (9) changes given an increase in t_c. To answer it, we need to know the sign of $K'(\cdot)$. If (i) bubbles increase at a positive rate with the state of fundamentals (equivalent to $b\left(\theta, r_0, r_1, \alpha\right)$ being convex in θ) and (ii) the higher the interest rate, the lower the effect of changes in θ on the asset's fundamental value (i.e., $f_\theta\left(\theta, r^*\right) > f_\theta\left(\theta, r\right)$ for $r^* < r$), it follows that $K'(\cdot) > 0$. In this case, (9) implies that a higher t_c should be associated with a higher θ^*, i.e., a lower asset liquidity should command a higher threshold level determining the decision of the central bank to step in and decrease the interest rate in a bubble burst episode. In summary, the model predicts that bubbles should be smaller in less liquid assets.

An analysis analogous to the one done for changes in t_c could be applied in relation to the effect on θ^* from changes in p_0, the initial price of the asset. Following the same reasoning presented before, we would conclude that the higher the initial price of the asset, the more cautious investors would be to buy it and, in turn, leading to smaller bubbles. We can also entertain a liquidity premium by way of assuming that p_0 and t_c are inversely related (among other factors, higher initial prices could result from high demand which translates as lower transactions costs). In this scenario, rather than studying the effect of changes in p_0 and t_c in isolation we would look at the impact on θ^* from their joint behavior.[13]

The discussion above in relation to the effect on the magnitude of bubbles given changes in the model's parameters is summarized in the proposition below:

[12] Including political costs too.

[13] One testable implication from the analysis of the joint behavior of price and transaction cost would be, for example, that bubbles are more likely after rather than before a stock split.

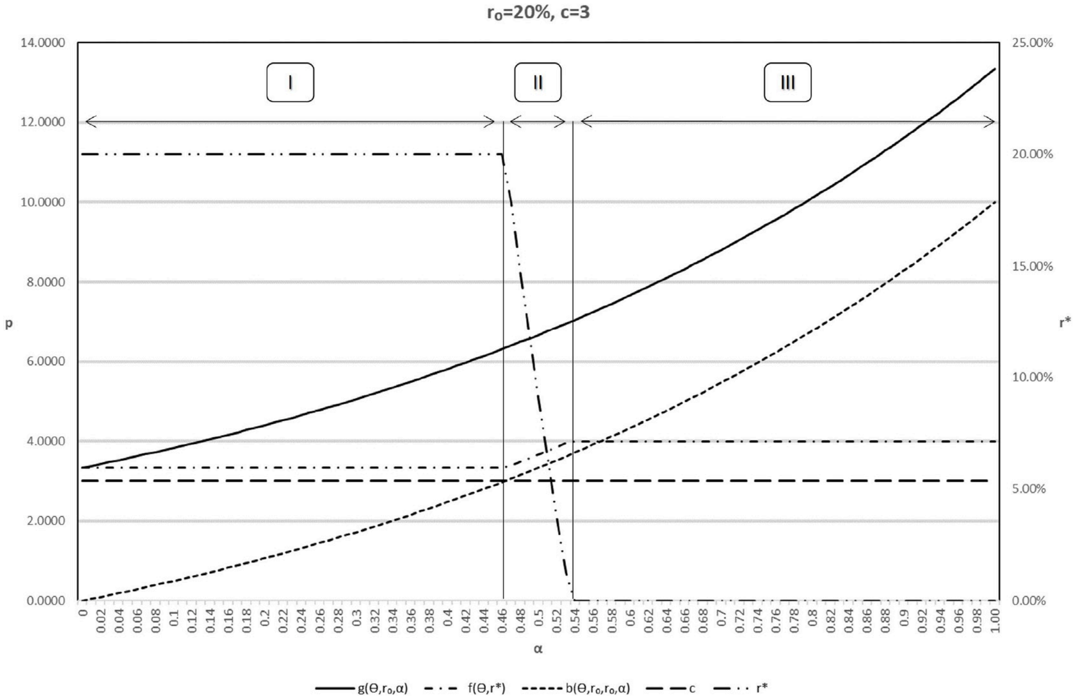

Fig. 1. Prices, bubble size, and interest rates for $c = 3$ and $r_0 = 20\%$.

Proposition 2. *In economies where bubbles are more likely to burst (higher L), investors are more cautious to buy the asset and do so only at relatively higher signals ($x > x^*$), hence leading to bubbles of smaller magnitude. Investors are also more cautious to buy the asset when it is more costly for the central bank to lower interest rates (higher c) and liquidity is lower (higher t_c), both scenarios also leading to smaller bubbles.*

3. Numerical example

In this section we provide a numerical illustration of the model in the scenario of perfect information presented in Section 2.1. Consider the following functions governing the observed and fundamental asset prices (and hence the bubble size):

$$g\left(\theta, r_0, \alpha\right) = \frac{\theta^{1+\alpha}}{1+r_0}; f\left(\theta, r_1\right) = \frac{\theta}{r_1} \Rightarrow b\left(\theta, r_0, r_1, \alpha\right) = \frac{\theta^{1+\alpha}}{1+r_0} - \frac{\theta}{r_1} \tag{10}$$

It is readily verifiable that the functions above satisfy the assumptions discussed in the beginning of Section 2. Without loss of generality, consider the state of fundamentals θ as following a uniform distribution $U[1, 10]$, the threshold of bubble bursts being $L = 5$, central bank cost of intervention $c = 3$, initial interest rate $r_0 = 20\%$, and $p_0 + t_c = 2.5$. Under these parameters, the minimum state of fundamentals for which a bubble burst would potentially warrant intervention (i.e., $\underline{\theta}$ as discussed in Section 2.1) is $\underline{\theta} = 2.46$. Hence, for a particular realization of $\theta = 4$, there is a bubble burst and intervention will be determined by the fraction α of investors buying the asset. In this context, the corresponding prices (observed and fundamental), bubble size, and optimal interest rates are displayed in Fig. 1.

Fig. 1 is divided in three regions, I, II and III. In region I, there is not enough investors buying the asset to make the magnitude of the bubble collapse (represented by the function $b\left(\theta, r_0, r_0, \alpha\right)$) to be higher than the cost of intervention c. Hence, in this region the interest rate remains constant at $r_0 = 20\%$ and the price of the asset drops to $f(4, 20\%) = 3.33$ (with the size of the collapse depending on the fraction of investors α buying the asset). In region II, α is sufficiently such that the cost of letting the bubble burst without decreasing the interest rate becomes larger than the cost of intervention. Hence, in this region the central bank lowers r_0 to r^*) (as a function of α and such that the bubble burst cost is now equal to c as discussed in footnote 5) causing the price of the asset to increase to $f(4, r^*)$. Finally, in region III the price of the asset and interest rate remains constant by virtue of the zero lower bound — mitigating the cost-impact of the bubble burst would warrant a further decrease in interest rates which is not feasible since r^* cannot be negative. Comparing the fundamental price with and without central bank intervention, i.e., $f(4, 20\%) = 3.33$ and $f(4, 0\%) = 4$, shows that a lowering of interest rate helped investors with a $0.67/3.33 = 20.12\%$ increase in prices.

In order to shed light on how the results depend on the parameters, Fig. 2 displays prices and central bank intervention (lowering of interest rates) for $c \in \{1, 3\}$ and $r_0 \in \{1\%, 10\%, 20\%\}$. First, it is noticeable that, with a higher cost of intervention (moving from $c = 1$ to $c = 3$), it takes more investors (α) to make a bubble large enough so that its burst would warrant a decrease in interest rates

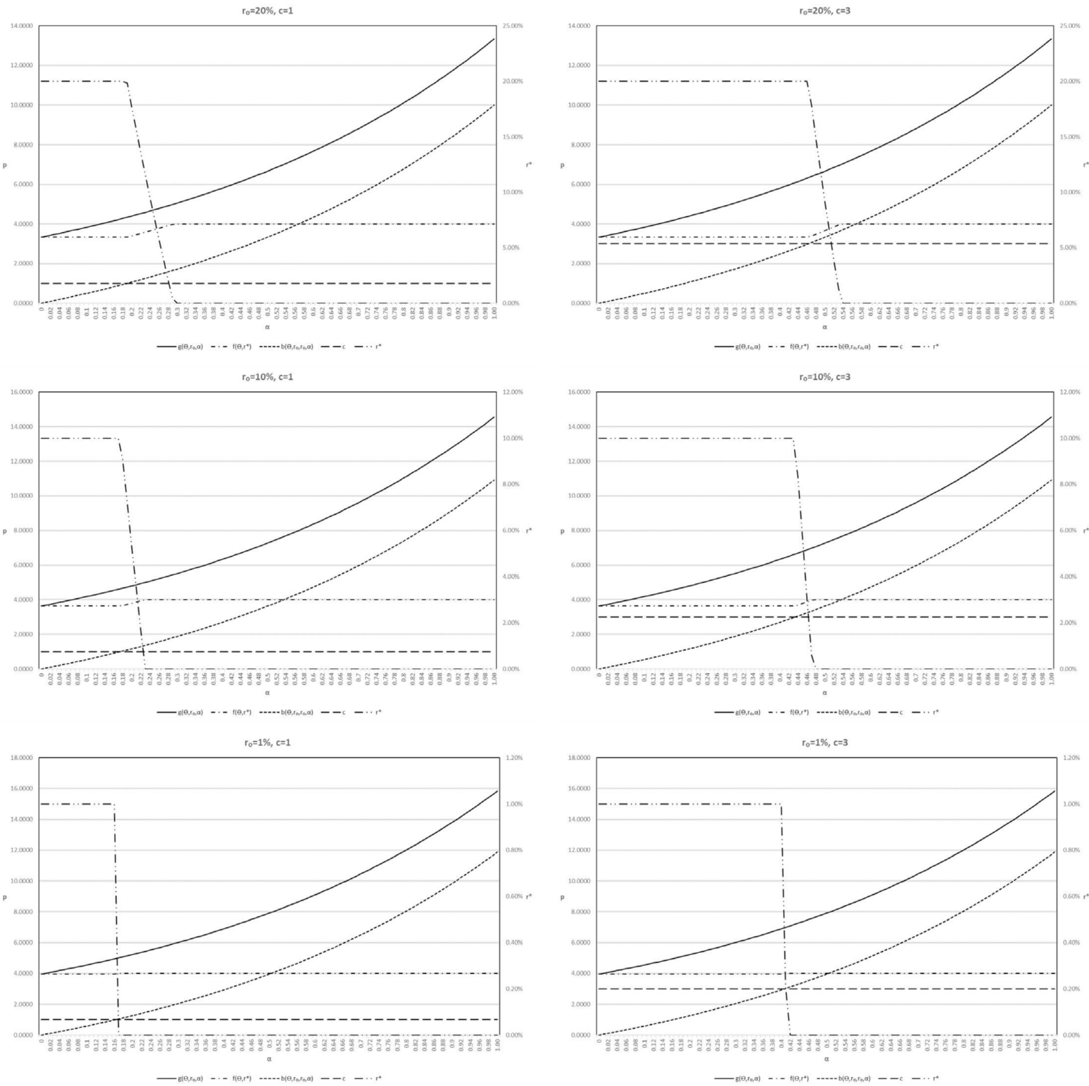

Fig. 2. Prices, bubble size, and interest rates for different parameters.

by the central bank. In this way, a higher cost of intervention means that it would be harder for investors to inflate bubbles since it would require more coordination among them; as for the cost of intervention, one could argue that it could be communicated by the central bank to investors through forward guidance.[14] Second, with a lower initial interest rate (moving from $r_0 = 20\%$ to $r_0 = 1\%$), intervention by the central bank to absorb the impact of bubbles loses its efficacy quite quickly with a faster arrival at the zero lower bound. This further highlights the point that (conventional) monetary policy might not by itself be enough to contain the negative side effects of bubble bursts and hence that a leaning against the wind policy would need to be in place so as to avoid bubbles getting too large in the first place.[15]

[14] The credibility of it would require the application of a dynamic model different from the one proposed here.

[15] Needless to say that we are abstracting from the side effects of raising rates beyond what would be required to maintain price stability and full employment which is one of the counterarguments against a 'leaning' policy.

4. Welfare and optimality of central bank intervention

The previous sections have shown that the possibility of central bank intervention in bubble burst episodes affects the behavior of investors and, as a result, the final price of the asset in the model. The question that we address now is whether such a policy is beneficial from a welfare perspective. Building on the setup discussed in Section 2.2, we turn now to the study of welfare enhancement from an ex-post perspective, subsequently analyzing the benefits of central bank intervention from an ex-ante standpoint too.

4.1. Ex-post optimality of intervention

For the purpose of analyzing the benefits of intervention, we consider as a metric of welfare the sum of investors' profits net of the costs imposed onto the economy following the central bank's intervention (or lack thereof) in a bubble burst episode. Hence, let W^{CB} denote the economy's welfare under the possibility of central bank intervention and W the analogous metric when such a possibility is precluded. We have:

$$W^{CB}(\theta) := \begin{cases} \int_{x\geq\theta^*_{CB}} \left[g\left(\theta, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right) - p_0 - t_c\right] dx & \text{if } \theta > L \\ \int_{x\geq\theta^*_{CB}} \left[f\left(\theta, r^*\right) - p_0 - t_c\right] dx - c & \text{if } \theta^*_{CB} \leq \theta \leq L \\ \int_{x\geq\theta^*_{CB}} \left[f\left(\theta, r_0\right) - p_0 - t_c\right] dx - b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right) & \text{if } \theta < \theta^*_{CB} \end{cases} \tag{11}$$

with θ^*_{CB} denoting the threshold determining central bank intervention and $\alpha\left(I_{\theta^*_{CB}}\right)$ the mass of investors buying the asset when the noisy in their signal dissipates (i.e., $\varepsilon \to 0$ implying $x^* \to \theta^*_{CB}$). Without intervention, analogously:

$$W(\theta) := \begin{cases} \int_{x\geq\theta^*} \left[g\left(\theta, r_0, \alpha\left(I_{\theta^*}\right)\right) - p_0 - t_c\right] dx & \text{if } \theta > L \\ \int_{x\geq\theta^*} \left[f\left(\theta, r_0\right) - p_0 - t_c\right] dx - b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right) & \text{if } \theta \leq L \end{cases} \tag{12}$$

with θ^* denoting the threshold to which the cut-off of investors' equilibrium strategy converge when $\varepsilon \to 0$. Before comparing the economy's welfare with and without the possibility of central bank intervention, from Proposition 7 in the Appendix it follows that θ^*_{CB} is implicitly given by:

$$\phi'_0\left(-\theta^*_{CB}\right)\left[f\left(\phi_0\left(-\theta^*_{CB}\right), r_0\right) - f\left(\phi_0\left(-\theta^*_{CB}\right), r^*\right)\right] + f\left(\theta^*_{CB}, r_0\right) + f\left(\theta^*_{CB}, r^*\right) = 2\left(p_0 + t_c\right), \tag{13}$$

whereas the analogous threshold θ^* without the possibility of a lowering in interest rates during bubble bursts is obtained from:

$$f\left(\theta^*, r_0\right) = p_0 + t_c. \tag{14}$$

From (8) and the fact that the fundamental value function is such that $f_\theta(\cdot) > 0$, it follows from (13) and (14) that:

$$\theta^* > \theta^*_{CB}. \tag{15}$$

Hence, since $x^*_{CB} \to \theta^*_{CB}$ and $x^* \to \theta^*$ with investors' signals becoming less noisy (i.e., $\varepsilon \to 0$), it follows that $x^* > x^*_{CB}$ and, therefore, without the possibility of central bank intervention investors will be more cautious to buy the asset — there will be a lower mass of them to inflate the bubble in this scenario. Hence, when the bubble does not burst (i.e., $\theta > L$), it follows from (13) and (14) that under the possibility of intervention the capital gains faced by investors will be higher and so will be their welfare:

$$W^{CB}(\theta) > W(\theta) \quad \text{if} \quad \theta > L. \tag{16}$$

In case of a bubble burst episode, the first relevant range for the comparison of the welfare measures is for $\theta \in \left[\theta^*_{CB}, L\right]$. In this scenario, from (11) and (12) it follows that intervention is welfare improving if the following condition is satisfied:

$$\int_{x\geq\theta^*_{CB}} \left[f\left(\theta, r^*\right) - p_0 - t_c\right] dx - c \geq \int_{x\geq\theta^*} \left[f\left(\theta, r_0\right) - p_0 - t_c\right] dx - b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right)$$

$$\Leftrightarrow$$

$$c \leq \int_{x\geq\theta^*_{CB}} f\left(\theta, r^*\right) dx - \int_{x\geq\theta^*} f\left(\theta, r_0\right) dx - \left(\theta^* - \theta^*_{CB}\right)\left(p_0 + t_c\right) + b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right). \tag{17}$$

Therefore, in the range of fundamentals where there the bubble bursts and the central bank lowers the interest rate, the answer to the question of whether such intervention is welfare improving depends on the parameter c, i.e., the magnitude of the side effects brought about by a laxer monetary policy. If c is high enough, the economy would be better-off in the absence of central bank intervention or, in other words, lowering the interest rate would be inefficient. Else, we could think of a high c as representing constraints for a decrease in interest rates to take place, e.g. inflation spiraling.

Lastly, in case there is a bubble burst and the central bank does not step in, i.e., $\theta \in \left[0, \theta^*_{CB}\right)$, from (15) it follows that:

$$\int_{x\geq\theta^*} \left[f\left(\theta, r_0\right) - p_0 - t_c\right] dx - b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right) >$$

$$\int_{x\geq\theta^*_{CB}} \left[f\left(\theta, r_0\right) - p_0 - t_c\right] dx - b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right), \tag{18}$$

which from (11) and (12) implies that $W(\theta) > W^{CB}(\theta)$. Hence, intervention reduces welfare in those states where the presence of the central bank induces more investors to buy the asset but with those very same investors left at their own when a bubble burst materializes without intervention being warranted. In this scenario, therefore, the existence of a policy that alleviates losses to investors is also inefficient.

4.2. Ex-ante optimality of intervention

We now turn to the analysis of the optimality of central bank's intervention policy from an ex-ante perspective. Hence, in what follows neither the central bank itself nor investors know the state of fundamentals, i.e., the realization of θ. To be precise, the question we want to shed light on is whether there the central bank standing by to lower the interest in case of large bubble bursts is justified to begin with. With uncertainty in relation to the state of fundamentals, the expected utility of investors upon buying the asset is given by (24) and, accordingly, we define the ex-ante, expected welfare metric in the economy with the possibility of central bank intervention, W_E^{CB}, as:

$$W_E^{CB}(\theta) := \int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*_{CB}}\right) dx \\ - Pr(\theta \le L)\left[Pr\left(\theta^*_{CB} \le \theta \le L\right) c - Pr\left(\theta < \theta^*_{CB}\right) E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right) \middle| \theta < \theta^*_{CB}\right]\right] \tag{19}$$

where Pr denotes the probability distribution of θ and E is the expected value operator (i.e., ex-ante investors form beliefs about the state of fundamentals based on private signals x that are uniformly distributed on $[\theta-\varepsilon, \theta+\varepsilon]$). Analogously, the expected welfare in case of no possibility of intervention is:

$$W_E(\theta) := \int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*}\right) dx - Pr(\theta \le L) E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right) | \theta \le L\right]. \tag{20}$$

Therefore, the intervention policy of the central bank is ex-ante efficient if and only if $W_E^{CB}(\theta) > W_E(\theta)$ which from (19) and (20) is equivalent to:

$$\int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*_{CB}}\right) dx - \int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*}\right) dx \\ > \\ Pr(\theta \le L)\left[Pr\left(\theta^*_{CB} \le \theta \le L\right) c - Pr\left(\theta < \theta^*_{CB}\right) E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right) \middle| \theta < \theta^*_{CB}\right]\right] \\ - Pr(\theta \le L) E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right) | \theta \le L\right]. \tag{21}$$

Hence, the condition for central bank intervention to be ex-ante welfare improving involves now the expected cost of intervention as captured by the term $Pr\left(\theta^*_{CB} \le \theta \le L\right) c$ - the higher it is, the less warranted the intervention policy will be. Hence, if the expected cost imposed into the economy from a decrease in interest rates during a bubble burst is sufficiently small, such policy is vindicated ex-ante as well. Else, the best alternative for the central bank is to commit to a non-intervention policy regardless of the size of bubble bursts.

Recall from Section 2.2.1 that the parameter L can be interpreted as the fragility of the economy, i.e., the degree of its exposure to bubble bursts: a higher L means that bubbles are more likely to burst. In this case, as argued before investors will be more cautious to buy the asset which in turn increases the cut-off level of the threshold strategy they adopt, x^*, and particularly so when there is no possibility of central bank intervention. Hence, it follows that changes in L have a concomitant effect on both sides of (21) and the dominating effect will be determined by the shape of the distribution of θ and the functional forms of the functions f and g. That said, suppose as a thought exercise that $L = 1$, i.e., the economy has the highest degree of fragility (and thus bubbles always burst). In this scenario, intervention is highly valued by investors and the utility from buying the asset should be strictly higher compared to the case where such policy cannot be carried by out by the central bank. This should make the left-hand side of (21) strictly positive and, recalling that θ follows a uniform distribution $U[0,1]$, the condition for $W^{CB}(\theta) > W(\theta)$ can be rewritten as:

$$\int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*_{CB}}\right) dx - \int_{\theta-\varepsilon}^{\theta+\varepsilon} u\left(x, I_{\theta^*}\right) dx \\ > \\ c\left(1-\theta^*_{CB}\right) - E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*_{CB}}\right)\right) \middle| \theta < \theta^*_{CB}\right] - E\left[b\left(\theta, r_0, r_0, \alpha\left(I_{\theta^*}\right)\right) | \theta \le L\right]. \tag{22}$$

Therefore, for c sufficiently small, the right-hand side of (22) is negative whereas the right-hand side is guaranteed to be positive, hence the inequality holds. In other words, for economies with the highest level of fragility ($L = 1$) and with the central bank facing a small cost c to step in during bubble bursts, intervention would be welfare enhancing.

The following proposition summarizes the analysis presented above contrasting the optimality of intervention ex-post and ex-ante:

Proposition 3. *A policy whereby the central bank lowers the interest rate to r^* is ex-post efficient (i.e., welfare enhancing) if the cost of intervention, c, is sufficiently small. If in addition to that the expected cost is also small, the same policy is ex-ante efficient too.*

5. Discussion

The comparison of the model under the assumptions of perfect and imperfect information allows us to draw analogies with the way that central banks operate in the real world. In particular, Propositions 2 and 3 emphasize the role played by c, i.e., the cost imposed into the economy from a decrease in the interest rate, in the central bank's intervention decision. In our setup, c is assumed to be common knowledge however we would expect investors to be aware of it upon communication from the central bank. Hence, our model lends support to forward guidance as an effective instrument in central bank's toolbox. The same applies to the parameter capturing market liquidity (t_c) which is yet another variable that the central bank exerts a considerable degree of influence over and, therefore, one that also demands forward guidance. As an illustration of this point, the taper tantrum episode of 2013 (whereby the Federal Reserve in the U.S. was perceived as transitioning from a quantitative easing to quantitative tightening policy) has brought a lot of volatility to financial markets; in our model, changes in t_c as impacted by central bank policy could also sway investors from participating in the market and hence also increase volatility.

Another reason why our model lends support to forward guidance is that, although the framework developed consists of fixed parameters (e.g. c and t_c), in reality the phenomena that they capture are dynamic. Hence, the central bank might be forced to lower the interest rate in circumstances where it was not expected to (e.g., when c transitions from high to low as it might happen with inflation) or to not intervene when investors would have otherwise expected a decrease in the interest rate. Without forward guidance whereby the central bank communicates to the market how the relevant variables are evolving and the corresponding impact on the implementation of the intervention policy, central bank's decision might be seen as time-inconsistent (irrespective of the fact that the policy being applied is the same).

In relation to the optimality of central bank intervention, it is important to emphasize that the model considered focuses only on the impact of monetary policy when bubbles burst. In the 'lean' vs. 'clean' debate, the main point of contention is in relation to the side effects of raising rates when prices are increasing, i.e., bubbles forming; in this sense, our analysis is not informative since, by default, we only consider interventions in bust episodes. Allen, Barlevy, and Gale (2018) show that, for credit-driven bubbles (as implicitly assumed in our setup) coupled with default costs (analogous to our parameter 'c'), a leaning against the wind policy is welfare improving in case default costs are sufficiently high; our analysis points to the same result by showing that a lowering of interest rates might not be effective to contain costly bursts. Hence, there would be room for preempting the formation of large bubbles through a leaning policy. Moreover, by way of not explicitly considering price stability and full employment in the model proposed, the paper is silent in relation to the side-effects of low interest rates in causing the formation of asset price bubbles in an inflation targeting regime. Our mechanism driving bubbles is only a decrease of interest rates as a response to episodes of financial distress that might lead to instability rather than monetary policy conducted during the normal course of affairs.

Another aspect of the modeling applied that deserves attention is in relation to the informational requirements impinged on the actors embedded in our setup, i.e., investors and the central bank. Although we assume that central banks are able to observe the magnitude of bubbles, they only act upon bust episodes (the ramifications of which being all that we are interested in). Hence, we could instead consider a policy whereby the central banks reacts to significant asset price reversals without entering into the merits of whether those represent bubble bursts or large variations in fundamental values. Not only that, we could also argue that, as far as its reaction function is concerned, all that matters for central banks is to grasp the impact on the economy following a sharp collapse in prices (with the implicitly assumption being that investors understand that too, e.g. through forward guidance as previously discussed).

The model presented provides insights into one factor that might give rise to asset price bubbles, i.e. central bank accommodative policy during crises episodes. However, it does not propose an endogenous mechanism through which bubbles burst. The parameters of the model shed light on the economy's characteristics and state of fundamentals that make bubbles more likely to burst although without specifying what exactly triggers the bust. Hence, we are not offering a full fledged framework that can capture the impact of (conventional) monetary policy in taming and inflating bubbles across cycles (business and financial) but, rather, only a simple one addressing how a decrease in interest rates during bursts induces investors to inflate prices in the first place. Some issues considered (e.g., future costs caused by intervention and credibility of central bank policy including its communication) would require a dynamic setup to be studied in more details.

Finally, the framework adopted does not explicitly consider financial intermediation which in turn precludes us from elaborating on the role of macroprudential regulation. This is an important point since a macroprudential approach is often viewed by policymakers as the way to fight bubbles (Barlevy, 2018; Biljanovska, Górnicka, & Vardoulakis, 2021). We do not see this is a shortcoming of the analysis since we not specifically focus on ways to prevent bubbles but rather whether central bank can induce them. In a framework where macroprudential policy can be effectively used to prevent the inflation of asset prices, a lowering of interest rates to absorb losses during bust episodes would be less conducive to moral hazard since bubbles would be smaller to start off. Not only that, by way of macroprudential policies preventing bubbles from getting too large, conventional monetary policy in the 'clean' stage of crises would have a better chance of doing its job (e.g., the zero lower bound would be less of an issue).

6. Concluding remarks

This paper develops a model to analyze the impact on the behavior of investors of a central bank policy whereby the interest rate is decreased to absorb losses resulting from a bubble burst. We show that such policy creates incentives for investors to magnify bubbles which in turn exacerbates the impact of bursts thus forcing the central bank to intervene. The intuition behind investors' behavior in face of central bank's policy is that, by buying the asset and fueling the bubble, capital gains in the states where the bubble does not burst will be large whereas in those states where there is a crash, intervention is more likely and investors' losses are absorbed.

By framing the game played between investors and the central bank similarly to the one between speculators and the government in the currency attack model of Morris and Shin (1998), we derive a unique equilibrium showing that investors' decision to buy the asset (which in our framework is the factor that fuels bubbles) follows a cut-off rule according to which buying occurs if investor's signal is higher than a specific threshold. Based on this unique equilibrium, a comparative statics analysis shows that (i) in economies intrinsically more fragile (bubbles more likely to burst), investors are more cautious to buy the asset causing smaller bubbles that are less likely to vindicate intervention in case they burst; (ii) investors discount the possibility of intervention if it is too costly for the central bank to decrease interest rates, thus leading to smaller bubbles too; (iii) investors are more cautious to buy illiquid assets (higher transactions costs) and, accordingly, smaller bubbles apply to this scenario as well (or, equivalently, a scenario where central bank policy reduces market or asset specific liquidity, e.g. increasing margin requirements).

The paper also addresses the question of whether intervention of the type postulated in our model (decrease in interest rates following bubble bursts) is optimal. In particular, by defining welfare as the sum of investors' profits from their investment strategies' plus the cost of central bank intervention (if it occurs at all), we show that intervention is welfare enhancing ex-post if the cost c imposed on the economy is not too high and, ex-ante, if the *expected* cost is low enough. The cost parameter should represent not only the direct effects of intervention but also those that are incurred to bring intervention into place (e.g. political costs). In those cases where the intervention cost is prohibitively high, it follows that the central bank is better-off letting bubbles burst (which should also be signaled to investors ex-ante if the economic environment is such that a cut in rates should not be expected).

As a venue for future research, we suggest further empirical investigation to shed light on the characteristics associated with different financial markets and economic environments that lead to the inflation of bubbles. Our model suggests that it is the combination of institutional arrangements (e.g., lending standards) and economic performance that drives the likelihood of bubbles forming (and bursting): this is, in essence, an empirical question. Not only that, the model presented proposes an exogenous mechanism triggering bursts that could be modified to incorporate the notion that, the more inflated a bubble is, the more likely it is to burst. This would allow us to account for a positive correlation between bubble size and busts evidenced in the financial cycles literature. The other dimensions in which the model could be modified is in terms of having financial intermediaries explicitly present and, therefore, allowing for a richer study of the impact of monetary and macroprudential policies (and their interplay) in the scenario of bubbles in asset prices. The notion that rational asset prices bubbles can be driven by the central bank being taken as a lender (or market maker) of last resort does not seem to be explored in the theoretical literature[16] - a dynamic model along these lines would be beneficial too. Finally, the model could be extended to a dynamic setup involving learning from investors about central bank's intervention policy (and associated feedback effects) towards a better understanding of how equilibrium is reached and the role played by forward guidance on that along the lines of Angeletos and Lian (2018).

Data availability

Data will be made available on request.

Appendix. Proof of Proposition 1

Proposition 1 is proved in three steps: first we show that investors' buying decision satisfies the property of strategic complementarity; second, that the function $u(x, \pi)$ (utility of an investor following strategy π and having received signal x) is continuous and monotonic in its first argument; finally, that the equilibrium strategy π (probability of buying the asset) is given by a step function with a unique x^* such that the asset is purchased if and only if the signal received is above a specific cut-off level, i.e., $x \geq x^*$. We start with the strategic complementarity property in the following lemma:

Lemma 4. *If $\pi(x) \geq \pi'(x)$ for any x (i.e., if the probability of buying the asset under strategy π is higher than the probability under any alternative strategy π'), then $u(x, \pi) \geq u(x, \pi')$ for any x.*

Proof. Since $\pi(x) \geq \pi'(x)$, one has $s(\theta, \pi) \geq s(\theta, \pi')$, for any θ, from the definition of s given by (2). Thus, from (3) it follows that:

$$A(\pi) \supseteq A(\pi') \Rightarrow A(\pi)^c \subseteq A(\pi')^c. \tag{23}$$

In other words, the event in which the central bank intervenes setting $r = r^*$ is more likely under π (recall the definition of $A(\cdot)$ from (3)). In conjunction with the assumptions related to the observed price ($g_\theta > 0$, $g_r < 0$, and $g_\alpha > 0$) and the fundamental value ($f_\theta > 0$ and $f_r < 0$), it holds that:

$$\begin{aligned} u(x, \pi) = \frac{1}{2\epsilon} \Bigg[& \int_{[L, x+\epsilon]} g(\theta, r_0, r_0, s(\theta, \pi))\, d\theta + \int_{[x-\epsilon, L] \cap A(\pi)^c} f(\theta, r_0)\, d\theta \\ & + \int_{[x-\epsilon, L] \cap A(\pi)} f(\theta, r^*)\, d\theta \Bigg] - p_0 - t_c \end{aligned}$$

[16] See the discussion in Barlevy (2018) for the traditional ways in which economists model bubbles.

$$\geq \frac{1}{2\varepsilon}\left[\int_{[L,x+\varepsilon]} g\left(\theta, r_0, r_0, s\left(\theta, \pi'\right)\right) d\theta + \int_{[x-\varepsilon,L]\cap A(\pi')^c} f\left(\theta, r_0\right) d\theta + \int_{[x-\varepsilon,L]\cap A(\pi')} f\left(\theta, r^*\right) d\theta\right] - p_0 - t_c = u\left(x, \pi'\right), \tag{24}$$

which proves the lemma. ■

We considers now the strategy profile where every investor buys the asset if and only if the signal x is greater than an arbitrary k. In this case, aggregate buying activity π will be given by the indicator function I_k defined as:

$$I_k := \begin{cases} 1 & \text{if } x \geq k \\ 0 & \text{if } x < k. \end{cases} \tag{25}$$

Under this rule, the expected payoff to buying the asset satisfies the following property:

Lemma 5. *$u\left(k, I_k\right)$ is continuous and strictly increasing in k.*

Proof. Consider the function $s\left(\theta, I_k\right)$, the fraction of investors who buy the asset at θ when the aggregate buying activity is given by the function I_k. Since by definition x is uniformly distributed over $[\theta-\varepsilon, \theta+\varepsilon]$, it follows that:

$$s\left(\theta, I_k\right) = \begin{cases} 0 & \text{if } k \geq \theta+\varepsilon \Leftrightarrow \theta \leq k-\varepsilon \\ \frac{1}{2} + \frac{(\theta-k)}{2\varepsilon} & \text{if } k-\varepsilon \leq \theta \leq k+\varepsilon \\ 1 & \text{if } k \leq \theta-\varepsilon \Leftrightarrow \theta \geq k+\varepsilon \end{cases} \tag{26}$$

where the second line is the result of the integral $\left[1/(2\varepsilon)\right]\int_k^{\theta+\varepsilon} d\theta$. Given I_k, there is a unique θ (which depends on k) such that the fraction of investors buying the asset equals the critical mass triggering central bank intervention during a bubble burst, i.e., under which[17] $s\left(\theta, I_k\right) = a(\theta)$. Let $\psi(k)$ be the (unique) value of ψ solving $s\left(k-\psi, I_k\right) = a(k-\psi)$; since the central bank lowers the interest rate if and only if θ lies in the interval $[k-\psi(k), L]$, investors' utility function $u\left(k, I_k\right)$ can be written as:

$$u\left(k, I_k\right) = \frac{1}{2\varepsilon}\left\{\int_L^{k+\varepsilon} g\left(\theta, r_0, r_0, s\left(\theta, I_k\right)\right) d\theta + \int_{k-\varepsilon}^{k-\psi(k)} f\left(\theta, r_0\right) d\theta + \int_{k-\psi(k)}^{L} f\left(\theta, r^*\right) d\theta\right\} - p_0 - t_c. \tag{27}$$

Taking the partial derivative of $u\left(k, I_k\right)$ with respect to k yields:

$$\begin{aligned} &\frac{d}{dk}\left\{\int_L^{k+\varepsilon} g\left(\theta, r_0, r_0, s\left(\theta, I_k\right)\right) d\theta + \int_{k-\varepsilon}^{k-\psi(k)} f\left(\theta, r_0\right) d\theta + \int_{k-\psi(k)}^{L} f\left(\theta, r^*\right) d\theta\right\} \\ &= g\left(k+\varepsilon, r_0, r_0, \underbrace{s\left(k+\varepsilon, I_k\right)}_{=1}\right) + \int_L^{k+\varepsilon} g_\alpha\left(\theta, r_0, r_0, s\left(\theta, I_k\right)\right)(-1/2\varepsilon)\, d\theta \\ &\quad + f\left(k-\psi(k), r_0\right)\left(1-\psi'(k)\right) - f\left(k-\varepsilon, r_0\right) - f\left(k-\psi(k), r^*\right)\left(1-\psi'(k)\right) \\ &= g\left(k+\varepsilon, r_0, r_0, 1\right) - f\left(k-\varepsilon, r_0\right) + \left[f\left(k-\psi(k), r_0\right) - f\left(k-\psi(k), r^*\right)\right]\left(1-\psi'(k)\right) \\ &\quad - \frac{1}{2\varepsilon}\int_L^{k+\varepsilon} g_\alpha\left(\theta, r_0, r_0, s\left(\theta, I_k\right)\right) d\theta \end{aligned} \tag{28}$$

where we used the fact that $s\left(\theta, I_k\right) = 1/2 + (\theta-k)/2\varepsilon$ for $k-\varepsilon \leq \theta \leq k+\varepsilon$, in turn implying that $1/2 - \psi(k)/2\varepsilon = a(k-\psi(k))$ whenever $\theta = k-\psi(k)$. Hence, $\psi(k)$ is negligible for ε sufficiently small which, combined with $1-\psi'(k) \geq 0$, leads to:

$$\underbrace{g\left(k+\varepsilon, r_0, r_0, 1\right) - f\left(k, r^*\right)}_{>0} + \underbrace{f\left(k, r_0\right) - f\left(k-\varepsilon, r_0\right)}_{>0} > 0. \tag{29}$$

Plugging this result into (28) allows us to conclude that $u\left(k, I_k\right)$ is monotonically increasing in k. Finally, $u\left(k, I_k\right)$ is continuous since it is an integral in which the limits of integration are themselves continuous in k, concluding then the proof of the lemma. ■

The intuition behind Lemma 5 is as follows. Recall that, if the buying strategy π of investors is given by the indicator function I_k, then $\pi(x) = 1$ whenever $x \geq k$ and zero otherwise. Hence, investors will buy the asset only when their signal indicates that the state of fundamentals is sufficiently high as otherwise they risk being caught in a bubble burst episode where the central bank

[17] Recall that, in a bubble burst episode, for a given θ, $a(\theta)$ is the value of α such that $b(\theta, \alpha) = c$; the assumption that $b_\theta > 0$ implies that $a' < 0$ and hence there is a unique θ at which $s\left(\theta, I_k\right) = a(\theta)$.

does not intervene (which happens when the state of fundamentals is too low or, else, if the bubble is not large enough). In this way, being more cautious in the investment decision (adopting a higher k) means that investors buy the asset only at states where bubbles are less likely to burst and, if they do, the central bank is more likely to step in and lower the interest rate.

The last result necessary to prove Proposition 1 is the following:

Lemma 6. *There is a unique x^* such that, in any equilibrium of the game with imperfect information about the state of fundamentals, an investor with signal x buys the asset if and only if $x > x^*$.*

Proof. First we show that there is a unique k such that:

$$u\left(k, I_k\right) = 0. \tag{30}$$

Under the investment strategy I_k, define the marginal investor as the one receiving signal $x = k$. When k is sufficiently large, i.e., $k \geq L+\varepsilon$, the marginal investor knows that the bubble will not burst (as that would happen only if $\theta \in [L, 1]$ which is inconsistent with the signal received). Since the payoff to buying the asset is always positive when the bubble does not burst, it follows that $u\left(k, I_k\right) > 0$. Conversely, when k is sufficiently small, i.e., $k \leq \underline{\theta} - \varepsilon$, the marginal investor is aware that θ lies in the interval $\left[0, \underline{\theta}\right]$ in which the central bank does not intervene and, accordingly, the expected utility from buying the asset is negative, $u\left(k, I_k\right) < 0$. Therefore, as a corollary of Lemma 5, there exists a unique k such that $u\left(k, I_k\right) = 0$ which we denote by x^*.

Consider now an arbitrary equilibrium of the game and define $\underline{x}$ and $\overline{x}$ by:

$$\underline{x} := \inf \{ x | \pi(x) > 0\} \tag{31}$$

$$\overline{x} := \sup \{ x | \pi(x) < 1\}. \tag{32}$$

Based on these definitions, it follows that:

$$\overline{x} \geq \sup \{ x | 0 < \pi(x) < 1\} \geq \inf \{ x | 0 < \pi(x) < 1\} \geq \underline{x} \tag{33}$$

where the first inequality due to $\{ x | 0 < \pi(x) < 1\} \subseteq \{ x | \pi(x) < 1\}$ and the last as a result of $\{ x | 0 < \pi(x) < 1\} \subseteq \{ x | \pi(x) > 0\}$. Therefore:

$$\underline{x} \leq \overline{x}. \tag{34}$$

$\pi(x) < 1$ implies that some investors do not buy the asset which is consistent with an equilibrium behavior only if the payoff from not buying is at least as high as the payoff from buying conditional on signal x. By continuity, this is also true at $\overline{x}$, i.e.:

$$u\left(\overline{x}, \pi\right) \leq 0. \tag{35}$$

Since[18] $I_{\overline{x}} \leq \pi$, it follows from Lemma 4 (strategic complementarity) and (35) that $u\left(\overline{x}, I_{\overline{x}}\right) \leq u\left(\overline{x}, \pi\right) \leq 0$, thus $u\left(\overline{x}, I_{\overline{x}}\right) \leq 0$. Knowing from Lemma 5 that $u\left(k, I_k\right)$ is increasing in k and x^* is the unique value of k such that $u\left(k, I_k\right) = 0$ leads to:

$$x^* \geq \overline{x}. \tag{36}$$

Conversely, $\pi(x) > 0$ implies that some investors do buy the asset which is consistent with an equilibrium behavior only if the payoff from buying is at least as high as the payoff from not buying conditional on signal x. By continuity, this should also be true at $\underline{x}$, i.e.:

$$u\left(\underline{x}, \pi\right) \geq 0. \tag{37}$$

With $I_{\underline{x}} \geq \pi$, Lemma 4 and (37) imply that $u\left(\underline{x}, I_{\underline{x}}\right) \geq u\left(\underline{x}, \pi\right) \geq 0$, thus $u\left(\underline{x}, I_{\underline{x}}\right) \geq 0$. Since from Lemma 5 $u\left(k, I_k\right)$ is increasing in k and x^* is the unique value of k such that $u\left(k, I_k\right) = 0$, it holds that:

$$\underline{x} \geq x^*. \tag{38}$$

Thus, from (36) and (38), $\underline{x} \geq x^* \geq \overline{x}$. However, in conjunction with (34) this implies that:

$$\underline{x} = x^* = \overline{x}. \tag{39}$$

Thus, the equilibrium π is given by the step function, I_{x^*}, completing the proof. ■

We are now in the position to prove Proposition 1. Given that the equilibrium buying strategy π is given by the step function I_{x^*}, aggregate buying activity at an arbitrary state θ is given by:

$$s\left(\theta, I_{x^*}\right) = \begin{cases} 0 & \text{if } \theta \leq x^* - \varepsilon \\ \frac{1}{2} + \frac{(\theta - x^*)}{2\varepsilon} & \text{if } x^* - \varepsilon \leq \theta \leq x^* + \varepsilon \\ 1 & \text{if } \theta \geq x^* + \varepsilon. \end{cases} \tag{40}$$

[18] As the indicator function is just a particular case of π and by the definition of $\overline{x}$.

For $\theta \in [0,1]$, therefore, (40) implies that $s\left(\theta, I_{x^*}\right)$ is increasing in θ whereas $a(\theta)$ is decreasing in it (see footnote 17). Moreover, it must be true that $x^* > \underline{\theta} - \varepsilon$ since otherwise not buying the asset would be a strictly better action contradicting the fact that x^* is the switching point associated with the equilibrium strategy as shown in Lemma 6. Thus, $s\left(\theta, I_{x^*}\right)$ and $a(\theta)$ must cross once (and only once)and define by θ^* the value of θ at which those two curves cross. Then, $s\left(\theta, I_{x^*}\right) \geq a(\theta)$ if and only if $\theta \geq \theta^*$, so that the central bank lowers the interest rate if and only if $\theta \geq \theta^*$. This concludes the proof of Proposition 1.

Proposition 7. *In the limit as ε tends to zero, θ^* is implicitly given by:*

$$\phi_0'\left(-\theta^*\right)\left[f\left(\phi_0\left(-\theta^*\right), r\right) - f\left(\phi_0\left(-\theta^*\right), r^*\right)\right] + f\left(\theta^*, r\right) + f\left(\theta^*, r^*\right) = 2\left(p_0 + t_c\right) \tag{41}$$

Proof. Let x^* be the solution to $u\left(x^*, I_{x^*}\right) = 0$ where I_x^* denotes the step function characterizing investors' equilibrium strategy (see Lemma 6) and with:

$$\begin{aligned} u\left(x^*, I_{x^*}\right) &= \frac{1}{2\varepsilon}\left\{\int_{[L,x^*+\varepsilon]} g\left(\theta, r_0, r_0, s\left(\theta, I_{x^*}\right)\right) d\theta + \int_{[x^*-\varepsilon,L]\cap A\left(I_{x^*}\right)^c} f\left(\theta, r_0\right) d\theta \right.\\ &\quad \left. + \int_{[x^*-\varepsilon,L]\cap A\left(I_{x^*}\right)} f\left(\theta, r^*\right) d\theta\right\} - p_0 - t_c \end{aligned} \tag{42}$$

From the definition of $A(\cdot)$ in (3) and using expression (40) we have:

$$\begin{aligned} A\left(I_{x^*}\right) &= \left\{\theta \mid s\left(\theta, I_{x^*}\right) \geq a(\theta)\right\} \\ &= \left\{\theta \mid \frac{1}{2} + \frac{\theta - x^*}{2\varepsilon} \geq a(\theta)\right\}. \end{aligned} \tag{43}$$

From:

$$\begin{aligned} &\frac{1}{2} + \frac{\theta - x^*}{2\varepsilon} \geq a(\theta) \\ \Leftrightarrow\ &\varepsilon + \theta - x^* \geq 2\varepsilon a(\theta) \\ \Leftrightarrow\ &\varepsilon - x^* \geq -\theta + 2\varepsilon a(\theta) \end{aligned} \tag{44}$$

and by defining $\Phi(\theta, \varepsilon) := -\theta + 2\varepsilon a(\theta)$, it follows that:

$$\Phi_\theta(\theta, \varepsilon) = -1 + 2\varepsilon \overbrace{a'(\theta)}^{<0} < 0. \tag{45}$$

Therefore, for a fixed ε, there exists an inverse function of $\Phi(\cdot, \varepsilon)$ which we denote by ϕ_ε. With:

$$\begin{aligned} &\varepsilon - x^* \geq \Phi(\theta, \varepsilon) \\ \Leftrightarrow\ &\theta \geq \phi_\varepsilon\left(\varepsilon - x^*\right), \end{aligned} \tag{46}$$

the set $A\left(I_{x^*}\right)$ in (43) can be rewritten as:

$$A\left(I_{x^*}\right) = \left\{\theta \mid \theta \geq \phi_\varepsilon\left(\varepsilon - x^*\right)\right\} \tag{47}$$

allowing us to rewrite the domains of integration in (42) as:

$$\left[x^* - \varepsilon, L\right] \cap A\left(I_{x^*}\right)^c = \left\{\theta \mid x^* - \varepsilon < \theta < \phi_\varepsilon\left(\varepsilon - x^*\right)\right\} \tag{48}$$

$$\left[x^* - \varepsilon, L\right] \cap A\left(I_{x^*}\right) = \left\{\theta \mid \phi_\varepsilon\left(\varepsilon - x^*\right) < \theta < L\right\}. \tag{49}$$

In conjunction with the fact that for ε sufficiently small one has $x^* + \varepsilon < L$, this allows (42) to equivalently be expressed as:

$$u\left(x^*, I_{x^*}\right) = \frac{1}{2\varepsilon}\left\{\int_{\left\{\theta \mid x^*-\varepsilon<\theta<\phi_\varepsilon(\varepsilon-x^*)\right\}} f\left(\theta, r_0\right) d\theta + \int_{\left\{\theta \mid \phi_\varepsilon(\varepsilon-x^*)<\theta<L\right\}} f\left(\theta, r^*\right) d\theta\right\} - p_0 - t_c. \tag{50}$$

Defining the function F by:

$$F(\varepsilon) := \int_{\left\{\theta \mid x^*-\varepsilon<\theta<\phi_\varepsilon(\varepsilon-x^*)\right\}} f(\theta, r)\, d\theta + \int_{\left\{\theta \mid \phi_\varepsilon(\varepsilon-x^*)<\theta<L\right\}} f\left(\theta, r^*\right) d\theta \tag{51}$$

allows us to state the condition $u\left(x^*, I_{x^*}\right) = 0$ as:

$$\frac{F(\varepsilon)}{2\varepsilon} - p_0 - t_c = 0. \tag{52}$$

Since[19] $F(\varepsilon) \to 0$ as $\varepsilon \to 0$, using L'Hospital rule yields:

$$\begin{aligned}\lim_{\varepsilon\to 0}\frac{F(\varepsilon)}{2\varepsilon} &= \frac{F'(0)}{2}\\ &= \frac{f\left(\phi_0(-x^*),r_0\right)\phi_0'(-x^*)+f\left(x^*,r_0\right)+f\left(x^*,r^*\right)-f\left(\phi_0(-x^*),r^*\right)\phi_0'(-x^*)}{2}\\ &= \frac{\phi_0'(-x^*)\left[f\left(\phi_0(-x^*),r_0\right)-f\left(\phi_0(-x^*),r^*\right)\right]}{2}+\frac{f\left(x^*,r_0\right)+f\left(x^*,r^*\right)}{2},\end{aligned} \tag{53}$$

where $\phi_0(\cdot) = \phi_\varepsilon(\cdot)\,|_{\varepsilon=0}$. Thus, in the limit as $\varepsilon \to 0$, expression (52) is given by:

$$\phi_0'\left(-x^*\right)\left[f\left(\phi_0\left(-x^*\right),r_0\right)-f\left(\phi_0\left(-x^*\right),r^*\right)\right]+f\left(x^*,r_0\right)+f\left(x^*,r^*\right)=2\left(p_0+t_c\right) \tag{54}$$

which completes the proof.[20] ■

References

Adrian, T., & Liang, N. (2016). *Monetary policy, financial conditions, and financial stability*: *Federal reserve bank of New York staff report 690*.
Adrian, T., & Shin, H. (2014). Procyclical leverage and value-at-risk. *The Review of Financial Studies*, *27*, 373–403.
Allen, F., Barlevy, G., & Gale, D. (2018). *WP 2017-16*, *A theoretical model of leaning against the wind*: *FRB of Chicago working paper*.
Allen, F., & Gale, D. (1998). Optimal financial crises. *The Journal of Finance*, *53*, 1245–1284.
Allen, F., & Gale, D. (1999). Bubbles, crises, and policy. *Oxford Review of Economic Policy*, *15*, 9–18.
Allen, F., & Gale, D. (2000). Bubbles and crises. *The Economic Journal*, *110*, 236–255.
Angeletos, G., & Lian, C. (2018). Forward guidance without common knowledge. *American Economic Review*, *108*, 2477–2512.
Baker, D. (2013). *Bubble bursting and central bank forward guidance*. Project Syndicate Blog Post.
Bao, T., & Zong, J. (2019). The impact of interest rate policy on individual expectations and asset bubbles in experimental markets. *Journal of Economic Dynamics & Control*, *107*, 1–20.
Barlevy, G. (2018). Bridging between policymakers' and economists' views on bubbles. *Economic Perspectives*, *42*.
Ben Naceur, S., Candelon, B., & Lajaunie, Q. (2019). *WP/19/94*, *Taming financial development to reduce crises*: *IMF working paper*.
Bernanke, B. (2015). *Should monetary policy take into account risks to financial stability?* Brookings Institution Blog Post.
Bernanke, B. (2020). The new tools of monetary policy. *American Economic Association Presidential Address*, 1–36.
Bernanke, B., & Gertler, M. (2000). *Working Paper 7559*, *Monetary policy and asset price volatility*: *NBER working paper series*.
Bernanke, B., & Gertler, M. (2001). Should central banks respond to movements in asset prices? *American Economic Review Papers & Proceedings*, *91*, 253–257.
Biljanovska, N., Górnicka, L., & Vardoulakis, A. (2021). *WP/19/184*, *Optimal macroprudential policy and asset price bubbles*: *IMF working paper*.
Blinder, A., & Reis, R. (2005). Understanding the greenspan standard. In *Proceedings of the economic policy symposium, Jackson Hole* (pp. 11–96).
Book, J. (2021). *To lean, clean, or reign supreme*. American Institute for Economic Research Blog Post.
Bordo, M., & Jeanne, O. (2002). *Working Paper 8966*, *Boom-busts in asset prices, economic instability, and monetary policy*: *NBER working paper series*.
Borio, C. (2012). *The financial cycle and macroeconomics: What have we learnt? BIS working paper no. 395*.
Borio, C., Drehmann, M., & Xia, D. (2018). The financial cycle and recession risk. *BIS Quarterly Review*, 59–71.
Borio, C., & Lowe, P. (2005). *Asset prices, financial and monetary stability: Exploring the nexus*: *BIS working paper no. 114*.
Boyarchenko, N., Favara, G., & Schularik, M. (2022). *2022–006*, *Financial stability considerations for monetary policy: Empirical challenges and evidence*: *FRB finance and economics discussion series*.
Brock, W., Lakonishok, J., & LeBaron, B. (1992). Simple technical trading rules and the stochastic properties of stock returns. *The Journal of Finance*, *47*, 1731–1764.
Brunnermeier, M. (2009). Deciphering the liquidity and credit crunch 2007-08. *Journal of Economic Perspectives*, *23*, 77–100.
Buiter, W., Cecchetti, S., Dominguez, K., & Sanchez Serrano, A. (2023). *Stabilising financial markets: Lending and market making as a last resort*: *ESRB reports of the advisory scientific committee no. 13*.
Cantú, C., Cavallino, P., De Fiore, F., & Yetman, J. (2021). *A global database on central banks' monetary responses to Covid-19*: *BIS working paper no. 934*.
Cao, J., & Chollete, L. (2017). Monetary policy and financial stability in the long-run: a simple game-theoretic approach. *Journal of Financial Stability*, *28*, 125–142.
Carlsson, H., & van Damme, E. (1993). Global games and equilibrium selection. *Econometrica*, *61*, 989–1018.
Carstens, A. (2019). The new role of central banks. In *Speech at financial stability institute's 20th anniversary conference*.
Cecchetti, S. (2004). How should monetary policy respond to asset price bubbles? *VOXEU Column*.
Cecchetti, S., Genberg, H., Lipsky, J., & Wadhwani, S. (2000). Asset prices and central bank policy. *Geneva Reports on the World Economy*, *2*.
Cieslak, A., & Vissing-Jorgensen, A. (2021). The economics of the fed put. *The Review of Financial Studies*, *34*, 4045–4089.
Claessens, S., Kose, M., & Terrones, M. (2011). *WP/11/76*, *Financial cycles: What? how? when? IMF working paper*.
Dell'Ariccia, G., Habermeier, K., Haksar, V., & Mancini-Griffoli, T. (2015). Monetary policy and financial stability. *IMF Staff Report*.
Dell'Ariccia, G., Igan, D., Laeven, L., & Tong, H. (2016). Credit booms and macrofinancial stability. *Economic Policy*, *31*, 299–355.
Dell'Ariccia, G., Laeven, L., & Suarez, G. (2017). Bank leverage and monetary policy's risk-taking channel: Evidence from the United States. *The Journal of Finance*, *72*, 613–654.
Dell'Ariccia, G., & Marquez, R. (2006). Lending booms and lending standards. *The Journal of Finance*, *61*, 2511–2546.
Diamond, D., & Dybvig, P. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, *91*, 401–419.
Drechsler, I., Savov, A., & Schnabl, P. (2018). A model of monetary policy and risk premia. *The Journal of Finance*, *73*, 317–373.
Evans, C. (2014). Thoughts on accommodative monetary policy, inflation and financial instability. In *Speech at the credit suisse Asian investment conference in Hong Kong*.
Filardo, A., Hubert, P., & Rungcharoenkitkul, P. (2022). Monetary policy reaction function and the financial cycle. *Journal of Banking & Finance*, *142*, 1–13.
Filardo, A., & Rungcharoenkitkul, P. (2016). *A quantitative case for leaning against the wind*: *BIS working paper no. 594*.

[19] Notice that the sets $\left\{\theta|\,x^*-\varepsilon<\theta<\phi_\varepsilon(\varepsilon-x^*)\right\}$ and $\left\{\theta|\,\phi_\varepsilon(\varepsilon-x^*)<\theta<L\right\}$ can be equivalently written as $\{\theta|\,x^*-\varepsilon<\theta<2\varepsilon(a(\theta)-1/2)+x^*\}$ and $\{\theta|\,2\varepsilon(a(\theta)-1/2)+x^*<\theta<x^*+\varepsilon\}$, respectively (the last using the fact that, for small ε, $x^*+\varepsilon<L$). It then follows that both collapse to the empty set as $\varepsilon\to 0$ and, therefore, $F(\varepsilon)\to 0$.

[20] Notice also that x^* converges to θ^* with $\varepsilon\to 0$ as $s\left(\theta, I_{x^*}\right)=I_{x^*}$.

Fischbacher, U., Hens, T., & Zeisberger, S. (2013). The impact of monetary policy on stock market bubbles and trading behavior: Evidence from the lab. *Journal of Economic Dynamics & Control*, *130*, 2104–2122.
Galí, J. (2014). Monetary policy and rational asset price bubbles. *American Economic Review*, *104*, 721–752.
Galí, J., Giusti, G., & Nousssair, C. (2021). Monetary policy and asset price bubbles: a laboratory experiment. *Journal of Economic Dynamics & Control*, *130*, 1–15.
Geanakoplos, J. (1997). Promises, promises. In S. Arthur, S. Durlauf, & D. Lane (Eds.), *The economy as an evolving complex system, II*: *vol. 2*, (pp. 285–320). Addison-Wesley.
Geanakoplos, J. (2003). Liquidity, default and crashes: Endogenous contracts in general equilibrium. In M. Dewatripont, P. Hansen, & S. Turnovsky (Eds.), *Vol. 2*, *Advances in economics and econometrics: theory and applications, eighth world congress* (pp. 170–205). Cambridge University Press.
Gilchrist, S., & Leahy, J. (2002). Monetary policy and asset prices. *Journal of Monetary Economics*, *49*, 75–97.
Gorton, G., & Ordoñez, G. (2020). Good booms, bad booms. *Journal of the European Economic Association*, *18*, 618–665.
Gourinchas, P., Valdes, R., & Landerretche, O. (2001). *Lending booms: Latin America and the world*: *NBER working paper series*, *Working Paper 8249*.
Gourio, F., Kashyap, A., & Sim, J. (2017). *Working Paper 23658*, *The tradeoffs in leaning against the wind*: *NBER working paper series*.
Greenlaw, D., Hatzius, J., Kashyap, A., & Shin, H. (2008). *Leveraged losses: Lessons from the mortgage market meltdown*: *US monetary policy forum report 2*.
Greenwood, R., Hanson, S., Shleifer, A., & Sørensen, J. (2022). Predictable financial crises. *The Journal of Finance*, *77*, 863–921.
Grimm, M., Jordà, O., Schularick, M., & Taylor, A. (2023). *Working Paper 30958*, *Loose monetary policy and financial instability*: *NBER working paper series*.
Hoffmann, A. (2013). Did the fed and ECB react asymmetrically with respect to asset market developments? *Journal of Policy Modeling*, *35*, 197–211.
Ikeda, D. (2022). Monetary policy, inflation, and rational asset price bubbles. *Journal of Money, Credit and Banking*, *54*, 1569–1603.
Jones, E., & Altunbas, Y. (2022). Stock markets have been a one-way bet for many years thanks to the 'fed put' - but those days are over. *The Conversation*.
Jordà, O., Schularik, M., & Taylor, A. (2013). When credit bites back. *Journal of Money, Credit and Banking*, *45*, 3–28.
Jordà, O., Schularik, M., & Taylor, A. (2015). Leveraged bubbles. *Journal of Monetary Economics*, *76*, S1–S20.
Klein, M. (2016). Don't clean - lean! *Financial Times*.
Kocherlakota, N. (2014). Low real interest rates. In *Speech at the ninth annual finance conference*. Carroll School of Management, Boston College.
Kocherlakota, N. (2016). Rules vs discretion: a reconsideration. *Brookings Papers on Economic Activity*, 1–36.
Kontonikas, A., MacDonald, R., & Saggu, A. (2013). Stock market reaction to fed funds rate surprises: State dependence and the financial crisis. *Journal of Banking & Finance*, *37*, 4025–4037.
Kurov, A., Olson, E., & Zaynutdinova, G. (2022). When does the fed care about stock prices. *Journal of Banking & Finance*, *142*, 1–16.
Loayza, N., Ouazad, A., & Rancière, R. (2018). *Working Paper 24474*, *Financial development, growth, and crisis: Is there a trade-off? NBER working paper series*.
Loayza, N., & Rancière, R. (2005). *WP/05/170*, *Financial development, financial fragility, and growth*: *IMF working paper*.
Lopomo Beteto Wegner, D. (2020). Liquidity policies and financial fragility. *International Review of Economics & Finance*, *70*, 135–153.
Mendoza, E. (2010). Sudden stops, financial crises and leverage. *American Economic Review*, *100*, 1941–1966.
Menon, R. (2021). *Monetary policy at the crossroads*. Chicago Booth Review Blog Post.
Miller, M., Weller, P., & Zhang, L. (2002). Moral hazard and the US stock market: Analysing the 'greenspan put'. *The Economic Journal*, *112*, 171–186.
Minsky, H. (1986). *Stabilizing an unstable economy*. New Haven, CT: Yale University Press.
Mishkin, F. (2011). How should central banks respond to asset-price bubbles? The 'lean' versus 'clean' debate after the GFC. *Reserve Bank of Australia Bulletin*, *June*, 59–69.
Mishkin, F., & White, E. (2003a). Stock market bubbles: When does intervention work? *Milken Institute Review*, *Second Quarter*, 44–52.
Mishkin, F., & White, E. (2003b). U.S. stock market crashes and their aftermath: Implications for monetary policy. In W. Hunter, G. Kaufman, & M. Pormerleano (Eds.), *Asset price bubbles: the implications for monetary, regulatory and international policies* (pp. 53–79). MIT Press.
Morris, S., & Shin, H. (1998). Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review*, *88*, 587–597.
Morris, S., & Shin, H. (2000). Rethinking multiple equilibria in macroeconomic modeling. *NBER Macroeconomics Annual*, *15*, 139–161.
Morris, S., & Shin, H. (2003). Global games: Theory and applications. In M. Dewatripont, P. Hansen, & S. Turnovsky (Eds.), *Vol. 1*, *Advances in economics and econometrics: theory and applications, eighth world congress* (pp. 56–114). Cambridge, UK: Cambridge University Press.
Putniņš, T. (2022). Free markets to fed markets: How modern monetary policy impacts equity markets. *Financial Analysts Journal*, *78*.
Rajan, R. (2005). *Working Paper 11728*, *Has financial development made the world riskier?*: *NBER working paper series*.
Ravn, S. (2012). Has the fed reacted asymmetrically to stock prices? *The B.E. Journal of Macroeconomics*, *12*, 1–34.
Restoy, F. (2020). *Central banks and financial stability: A reflection after the Covid-19 outbreak*: *Financial instability institute occasional paper no. 16*.
Rigobon, R., & Sack, B. (2003). Measuring the reaction of monetary policy to the stock market. *Quarterly Journal of Economics*, *118*, 639–669.
Schularik, M., & Taylor, A. (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, *102*, 1029–1061.
Shiller, R. (2005). *Irrational exuberance* (2nd ed.). New York: Doubleday.
Smets, F. (2014). Financial stability and monetary policy: How closely interlinked? *International Journal of Central Banking*, *35*, 263–300.
Stein, J. (2014). *Incorporating financial stability considerations into a monetary policy framework*. Washington D.C.: Speech at the International Research Forum on Monetary Policy.
Svensson, L. (2017). Cost-benefit analysis of leaning against the wind. *Journal of Monetary Economics*, *90*, 193–213.
Svensson, L. (2019). The relation between monetary policy and financial-stability policy. In A. Aguirre, M. Brunnermeier, & D. Saravia (Eds.), *Monetary policy and financial stability: transmission mechanisms and policy implications* (pp. 283–310). Central Bank of Chile.
The Economist (2013). Jeremy stein leans against the wind. February 7th.
Thorbecke, W. (1997). On stock market returns and monetary policy. *The Journal of Finance*, *52*.
Vickers, J. (2000). Monetary policy and asset prices. *The Manchester School*, *68*, 1–22.
White, W. (2009). Should monetary policy lean or clean: a reassessment. *Central Banking*, *19*, 32–42.