

Dharmaraja Selvamuthu · Dipayan Das

# Introduction to Statistical Methods, Design of Experiments and Statistical Quality Control

 Springer

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