

# **ECONOMETRIC ANALYSIS OF STOCHASTIC DOMINANCE**

Concepts, Methods, Tools, and Applications

YOON-JAE WHANG

*Seoul National University*



**CAMBRIDGE**  
UNIVERSITY PRESS

# Contents

|                                   |                |
|-----------------------------------|----------------|
| <i>List of figures</i>            | <i>page</i> xi |
| <i>List of tables</i>             | xii            |
| <i>Preface</i>                    | xiii           |
| <i>Abbreviations and Notation</i> | xv             |

|   |    |
|---|----|
| <b>1 Introduction</b>                                 | 1  |
| 1.1 Concepts of Stochastic Dominance                  | 1  |
| 1.1.1 Definitions                                     | 1  |
| 1.1.2 Basic Properties of Stochastic Dominance        | 7  |
| 1.1.3 A Numerical Example                             | 9  |
| 1.1.4 Extensions and Some Related Concepts            | 11 |
| 1.2 Applications of Stochastic Dominance              | 15 |
| 1.2.1 Welfare Analysis                                | 15 |
| 1.2.2 Finance   | 17 |
| 1.2.3 Industrial Organization                         | 18 |
| 1.2.4 Labor Economics                                 | 20 |
| 1.2.5 International Economics                         | 20 |
| 1.2.6 Health Economics                                | 21 |
| 1.2.7 Agricultural Economics                          | 21 |
| 1.3 Outline of Subsequent Chapters                    | 22 |
| <b>2 Tests of Stochastic Dominance: Basic Results</b> | 24 |
| 2.1 Introduction                                      | 24 |
| 2.2 Null of Dominance against Non-Dominance           | 25 |
| 2.2.1 Tests Based on Multiple Comparisons             | 26 |
| 2.2.2 Supremum-Type Tests                             | 32 |
| 2.2.3 Integral-Type Tests                             | 45 |
| 2.2.4 Quantile-Based Tests                            | 48 |
| 2.2.5 Neyman's Smooth Tests                           | 53 |
| 2.3 Null of Non-Dominance against Dominance           | 58 |

|          |  |            |
|----------|--|------------|
| 2.3.1    | Infimum t-Test   | 59         |
| 2.3.2    | Empirical Likelihood Test                                      | 61         |
| 2.4      | Null of Equality against Dominance                             | 64         |
| 2.5      | Empirical Examples   | 65         |
| 2.5.1    | Comparison of Income Distributions                             | 65         |
| 2.5.2    | Testing for Monday Effects in Stock Markets                    | 67         |
| <b>3</b> | <b>Tests of Stochastic Dominance: Further Results</b>          | <b>73</b>  |
| 3.1      | Stochastic Dominance Tests with Improved Power                 | 73         |
| 3.1.1    | The Contact Set Approach                                       | 73         |
| 3.1.2    | The Selective Recentering Approach                             | 80         |
| 3.1.3    | Remarks  | 83         |
| 3.1.4    | A Numerical Example  | 85         |
| 3.2      | Program Evaluation and Stochastic Dominance                    | 87         |
| 3.2.1    | Distributional Treatment Effects                               | 87         |
| 3.2.2    | Counterfactual Policy Analysis                                 | 98         |
| 3.3      | Some Issues of Stochastic Dominance Tests                      | 104        |
| 3.3.1    | Stochastic Dominance Tests with Unbounded Supports             | 104        |
| 3.3.2    | Classification of Stochastic Dominance Relations               | 106        |
| 3.3.3    | Large Deviation Approximation                                  | 108        |
| 3.4      | Empirical Examples   | 109        |
| 3.4.1    | Distributional Treatment Effects of Veteran Status             | 109        |
| 3.4.2    | Returns to Schooling: Quantile Treatment Effects               | 110        |
| <b>4</b> | <b>Stochastic Dominance with Covariates</b>                    | <b>113</b> |
| 4.1      | Introduction   | 113        |
| 4.2      | Conditional Stochastic Dominance at Fixed Values of Covariates | 114        |
| 4.2.1    | Quantile-Based Tests   | 114        |
| 4.2.2    | Cumulative Distribution Function-Based Tests                   | 117        |
| 4.3      | Conditional Stochastic Dominance at All Values of Covariates   | 119        |
| 4.3.1    | The Poissonization Approach                                    | 119        |
| 4.3.2    | The Least Concave Majorant Approach                            | 122        |
| 4.3.3    | The Strong Approximation Approach                              | 124        |
| 4.3.4    | The Unconditional Moment Representation Approach               | 127        |
| 4.4      | Stochastic Monotonicity  | 130        |
| 4.5      | Empirical Examples   | 134        |
| 4.5.1    | Testing for Conditional Treatment Effects                      | 134        |
| 4.5.2    | Testing for a Strong Leverage Hypothesis                       | 136        |
| <b>5</b> | <b>Extensions of Stochastic Dominance</b>                      | <b>141</b> |
| 5.1      | Multivariate Stochastic Dominance                              | 141        |
| 5.2      | Analysis of Economic Inequality and Poverty                    | 143        |
| 5.2.1    | Lorenz Dominance   | 143        |

|                   |  |     |
|-------------------|--|-----|
| 5.2.2             | Poverty Dominance  | 146 |
| 5.2.3             | Initial Dominance  | 147 |
| 5.3               | Analysis of Portfolio Choice Problems                              | 149 |
| 5.3.1             | Marginal Conditional Stochastic Dominance                          | 149 |
| 5.3.2             | Stochastic Dominance Efficiency                                    | 151 |
| 5.3.3             | Convex Stochastic Dominance and Stochastic<br>Dominance Optimality | 154 |
| 5.4               | Weaker Notions of Stochastic Dominance                             | 159 |
| 5.4.1             | Almost Stochastic Dominance  | 159 |
| 5.4.2             | Approximate Stochastic Dominance                                   | 162 |
| 5.4.3             | Infinite-Order Stochastic Dominance                                | 166 |
| 5.5               | Related Concepts of Stochastic Dominance                           | 169 |
| 5.5.1             | Density Ratio Dominance  | 169 |
| 5.5.2             | Uniform Stochastic Ordering  | 172 |
| 5.5.3             | Positive Quadrant Dependence                                       | 176 |
| 5.5.4             | Expectation Dependence Dominance                                   | 178 |
| 5.5.5             | Central Dominance  | 180 |
| 5.5.6             | Spatial Dominance  | 186 |
| <b>6</b>          | <b>Some Further Topics</b>   | 187 |
| 6.1               | Distributional Overlap Measure                                     | 187 |
| 6.2               | Generalized Functional Inequalities                                | 189 |
| 6.3               | Distributions with Measurement Errors                              | 192 |
| 6.4               | Stochastic Dominance Tests with Many Covariates                    | 195 |
| 6.5               | Robust Forecasting Comparisons                                     | 198 |
| <b>7</b>          | <b>Conclusions</b>   | 200 |
| <b>Appendix A</b> | <b>Basic Technical Tools</b>                                       | 202 |
| A.1               | A Probability Background   | 202 |
| A.2               | Empirical Processes  | 205 |
| A.2.1             | Independent Observations   | 205 |
| A.2.2             | Dependent Observations   | 209 |
| A.3               | Poissonization Methods   | 213 |
| <b>Appendix B</b> | <b>Computer Code for Stochastic Dominance Tests</b>                | 218 |
| B.1               | Introduction   | 218 |
| B.2               | Supremum-Type Tests  | 221 |
| B.2.1             | McFadden Test  | 221 |
| B.2.2             | Barrett–Donald Test  | 222 |
| B.2.3             | Linton–Maasoumi–Whang Test   | 225 |
| B.2.4             | Donald–Hsu Test  | 227 |
| B.3               | Integral-Type Test   | 230 |
| B.3.1             | Hall–Yatchew Test  | 230 |

|                     |  |     |
|---------------------|--|-----|
| B.3.2               | Bennett Test   | 232 |
| B.3.3               | Linton–Song–Whang Test                               | 235 |
| B.4                 | Stochastic Dominance with Covariates: Lee–Whang Test | 237 |
| <i>Bibliography</i> |  | 242 |
| <i>Index</i>        |  | 259 |