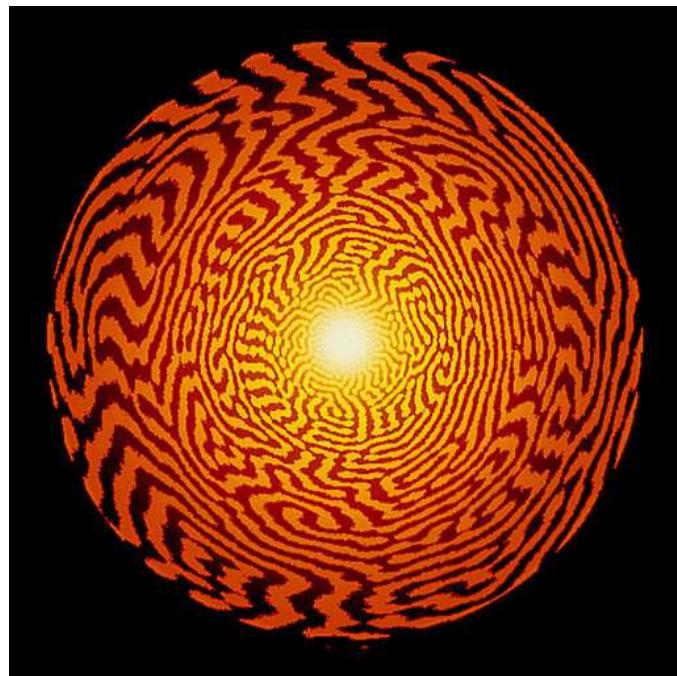


Forecasting  
*in Economics, Business, Finance and Beyond*



Francis X. Diebold  
University of Pennsylvania

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