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# **Non-Life Insurance Mathematics**

An Introduction with the Poisson Process

Second Edition



Springer

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## Part I Collective Risk Models

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