# An Introduction to Mathematical Statistics and Its Applications

Sixth Edition

## Richard J. Larsen

Vanderbilt University

## Morris L. Marx

University of West Florida



# Contents

#### Preface viii

#### 1 Introduction 1

- I.I An Overview I
- I.2 Some Examples 2
- 1.3 A Brief History 6
- I.4 A Chapter Summary I4

## 2 Probability 15

- 2.1 Introduction 15
- 2.2 Sample Spaces and the Algebra of Sets 17
- 2.3 The Probability Function 26
- 2.4 Conditional Probability 31
- 2.5 Independence 50
- 2.6 Combinatorics 65
- 2.7 Combinatorial Probability 89
- 2.8 Taking a Second Look at Statistics (Monte Carlo Techniques) 99

### 3 Random Variables 102

- 3.1 Introduction 102
- 3.2 Binomial and Hypergeometric Probabilities 103
- 3.3 Discrete Random Variables 116
- 3.4 Continuous Random Variables 127
- 3.5 Expected Values 137
- 3.6 The Variance 153
- 3.7 Joint Densities 160
- 3.8 Transforming and Combining Random Variables 174
- 3.9 Further Properties of the Mean and Variance 182
- 3.10 Order Statistics 192
- 3.11 Conditional Densities 199
- 3.12 Moment-Generating Functions 206
- 3.13 Taking a Second Look at Statistics (Interpreting Means) 215

#### 4 Special Distributions 218

- 4.1 Introduction 218
- 4.2 The Poisson Distribution 219
- 4.3 The Normal Distribution 235
- 4.4 The Geometric Distribution 257
- 4.5 The Negative Binomial Distribution 259
- 4.6 The Gamma Distribution 267
- 4.7 Taking a Second Look at Statistics (Monte Carlo Simulations) 271

Appendix 4.A.I Properties of Frequently-Used pdfs 274

Appendix 4.A.2 A Proof of the Central Limit Theorem 276

#### 5 Estimation 278

- 5.1 Introduction 278
- **5.2** Estimating Parameters: The Method of Maximum Likelihood and the Method of Moments **280**
- 5.3 Interval Estimation 293
- 5.4 Properties of Estimators 308
- 5.5 Minimum-Variance Estimators: The Cramér-Rao Lower Bound 316
- 5.6 Sufficient Estimators 319
- 5.7 Consistency 326
- 5.8 Bayesian Estimation 329
- 5.9 Taking a Second Look at Statistics (Beyond Classical Estimation) 341

#### 6 Hypothesis Testing 343

- 6.1 Introduction 343
- 6.2 The Decision Rule 344
- **6.3** Testing Binomial Data— $H_0$ :  $p = p_0$  **353**
- 6.4 Type I and Type II Errors 359
- 6.5 A Notion of Optimality: The Generalized Likelihood Ratio 375
- 6.6 Taking a Second Look at Hypothesis Testing (Statistical Significance versus "Practical" Significance) 378

/ INFERENCES BASED ON THE NORMAL DISTRIBUTION 380

- 7.1 Introduction 380
- **7.2** Comparing  $\frac{\overline{Y}-\mu}{\sigma/\sqrt{n}}$  and  $\frac{\overline{Y}-\mu}{S/\sqrt{n}}$  **381**
- **7.3** Deriving the Distribution of  $\frac{\overline{Y} \mu}{\overline{Y} \sqrt{n}}$  **383**

- 7.4 Drawing Inferences About  $\mu$  389
- **7.5** Drawing Inferences About  $\sigma^2$  **404**
- 7.6 Taking a Second Look at Statistics (Type II Error) 412
- **Appendix 7.A.1** Some Distribution Results for  $\overline{Y}$  and  $S^2$  **414**

Appendix 7.A.2 A Proof That the One-Sample t Test Is a GLRT 416

Appendix 7.A.3 A Proof of Theorem 7.5.2 418

#### 8 Types of Data: A Brief Overview 421

- 8.1 Introduction 421
- 8.2 Classifying Data 427
- 8.3 Taking a Second Look at Statistics (Why Samples Are Not "Valid"!) 448

#### 9 Two-Sample Inferences 450

- 9.1 Introduction 450
- **9.2** Testing  $H_0: \mu_X = \mu_Y$  **451**
- **9.3** Testing  $H_0$ :  $\sigma_{\chi}^2 = \sigma_{\gamma}^2$ —The *F* Test **463**
- **9.4** Binomial Data: Testing  $H_0$ :  $p_{\chi} = p_{\gamma}$  **468**
- 9.5 Confidence Intervals for the Two-Sample Problem 473
- 9.6 Taking a Second Look at Statistics (Choosing Samples) 478

Appendix 9.A.IA Derivation of the Two-Sample t Test (A Proof of<br/>Theorem 9.2.2)480

### 10 Goodness-of-Fit Tests 483

- IO.I Introduction 483
- **10.2** The Multinomial Distribution **484**
- 10.3 Goodness-of-Fit Tests: All Parameters Known 488
- 10.4 Goodness-of-Fit Tests: Parameters Unknown 498
- 10.5 Contingency Tables 507
- 10.6 Taking a Second Look at Statistics (Outliers) 517

## 11 Regression 520

- II.I Introduction 520
- **II.2** The Method of Least Squares **520**
- 11.3 The Linear Model 543
- 11.4 Covariance and Correlation 563

- **11.5** The Bivariate Normal Distribution **570**
- 11.6 Taking a Second Look at Statistics (How Not to Interpret the Sample Correlation Coefficient) 576

Appendix II.A.I A Proof of Theorem 11.3.3 577

## 12 THE ANALYSIS OF VARIANCE 580

- 12.1 Introduction 580
- 12.2 The F Test 582
- 12.3 Multiple Comparisons: Tukey's Method 592
- 12.4 Testing Subhypotheses with Contrasts 596
- 12.5 Data Transformations 604
- 12.6 Taking a Second Look at Statistics (Putting the Subject of Statistics Together—The Contributions of Ronald A. Fisher) 606

Appendix 12.A.I A Proof of Theorem 12.2.2 608

**Appendix 12.A.2** The Distribution of  $\frac{SSTR/(k-1)}{SSE/(n-k)}$  When  $H_1$  Is True **608** 

## 13 RANDOMIZED BLOCK DESIGNS 613

- 13.1 Introduction 613
- **13.2** The F Test for a Randomized Block Design **614**
- **13.3** The Paired *t* Test **628**
- 13.4 Taking a Second Look at Statistics (Choosing between a Two-Sample t Test and a Paired t Test)634

## 14 Nonparametric Statistics 638

- 14.1 Introduction 638
- 14.2 The Sign Test 639
- 14.3 Wilcoxon Tests 645
- 14.4 The Kruskal-Wallis Test 658
- 14.5 The Friedman Test 662
- 14.6 Testing for Randomness 665
- 14.7 Taking a Second Look at Statistics (Comparing Parametric and Nonparametric Procedures) 669

## 15 Factorial Data (Available Online) 15-1

- 15.1 Introduction 15-1
- **15.2** The Two-Factor Factorial **15-4**
- 15.3 Sums of Squares for Two-Factor Factorials 15-16
- 15.4 Expected Mean Squares 15-26
- 15.5 Examples 15-30
- 15.6 The Three-Factor Factorial Design 15-40
- 15.7 2<sup>n</sup> Designs 15-51
- 15.8 Fractional Factorials 15-72

Appendix A: Statistical Tables 674

Answers to Selected Odd-Numbered Questions 701

Bibliography 725

Index 737