

OPTIONS, FUTURES, AND OTHER DERIVATIVES

John C. Hull

*Maple Financial Group Professor of Derivatives and Risk Management
Joseph L. Rotman School of Management
University of Toronto*

ELEVENTH EDITION
GLOBAL EDITION



Pearson

Harlow, England • London • New York • Boston • San Francisco • Toronto • Sydney • Dubai • Singapore • Hong Kong
Tokyo • Seoul • Taipei • New Delhi • Cape Town • São Paulo • Mexico City • Madrid • Amsterdam • Munich • Paris • Milan

CONTENTS IN BRIEF

| | |
|---|-----|
| List of business snapshots | 15 |
| List of technical notes | 16 |
| Preface | 17 |
| 1. Introduction | 23 |
| 2. Futures markets and central counterparties | 46 |
| 3. Hedging strategies using futures | 70 |
| 4. Interest rates | 98 |
| 5. Determination of forward and futures prices | 124 |
| 6. Interest rate futures | 152 |
| 7. Swaps | 172 |
| 8. Securitization and the financial crisis of 2007–8 | 201 |
| 9. XVAs | 216 |
| 10. Mechanics of options markets | 227 |
| 11. Properties of stock options | 247 |
| 12. Trading strategies involving options | 268 |
| 13. Binomial trees | 288 |
| 14. Wiener processes and Itô's lemma | 316 |
| 15. The Black–Scholes–Merton model | 338 |
| 16. Employee stock options | 371 |
| 17. Options on stock indices and currencies | 384 |
| 18. Futures options and Black's model | 401 |
| 19. The Greek letters | 417 |
| 20. Volatility smiles and volatility surfaces | 451 |
| 21. Basic numerical procedures | 470 |
| 22. Value at risk and expected shortfall | 514 |
| 23. Estimating volatilities and correlations | 542 |
| 24. Credit risk | 562 |
| 25. Credit derivatives | 587 |
| 26. Exotic options | 614 |
| 27. More on models and numerical procedures | 640 |
| 28. Martingales and measures | 670 |
| 29. Interest rate derivatives: The standard market models | 688 |
| 30. Convexity, timing, and quanto adjustments | 707 |
| 31. Equilibrium models of the short rate | 719 |
| 32. No-arbitrage models of the short rate | 732 |
| 33. Modeling forward rates | 755 |
| 34. Swaps revisited | 773 |
| 35. Energy and commodity derivatives | 785 |
| 36. Real options | 802 |
| 37. Derivatives mishaps and what we can learn from them | 815 |
| Glossary of terms | 827 |
| DerivaGem software | 851 |
| Exchanges trading futures and options | 856 |
| Table for $N(x)$ When $x \leq 0$ | 857 |
| Author index | 859 |
| Subject index | 863 |

CONTENTS

| | |
|--|-----------|
| List of business snapshots | 15 |
| List of technical notes | 16 |
| Preface | 17 |
| Chapter 1. Introduction | 23 |
| 1.1 Exchange-traded markets | 24 |
| 1.2 Over-the-counter markets | 25 |
| 1.3 Forward contracts | 28 |
| 1.4 Futures contracts | 30 |
| 1.5 Options | 31 |
| 1.6 Types of traders | 33 |
| 1.7 Hedgers | 34 |
| 1.8 Speculators | 36 |
| 1.9 Arbitrageurs | 39 |
| 1.10 Dangers | 39 |
| Summary | 41 |
| Further reading | 41 |
| Practice questions | 42 |
| Chapter 2. Futures markets and central counterparties | 46 |
| 2.1 Background | 46 |
| 2.2 Specification of a futures contract | 48 |
| 2.3 Convergence of futures price to spot price | 50 |
| 2.4 The operation of margin accounts | 51 |
| 2.5 OTC markets | 54 |
| 2.6 Market quotes | 57 |
| 2.7 Delivery | 60 |
| 2.8 Types of traders and types of orders | 61 |
| 2.9 Regulation | 62 |
| 2.10 Accounting and tax | 63 |
| 2.11 Forward vs. futures contracts | 64 |
| Summary | 65 |
| Further reading | 66 |
| Practice questions | 67 |
| Chapter 3. Hedging strategies using futures | 70 |
| 3.1 Basic principles | 70 |
| 3.2 Arguments for and against hedging | 72 |
| 3.3 Basis risk | 75 |
| 3.4 Cross hedging | 79 |
| 3.5 Stock index futures | 84 |
| 3.6 Stack and roll | 89 |
| Summary | 90 |

| | | |
|-------------------|--|------------|
| | Further reading | 92 |
| | Practice questions | 93 |
| | Appendix: Capital asset pricing model | 96 |
| Chapter 4. | Interest rates | 98 |
| 4.1 | Types of rates | 98 |
| 4.2 | Reference rates | 99 |
| 4.3 | The risk-free rate | 101 |
| 4.4 | Measuring interest rates | 101 |
| 4.5 | Zero rates | 104 |
| 4.6 | Bond pricing | 105 |
| 4.7 | Determining zero rates | 106 |
| 4.8 | Forward rates | 109 |
| 4.9 | Forward rate agreements | 110 |
| 4.10 | Duration | 112 |
| 4.11 | Convexity | 116 |
| 4.12 | Theories of the term structure of interest rates | 117 |
| | Summary | 119 |
| | Further reading | 120 |
| | Practice questions | 121 |
| Chapter 5. | Determination of forward and futures prices | 124 |
| 5.1 | Investment assets vs. consumption assets | 124 |
| 5.2 | Short selling | 125 |
| 5.3 | Assumptions and notation | 126 |
| 5.4 | Forward price for an investment asset | 127 |
| 5.5 | Known income | 130 |
| 5.6 | Known yield | 132 |
| 5.7 | Valuing forward contracts | 133 |
| 5.8 | Are forward prices and futures prices equal? | 135 |
| 5.9 | Futures prices of stock indices | 135 |
| 5.10 | Forward and futures contracts on currencies | 137 |
| 5.11 | Futures on commodities | 141 |
| 5.12 | The cost of carry | 143 |
| 5.13 | Delivery options | 144 |
| 5.14 | Futures prices and expected future spot prices | 144 |
| | Summary | 147 |
| | Further reading | 148 |
| | Practice questions | 149 |
| Chapter 6. | Interest rate futures | 152 |
| 6.1 | Day count and quotation conventions | 152 |
| 6.2 | Treasury bond futures | 155 |
| 6.3 | Eurodollar and SOFR futures | 160 |
| 6.4 | Duration-based hedging strategies using futures | 165 |
| 6.5 | Hedging portfolios of assets and liabilities | 167 |
| | Summary | 168 |
| | Further reading | 168 |
| | Practice questions | 169 |
| Chapter 7. | Swaps | 172 |
| 7.1 | Mechanics of interest rate swaps | 172 |
| 7.2 | Determining risk-free rates | 175 |

| | | |
|--------------------|--|------------|
| 7.3 | Reasons for trading interest rate swaps | 176 |
| 7.4 | The organization of trading | 178 |
| 7.5 | The comparative-advantage argument | 181 |
| 7.6 | Valuation of interest rate swaps | 183 |
| 7.7 | How the value changes through time | 185 |
| 7.8 | Fixed-for-fixed currency swaps | 186 |
| 7.9 | Valuation of fixed-for-fixed currency swaps | 190 |
| 7.10 | Other currency swaps | 192 |
| 7.11 | Credit risk | 193 |
| 7.12 | Credit default swaps | 193 |
| 7.13 | Other types of swaps | 194 |
| | Summary | 196 |
| | Further reading | 196 |
| | Practice questions | 197 |
| Chapter 8. | Securitization and the financial crisis of 2007–8 | 201 |
| 8.1 | Securitization | 201 |
| 8.2 | The U.S. housing market | 205 |
| 8.3 | What went wrong? | 209 |
| 8.4 | The aftermath | 211 |
| | Summary | 213 |
| | Further reading | 213 |
| | Practice questions | 215 |
| Chapter 9. | XVAs | 216 |
| 9.1 | CVA and DVA | 216 |
| 9.2 | FVA and MVA | 219 |
| 9.3 | KVA | 222 |
| 9.4 | Calculation issues | 223 |
| | Summary | 224 |
| | Further reading | 225 |
| | Practice questions | 226 |
| Chapter 10. | Mechanics of options markets | 227 |
| 10.1 | Types of options | 227 |
| 10.2 | Option positions | 229 |
| 10.3 | Underlying assets | 231 |
| 10.4 | Specification of stock options | 233 |
| 10.5 | Trading | 236 |
| 10.6 | Trading costs | 237 |
| 10.7 | Margin requirements | 237 |
| 10.8 | The options clearing corporation | 239 |
| 10.9 | Regulation | 239 |
| 10.10 | Taxation | 240 |
| 10.11 | Warrants, employee stock options, and convertibles | 241 |
| 10.12 | Over-the-counter options markets | 242 |
| | Summary | 243 |
| | Further reading | 243 |
| | Practice questions | 244 |
| Chapter 11. | Properties of stock options | 247 |
| 11.1 | Factors affecting option prices | 247 |
| 11.2 | Assumptions and notation | 251 |

| | | |
|--------------------|---|------------|
| 11.3 | Upper and lower bounds for option prices | 252 |
| 11.4 | Put–call parity | 255 |
| 11.5 | Calls on a non-dividend-paying stock | 257 |
| 11.6 | Puts on a non-dividend-paying stock | 260 |
| 11.7 | Effect of dividends | 262 |
| | Summary | 263 |
| | Further reading | 264 |
| | Practice questions | 265 |
| Chapter 12. | Trading strategies involving options | 268 |
| 12.1 | Principal-protected notes | 268 |
| 12.2 | Trading an option and the underlying asset | 270 |
| 12.3 | Spreads | 272 |
| 12.4 | Combinations | 280 |
| 12.5 | Other payoffs | 283 |
| | Summary | 284 |
| | Further reading | 285 |
| | Practice questions | 285 |
| Chapter 13. | Binomial trees | 288 |
| 13.1 | A one-step binomial model and a no-arbitrage argument | 288 |
| 13.2 | Risk-neutral valuation | 292 |
| 13.3 | Two-step binomial trees | 294 |
| 13.4 | A put example | 297 |
| 13.5 | American options | 298 |
| 13.6 | Delta | 299 |
| 13.7 | Matching volatility with u and d | 300 |
| 13.8 | The binomial tree formulas | 302 |
| 13.9 | Increasing the number of steps | 302 |
| 13.10 | Using DerivaGem | 303 |
| 13.11 | Options on other assets | 304 |
| | Summary | 308 |
| | Further reading | 308 |
| | Practice questions | 309 |
| | Appendix: Derivation of the Black–Scholes–Merton option-pricing formula from a binomial tree | 312 |
| Chapter 14. | Wiener processes and Itô’s lemma | 316 |
| 14.1 | The Markov property | 316 |
| 14.2 | Continuous-time stochastic processes | 317 |
| 14.3 | The process for a stock price | 322 |
| 14.4 | The parameters | 325 |
| 14.5 | Correlated processes | 326 |
| 14.6 | Itô’s lemma | 327 |
| 14.7 | The lognormal property | 328 |
| 14.8 | Fractional Brownian motion | 329 |
| | Summary | 330 |
| | Further reading | 332 |
| | Practice questions | 333 |
| | Appendix: A nonrigorous derivation of Itô’s lemma | 336 |
| Chapter 15. | The Black–Scholes–Merton model | 338 |
| 15.1 | Lognormal property of stock prices | 339 |
| 15.2 | The distribution of the rate of return | 340 |

| | | |
|--------------------|--|------------|
| 15.3 | The expected return | 341 |
| 15.4 | Volatility | 342 |
| 15.5 | The idea underlying the Black–Scholes–Merton differential equation | 346 |
| 15.6 | Derivation of the Black–Scholes–Merton differential equation | 348 |
| 15.7 | Risk-neutral valuation | 351 |
| 15.8 | Black–Scholes–Merton pricing formulas | 352 |
| 15.9 | Cumulative normal distribution function | 355 |
| 15.10 | Warrants and employee stock options | 356 |
| 15.11 | Implied volatilities | 358 |
| 15.12 | Dividends | 360 |
| | Summary | 363 |
| | Further reading | 364 |
| | Practice questions | 365 |
| | Appendix: Proof of the Black–Scholes–Merton formula using risk-neutral valuation | 369 |
| Chapter 16. | Employee stock options | 371 |
| 16.1 | Contractual arrangements | 371 |
| 16.2 | Do options align the interests of shareholders and managers? | 373 |
| 16.3 | Accounting issues | 374 |
| 16.4 | Valuation | 375 |
| 16.5 | The backdating scandal | 380 |
| | Summary | 381 |
| | Further reading | 381 |
| | Practice questions | 382 |
| Chapter 17. | Options on stock indices and currencies | 384 |
| 17.1 | Options on stock indices | 384 |
| 17.2 | Currency options | 386 |
| 17.3 | Options on stocks paying known dividend yields | 389 |
| 17.4 | Valuation of European stock index options | 391 |
| 17.5 | Valuation of European currency options | 394 |
| 17.6 | American options | 395 |
| | Summary | 396 |
| | Further reading | 397 |
| | Practice questions | 397 |
| Chapter 18. | Futures options and Black’s model | 401 |
| 18.1 | Nature of futures options | 401 |
| 18.2 | Reasons for the popularity of futures options | 404 |
| 18.3 | European spot and futures options | 404 |
| 18.4 | Put–call parity | 405 |
| 18.5 | Bounds for futures options | 406 |
| 18.6 | Drift of a futures price in a risk-neutral world | 407 |
| 18.7 | Black’s model for valuing futures options | 408 |
| 18.8 | Using Black’s model instead of Black–Scholes–Merton | 409 |
| 18.9 | Valuation of futures options using binomial trees | 410 |
| 18.10 | American futures options vs. American spot options | 412 |
| 18.11 | Futures-style options | 413 |
| | Summary | 413 |
| | Further reading | 414 |
| | Practice questions | 414 |

| | |
|---|------------|
| Chapter 19. The Greek letters | 417 |
| 19.1 Illustration | 417 |
| 19.2 Naked and covered positions | 418 |
| 19.3 Greek letter calculation | 420 |
| 19.4 Delta hedging | 421 |
| 19.5 Theta | 427 |
| 19.6 Gamma | 429 |
| 19.7 Relationship between delta, theta, and gamma | 433 |
| 19.8 Vega | 434 |
| 19.9 Rho | 436 |
| 19.10 The realities of hedging | 437 |
| 19.11 Scenario analysis | 437 |
| 19.12 Extension of formulas | 439 |
| 19.13 Portfolio insurance | 441 |
| 19.14 Application of machine learning to hedging | 443 |
| Summary | 444 |
| Further reading | 445 |
| Practice questions | 446 |
| Appendix: Taylor series expansions and Greek letters | 450 |
| Chapter 20. Volatility smiles and volatility surfaces | 451 |
| 20.1 Implied volatilities of calls and puts | 451 |
| 20.2 Volatility smile for foreign currency options | 453 |
| 20.3 Volatility smile for equity options | 456 |
| 20.4 Alternative ways of characterizing the volatility smile | 458 |
| 20.5 The volatility term structure and volatility surfaces | 458 |
| 20.6 Minimum variance delta | 460 |
| 20.7 The role of the model | 460 |
| 20.8 When a single large jump is anticipated | 460 |
| Summary | 462 |
| Further reading | 463 |
| Practice questions | 464 |
| Appendix: Determining implied risk-neutral distributions from volatility smiles | 467 |
| Chapter 21. Basic numerical procedures | 470 |
| 21.1 Binomial trees | 470 |
| 21.2 Using the binomial tree for options on indices, currencies, and futures contracts | 478 |
| 21.3 Binomial model for a dividend-paying stock | 480 |
| 21.4 Alternative procedures for constructing trees | 485 |
| 21.5 Time-dependent parameters | 488 |
| 21.6 Monte Carlo simulation | 489 |
| 21.7 Variance reduction procedures | 495 |
| 21.8 Finite difference methods | 498 |
| Summary | 508 |
| Further reading | 509 |
| Practice questions | 510 |
| Chapter 22. Value at risk and expected shortfall | 514 |
| 22.1 The VaR and ES measures | 514 |
| 22.2 Historical simulation | 517 |

| | | |
|--------------------|--|------------|
| 22.3 | Model-building approach | 521 |
| 22.4 | The linear model | 524 |
| 22.5 | The quadratic model | 530 |
| 22.6 | Monte Carlo simulation | 533 |
| 22.7 | Comparison of approaches | 533 |
| 22.8 | Back testing | 534 |
| 22.9 | Principal components analysis | 534 |
| | Summary | 537 |
| | Further reading | 538 |
| | Practice questions | 539 |
| Chapter 23. | Estimating volatilities and correlations | 542 |
| 23.1 | Estimating volatility | 542 |
| 23.2 | The exponentially weighted moving average model | 544 |
| 23.3 | The GARCH(1,1) model | 546 |
| 23.4 | Choosing between the models | 547 |
| 23.5 | Maximum likelihood methods | 548 |
| 23.6 | Using GARCH(1,1) to forecast future volatility | 553 |
| 23.7 | Correlations | 556 |
| | Summary | 558 |
| | Further reading | 559 |
| | Practice questions | 559 |
| Chapter 24. | Credit risk | 562 |
| 24.1 | Credit ratings | 562 |
| 24.2 | Historical default probabilities | 563 |
| 24.3 | Recovery rates | 564 |
| 24.4 | Estimating default probabilities from bond yield spreads | 564 |
| 24.5 | Comparison of default probability estimates | 567 |
| 24.6 | Using equity prices to estimate default probabilities | 570 |
| 24.7 | Credit risk in derivatives transactions | 571 |
| 24.8 | Default correlation | 577 |
| 24.9 | Credit VaR | 580 |
| | Summary | 582 |
| | Further reading | 583 |
| | Practice questions | 583 |
| Chapter 25. | Credit derivatives | 587 |
| 25.1 | Credit default swaps | 588 |
| 25.2 | Valuation of credit default swaps | 591 |
| 25.3 | Credit indices | 595 |
| 25.4 | The use of fixed coupons | 596 |
| 25.5 | CDS forwards and options | 597 |
| 25.6 | Basket credit default swaps | 597 |
| 25.7 | Total return swaps | 597 |
| 25.8 | Collateralized debt obligations | 599 |
| 25.9 | Role of correlation in a basket CDS and CDO | 601 |
| 25.10 | Valuation of a synthetic CDO | 601 |
| 25.11 | Alternatives to the standard market model | 608 |
| | Summary | 610 |
| | Further reading | 610 |
| | Practice questions | 611 |

| | |
|--|------------|
| Chapter 26. Exotic options | 614 |
| 26.1 Packages | 614 |
| 26.2 Perpetual American call and put options | 615 |
| 26.3 Nonstandard American options | 616 |
| 26.4 Gap options | 617 |
| 26.5 Forward start options | 618 |
| 26.6 Cliquet options | 618 |
| 26.7 Compound options | 618 |
| 26.8 Chooser options | 619 |
| 26.9 Barrier options | 620 |
| 26.10 Binary options | 622 |
| 26.11 Lookback options | 623 |
| 26.12 Shout options | 625 |
| 26.13 Asian options | 626 |
| 26.14 Options to exchange one asset for another | 627 |
| 26.15 Options involving several assets | 628 |
| 26.16 Volatility and variance swaps | 629 |
| 26.17 Static options replication | 632 |
| Summary | 634 |
| Further reading | 635 |
| Practice questions | 635 |
| Chapter 27. More on models and numerical procedures | 640 |
| 27.1 Alternatives to Black–Scholes–Merton | 641 |
| 27.2 Stochastic volatility models | 646 |
| 27.3 The IVF model | 649 |
| 27.4 Convertible bonds | 650 |
| 27.5 Path-dependent derivatives | 653 |
| 27.6 Barrier options | 656 |
| 27.7 Options on two correlated assets | 658 |
| 27.8 Monte Carlo simulation and American options | 660 |
| Summary | 665 |
| Further reading | 666 |
| Practice questions | 667 |
| Chapter 28. Martingales and measures | 670 |
| 28.1 The market price of risk | 671 |
| 28.2 Several state variables | 674 |
| 28.3 Martingales | 675 |
| 28.4 Alternative choices for the numeraire | 676 |
| 28.5 Extension to several factors | 679 |
| 28.6 Black’s model revisited | 680 |
| 28.7 Option to exchange one asset for another | 681 |
| 28.8 Change of numeraire | 682 |
| Summary | 684 |
| Further reading | 685 |
| Practice questions | 685 |
| Chapter 29. Interest rate derivatives: The standard market models | 688 |
| 29.1 Bond options | 688 |
| 29.2 Interest rate caps and floors | 693 |
| 29.3 European swap options | 699 |
| 29.4 Hedging interest rate derivatives | 703 |

| | |
|--|------------|
| Summary | 703 |
| Further reading | 704 |
| Practice questions | 704 |
| Chapter 30. Convexity, timing, and quanto adjustments | 707 |
| 30.1 Convexity adjustments | 707 |
| 30.2 Timing adjustments | 710 |
| 30.3 Quantos | 711 |
| Summary | 714 |
| Further reading | 715 |
| Practice questions | 715 |
| Appendix: Proof of the convexity adjustment formula | 718 |
| Chapter 31. Equilibrium models of the short rate | 719 |
| 31.1 Background | 719 |
| 31.2 One-factor models | 721 |
| 31.3 Real-world vs. risk-neutral processes | 726 |
| 31.4 Estimating parameters | 727 |
| 31.5 More sophisticated models | 728 |
| Summary | 729 |
| Further reading | 729 |
| Practice questions | 729 |
| Chapter 32. No-arbitrage models of the short rate | 732 |
| 32.1 Extensions of equilibrium models | 732 |
| 32.2 Options on bonds | 736 |
| 32.3 Volatility structures | 737 |
| 32.4 Interest rate trees | 738 |
| 32.5 A general tree-building procedure | 740 |
| 32.6 Calibration | 749 |
| 32.7 Hedging using a one-factor model | 751 |
| Summary | 752 |
| Further reading | 752 |
| Practice questions | 752 |
| Chapter 33. Modeling forward rates | 755 |
| 33.1 The Heath, Jarrow, and Morton model | 755 |
| 33.2 The BGM model | 758 |
| 33.3 Agency mortgage-backed securities | 768 |
| Summary | 770 |
| Further reading | 770 |
| Practice questions | 771 |
| Chapter 34. Swaps revisited | 773 |
| 34.1 Variations on the vanilla deal | 773 |
| 34.2 Compounding swaps | 775 |
| 34.3 Currency and nonstandard swaps | 776 |
| 34.4 Equity swaps | 777 |
| 34.5 Swaps with embedded options | 779 |
| 34.6 Other swaps | 781 |
| Summary | 782 |
| Further reading | 783 |
| Practice questions | 783 |

| | |
|--|------------|
| Chapter 35. Energy and commodity derivatives | 785 |
| 35.1 Agricultural commodities | 785 |
| 35.2 Metals | 786 |
| 35.3 Energy products | 787 |
| 35.4 Modeling commodity prices | 789 |
| 35.5 Weather derivatives | 795 |
| 35.6 Insurance derivatives | 796 |
| 35.7 Pricing weather and insurance derivatives | 797 |
| 35.8 How an energy producer can hedge risks | 798 |
| Summary | 799 |
| Further reading | 799 |
| Practice questions | 800 |
| Chapter 36. Real options | 802 |
| 36.1 Capital investment appraisal | 802 |
| 36.2 Extension of the risk-neutral valuation framework | 803 |
| 36.3 Estimating the market price of risk | 805 |
| 36.4 Application to the valuation of a business | 806 |
| 36.5 Evaluating options in an investment opportunity | 806 |
| Summary | 813 |
| Further reading | 813 |
| Practice questions | 814 |
| Chapter 37. Derivatives mishaps and what we can learn from them | 815 |
| 37.1 Lessons for all users of derivatives | 815 |
| 37.2 Lessons for financial institutions | 819 |
| 37.3 Lessons for nonfinancial corporations | 824 |
| Summary | 826 |
| Further reading | 826 |
| Glossary of terms | 827 |
| DerivaGem software | 851 |
| Exchanges trading futures and options | 856 |
| Table for $N(x)$ When $x \leq 0$ | 857 |
| Author index | 859 |
| Subject index | 863 |