

Economics: Current and Future Developments

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*Interest Rate Modeling for Risk Management
Market Price of Interest Rate Risk*

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