

OPTIONS, FUTURES, AND OTHER DERIVATIVES

John C. Hull

*Maple Financial Group Professor of Derivatives and Risk Management
Joseph L. Rotman School of Management
University of Toronto*

ELEVENTH EDITION



CONTENTS IN BRIEF

List of business snapshots	xv
List of technical notes	xvi
Preface	xvii
1. Introduction	1
2. Futures markets and central counterparties	24
3. Hedging strategies using futures	48
4. Interest rates	76
5. Determination of forward and futures prices	102
6. Interest rate futures	130
7. Swaps	150
8. Securitization and the financial crisis of 2007–8	179
9. XVA	194
10. Mechanics of options markets	205
11. Properties of stock options	225
12. Trading strategies involving options	246
13. Binomial trees	266
14. Wiener processes and Itô’s lemma	294
15. The Black–Scholes–Merton model	316
16. Employee stock options	349
17. Options on stock indices and currencies	362
18. Futures options and Black’s model	379
19. The Greek letters	395
20. Volatility smiles and volatility surfaces	429
21. Basic numerical procedures	448
22. Value at risk and expected shortfall	492
23. Estimating volatilities and correlations	520
24. Credit risk	540
25. Credit derivatives	565
26. Exotic options	592
27. More on models and numerical procedures	618
28. Martingales and measures	648
29. Interest rate derivatives: The standard market models	666
30. Convexity, timing, and quanto adjustments	685
31. Equilibrium models of the short rate	697
32. No-arbitrage models of the short rate	710
33. Modeling forward rates	733
34. Swaps revisited	751
35. Energy and commodity derivatives	763
36. Real options	780
37. Derivatives mishaps and what we can learn from them	793
Glossary of terms	805
DerivaGem software	829
Exchanges trading futures and options	834
Tables for $N(x)$	835
Author index	837
Subject index	841

CONTENTS

List of business snapshots	xv
List of technical notes	xvi
Preface	xvii
Chapter 1. Introduction	1
1.1 Exchange-traded markets	2
1.2 Over-the-counter markets	3
1.3 Forward contracts	6
1.4 Futures contracts	8
1.5 Options	9
1.6 Types of traders	11
1.7 Hedgers	12
1.8 Speculators	14
1.9 Arbitrageurs	17
1.10 Dangers	17
Summary	19
Further reading	19
Short concept questions	19
Practice questions	20
Chapter 2. Futures markets and central counterparties	24
2.1 Background	24
2.2 Specification of a futures contract	26
2.3 Convergence of futures price to spot price	28
2.4 The operation of margin accounts	29
2.5 OTC markets	32
2.6 Market quotes	35
2.7 Delivery	38
2.8 Types of traders and types of orders	39
2.9 Regulation	40
2.10 Accounting and tax	41
2.11 Forward vs. futures contracts	42
Summary	43
Further reading	44
Short concept questions	45
Practice questions	45
Chapter 3. Hedging strategies using futures	48
3.1 Basic principles	48
3.2 Arguments for and against hedging	50
3.3 Basis risk	53
3.4 Cross hedging	57

3.5	Stock index futures	62
3.6	Stack and roll	67
	Summary	68
	Further reading	70
	Short concept questions	71
	Practice questions	71
	Appendix: Capital asset pricing model	74
Chapter 4.	Interest rates	76
4.1	Types of rates	76
4.2	Reference rates	77
4.3	The risk-free rate	79
4.4	Measuring interest rates	79
4.5	Zero rates	82
4.6	Bond pricing	83
4.7	Determining zero rates	84
4.8	Forward rates	87
4.9	Forward rate agreements	88
4.10	Duration	90
4.11	Convexity	94
4.12	Theories of the term structure of interest rates	95
	Summary	97
	Further reading	98
	Short concept questions	98
	Practice questions	99
Chapter 5.	Determination of forward and futures prices	102
5.1	Investment assets vs. consumption assets	102
5.2	Short selling	103
5.3	Assumptions and notation	104
5.4	Forward price for an investment asset	105
5.5	Known income	108
5.6	Known yield	110
5.7	Valuing forward contracts	111
5.8	Are forward prices and futures prices equal?	113
5.9	Futures prices of stock indices	113
5.10	Forward and futures contracts on currencies	115
5.11	Futures on commodities	119
5.12	The cost of carry	121
5.13	Delivery options	122
5.14	Futures prices and expected future spot prices	122
	Summary	125
	Further reading	126
	Short concept questions	126
	Practice questions	127
Chapter 6.	Interest rate futures	130
6.1	Day count and quotation conventions	130
6.2	Treasury bond futures	133
6.3	Eurodollar and SOFR futures	138
6.4	Duration-based hedging strategies using futures	143
6.5	Hedging portfolios of assets and liabilities	145
	Summary	146
	Further reading	146
	Short concept questions	146
	Practice questions	147

Chapter 7. Swaps	150
7.1 Mechanics of interest rate swaps	150
7.2 Determining risk-free rates	153
7.3 Reasons for trading interest rate swaps	154
7.4 The organization of trading	156
7.5 The comparative-advantage argument	159
7.6 Valuation of interest rate swaps	161
7.7 How the value changes through time	163
7.8 Fixed-for-fixed currency swaps	164
7.9 Valuation of fixed-for-fixed currency swaps	168
7.10 Other currency swaps	170
7.11 Credit risk	171
7.12 Credit default swaps	171
7.13 Other types of swaps	172
Summary	174
Further reading	174
Short concept questions	174
Practice questions	175
Chapter 8. Securitization and the financial crisis of 2007–8	179
8.1 Securitization	179
8.2 The U.S. housing market	183
8.3 What went wrong?	187
8.4 The aftermath	189
Summary	191
Further reading	191
Short concept questions	192
Practice questions	192
Chapter 9. XVAs	194
9.1 CVA and DVA	194
9.2 FVA and MVA	197
9.3 KVA	200
9.4 Calculation issues	201
Summary	202
Further reading	203
Short concept questions	203
Practice questions	203
Chapter 10. Mechanics of options markets	205
10.1 Types of options	205
10.2 Option positions	207
10.3 Underlying assets	209
10.4 Specification of stock options	211
10.5 Trading	214
10.6 Trading costs	215
10.7 Margin requirements	215
10.8 The options clearing corporation	217
10.9 Regulation	217
10.10 Taxation	218
10.11 Warrants, employee stock options, and convertibles	219
10.12 Over-the-counter options markets	220
Summary	221
Further reading	221
Short concept questions	221
Practice questions	222

Chapter 11. Properties of stock options	225
11.1 Factors affecting option prices	225
11.2 Assumptions and notation	229
11.3 Upper and lower bounds for option prices	230
11.4 Put–call parity	233
11.5 Calls on a non-dividend-paying stock	235
11.6 Puts on a non-dividend-paying stock	238
11.7 Effect of dividends	240
Summary	241
Further reading	242
Short concept questions	243
Practice questions	243
Chapter 12. Trading strategies involving options	246
12.1 Principal-protected notes	246
12.2 Trading an option and the underlying asset	248
12.3 Spreads	250
12.4 Combinations	258
12.5 Other payoffs	261
Summary	262
Further reading	263
Short concept questions	263
Practice questions	263
Chapter 13. Binomial trees	266
13.1 A one-step binomial model and a no-arbitrage argument	266
13.2 Risk-neutral valuation	270
13.3 Two-step binomial trees	272
13.4 A put example	275
13.5 American options	276
13.6 Delta	277
13.7 Matching volatility with u and d	278
13.8 The binomial tree formulas	280
13.9 Increasing the number of steps	280
13.10 Using DerivaGem	281
13.11 Options on other assets	282
Summary	286
Further reading	286
Short concept questions	287
Practice questions	287
Appendix: Derivation of the Black–Scholes–Merton option-pricing formula from a binomial tree	290
Chapter 14. Wiener processes and Itô's lemma	294
14.1 The Markov property	294
14.2 Continuous-time stochastic processes	295
14.3 The process for a stock price	300
14.4 The parameters	303
14.5 Correlated processes	304
14.6 Itô's lemma	305
14.7 The lognormal property	306
14.8 Fractional Brownian motion	307
Summary	308
Further reading	310
Short concept questions	311

Practice questions	311
Appendix: A nonrigorous derivation of Itô's lemma	314
Chapter 15. The Black–Scholes–Merton model	316
15.1 Lognormal property of stock prices	317
15.2 The distribution of the rate of return	318
15.3 The expected return	319
15.4 Volatility	320
15.5 The idea underlying the Black–Scholes–Merton differential equation	324
15.6 Derivation of the Black–Scholes–Merton differential equation	326
15.7 Risk-neutral valuation	329
15.8 Black–Scholes–Merton pricing formulas	330
15.9 Cumulative normal distribution function	333
15.10 Warrants and employee stock options	334
15.11 Implied volatilities	336
15.12 Dividends	338
Summary	341
Further reading	342
Short concept questions	343
Practice questions	343
Appendix: Proof of the Black–Scholes–Merton formula using risk-neutral valuation	347
Chapter 16. Employee stock options	349
16.1 Contractual arrangements	349
16.2 Do options align the interests of shareholders and managers?	351
16.3 Accounting issues	352
16.4 Valuation	353
16.5 The backdating scandal	358
Summary	359
Further reading	359
Short concept questions	359
Practice questions	360
Chapter 17. Options on stock indices and currencies	362
17.1 Options on stock indices	362
17.2 Currency options	364
17.3 Options on stocks paying known dividend yields	367
17.4 Valuation of European stock index options	369
17.5 Valuation of European currency options	372
17.6 American options	373
Summary	374
Further reading	375
Short concept questions	375
Practice questions	375
Chapter 18. Futures options and Black's model	379
18.1 Nature of futures options	379
18.2 Reasons for the popularity of futures options	382
18.3 European spot and futures options	382
18.4 Put–call parity	383
18.5 Bounds for futures options	384
18.6 Drift of a futures price in a risk-neutral world	385
18.7 Black's model for valuing futures options	386
18.8 Using Black's model instead of Black–Scholes–Merton	387

18.9	Valuation of futures options using binomial trees	388
18.10	American futures options vs. American spot options	390
18.11	Futures-style options	391
	Summary	391
	Further reading	392
	Short concept questions	392
	Practice questions	392
Chapter 19. The Greek letters		395
19.1	Illustration	395
19.2	Naked and covered positions	396
19.3	Greek letter calculation	398
19.4	Delta hedging	399
19.5	Theta	405
19.6	Gamma	407
19.7	Relationship between delta, theta, and gamma	411
19.8	Vega	412
19.9	Rho	414
19.10	The realities of hedging	415
19.11	Scenario analysis	415
19.12	Extension of formulas	417
19.13	Portfolio insurance	419
19.14	Application of machine learning to hedging	421
	Summary	422
	Further reading	423
	Short concept questions	424
	Practice questions	424
	Appendix: Taylor series expansions and Greek letters	428
Chapter 20. Volatility smiles and volatility surfaces		429
20.1	Implied volatilities of calls and puts	429
20.2	Volatility smile for foreign currency options	431
20.3	Volatility smile for equity options	434
20.4	Alternative ways of characterizing the volatility smile	436
20.5	The volatility term structure and volatility surfaces	436
20.6	Minimum variance delta	438
20.7	The role of the model	438
20.8	When a single large jump is anticipated	438
	Summary	440
	Further reading	441
	Short concept questions	441
	Practice questions	442
	Appendix: Determining implied risk-neutral distributions from volatility smiles	445
Chapter 21. Basic numerical procedures		448
21.1	Binomial trees	448
21.2	Using the binomial tree for options on indices, currencies, and futures contracts	456
21.3	Binomial model for a dividend-paying stock	458
21.4	Alternative procedures for constructing trees	463
21.5	Time-dependent parameters	466
21.6	Monte Carlo simulation	467
21.7	Variance reduction procedures	473
21.8	Finite difference methods	476

Summary	486
Further reading	487
Practice questions	488
Chapter 22. Value at risk and expected shortfall	492
22.1 The VaR and ES measures	492
22.2 Historical simulation	495
22.3 Model-building approach	499
22.4 The linear model	502
22.5 The quadratic model	508
22.6 Monte Carlo simulation	511
22.7 Comparison of approaches	511
22.8 Back testing	512
22.9 Principal components analysis	512
Summary	515
Further reading	516
Practice questions	517
Chapter 23. Estimating volatilities and correlations	520
23.1 Estimating volatility	520
23.2 The exponentially weighted moving average model	522
23.3 The GARCH(1,1) model	524
23.4 Choosing between the models	525
23.5 Maximum likelihood methods	526
23.6 Using GARCH(1,1) to forecast future volatility	531
23.7 Correlations	534
Summary	536
Further reading	537
Practice questions	537
Chapter 24. Credit risk	540
24.1 Credit ratings	540
24.2 Historical default probabilities	541
24.3 Recovery rates	542
24.4 Estimating default probabilities from bond yield spreads	542
24.5 Comparison of default probability estimates	545
24.6 Using equity prices to estimate default probabilities	548
24.7 Credit risk in derivatives transactions	549
24.8 Default correlation	555
24.9 Credit VaR	558
Summary	560
Further reading	561
Practice questions	561
Chapter 25. Credit derivatives	565
25.1 Credit default swaps	566
25.2 Valuation of credit default swaps	569
25.3 Credit indices	573
25.4 The use of fixed coupons	574
25.5 CDS forwards and options	575
25.6 Basket credit default swaps	575
25.7 Total return swaps	575
25.8 Collateralized debt obligations	577
25.9 Role of correlation in a basket CDS and CDO	579
25.10 Valuation of a synthetic CDO	579
25.11 Alternatives to the standard market model	586

Summary	588
Further reading	588
Practice questions	589
Chapter 26. Exotic options	592
26.1 Packages	592
26.2 Perpetual American call and put options	593
26.3 Nonstandard American options	594
26.4 Gap options	595
26.5 Forward start options	596
26.6 Clique options	596
26.7 Compound options	596
26.8 Chooser options	597
26.9 Barrier options	598
26.10 Binary options	600
26.11 Lookback options	601
26.12 Shout options	603
26.13 Asian options	604
26.14 Options to exchange one asset for another	605
26.15 Options involving several assets	606
26.16 Volatility and variance swaps	607
26.17 Static options replication	610
Summary	612
Further reading	613
Practice questions	613
Chapter 27. More on models and numerical procedures	618
27.1 Alternatives to Black–Scholes–Merton	619
27.2 Stochastic volatility models	624
27.3 The IVF model	627
27.4 Convertible bonds	628
27.5 Path-dependent derivatives	631
27.6 Barrier options	634
27.7 Options on two correlated assets	636
27.8 Monte Carlo simulation and American options	638
Summary	643
Further reading	644
Practice questions	645
Chapter 28. Martingales and measures	648
28.1 The market price of risk	649
28.2 Several state variables	652
28.3 Martingales	653
28.4 Alternative choices for the numeraire	654
28.5 Extension to several factors	657
28.6 Black’s model revisited	658
28.7 Option to exchange one asset for another	659
28.8 Change of numeraire	660
Summary	662
Further reading	663
Practice questions	663
Chapter 29. Interest rate derivatives: The standard market models	666
29.1 Bond options	666
29.2 Interest rate caps and floors	671

29.3	European swap options	677
29.4	Hedging interest rate derivatives	681
	Summary	681
	Further reading	682
	Practice questions	682
Chapter 30. Convexity, timing, and quanto adjustments	685	
30.1	Convexity adjustments	685
30.2	Timing adjustments	688
30.3	Quantos	689
	Summary	692
	Further reading	693
	Practice questions	693
	Appendix: Proof of the convexity adjustment formula	696
Chapter 31. Equilibrium models of the short rate	697	
31.1	Background	697
31.2	One-factor models	699
31.3	Real-world vs. risk-neutral processes	704
31.4	Estimating parameters	705
31.5	More sophisticated models	706
	Summary	707
	Further reading	707
	Practice questions	707
Chapter 32. No-arbitrage models of the short rate	710	
32.1	Extensions of equilibrium models	710
32.2	Options on bonds	714
32.3	Volatility structures	715
32.4	Interest rate trees	716
32.5	A general tree-building procedure	718
32.6	Calibration	727
32.7	Hedging using a one-factor model	729
	Summary	730
	Further reading	730
	Practice questions	730
Chapter 33. Modeling forward rates	733	
33.1	The Heath, Jarrow, and Morton model	733
33.2	The BGM model	736
33.3	Agency mortgage-backed securities	746
	Summary	748
	Further reading	748
	Practice questions	749
Chapter 34. Swaps revisited	751	
34.1	Variations on the vanilla deal	751
34.2	Compounding swaps	753
34.3	Currency and nonstandard swaps	754
34.4	Equity swaps	755
34.5	Swaps with embedded options	757
34.6	Other swaps	759
	Summary	760
	Further reading	761
	Practice questions	761

Chapter 35. Energy and commodity derivatives	763
35.1 Agricultural commodities	763
35.2 Metals	764
35.3 Energy products	765
35.4 Modeling commodity prices	767
35.5 Weather derivatives	773
35.6 Insurance derivatives	774
35.7 Pricing weather and insurance derivatives	775
35.8 How an energy producer can hedge risks	776
Summary	777
Further reading	777
Practice questions	778
Chapter 36. Real options	780
36.1 Capital investment appraisal	780
36.2 Extension of the risk-neutral valuation framework	781
36.3 Estimating the market price of risk	783
36.4 Application to the valuation of a business	784
36.5 Evaluating options in an investment opportunity	784
Summary	791
Further reading	791
Practice questions	792
Chapter 37. Derivatives mishaps and what we can learn from them	793
37.1 Lessons for all users of derivatives	793
37.2 Lessons for financial institutions	797
37.3 Lessons for nonfinancial corporations	802
Summary	804
Further reading	804
Glossary of terms	805
DerivaGem software	829
Exchanges trading futures and options	834
Tables for $N(x)$	835
Author index	837
Subject index	841