

# Quantitative Business Valuation

*A Mathematical Approach for  
Today's Professionals*

**Second Edition**

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