

# Introductory Econometrics

A MODERN APPROACH

SEVENTH EDITION

**Jeffrey M. Wooldridge**

*Michigan State University*



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Australia • Brazil • Mexico • Singapore • United Kingdom • United States

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