
John D. Levendis

Time Series Econometrics

Learning Through Replication

Contents

1	Introduction	1
1.1	What Makes Time-Series Econometrics Unique?	1
1.2	Notation	3
1.3	Statistical Review	5
1.4	Specifying Time in Stata	7
1.5	Installing New Stata Commands	8
1.6	Exercises	9
2	ARMA(p,q) Processes	11
2.1	Introduction	11
2.1.1	Stationarity	12
2.1.2	A Purely Random Process	14
2.2	AR(1) Models	15
2.2.1	Estimating an AR(1) Model	15
2.2.2	Impulse Responses	20
2.2.3	Forecasting	24
2.3	AR(p) Models	25
2.3.1	Estimating an AR(p) Model	26
2.3.2	Impulse Responses	28
2.3.3	Forecasting	30
2.4	MA(1) Models	32
2.4.1	Estimation	33
2.4.2	Impulse Responses	33
2.4.3	Forecasting	34
2.5	MA(q) Models	37
2.5.1	Estimation	37
2.5.2	Impulse Responses	38
2.6	Non-zero ARMA Processes	39
2.6.1	Non-zero AR Processes	40
2.6.2	Non-zero MA Processes	41
2.6.3	Dealing with Non-zero Means	42
2.7	ARMA(p,q) Models	45
2.7.1	Estimation	46
2.8	Conclusion	46

3	Model Selection in ARMA(p,q) Processes	47
3.1	ACFs and PACFs	48
3.1.1	Theoretical ACF of an AR(1) Process	48
3.1.2	Theoretical ACF of an AR(p) Process	52
3.1.3	Theoretical ACF of an MA(1) Process	57
3.1.4	Theoretical ACF of an MA(q) Process	59
3.1.5	Theoretical PACFs	63
3.1.6	Summary: Theoretical ACFs and PACFs	64
3.2	Empirical ACFs and PACFs	64
3.2.1	Calculating Empirical ACFs	68
3.2.2	Calculating Empirical PACFs	69
3.3	Putting It All Together	72
3.4	Information Criteria	77
4	Stationarity and Invertibility	81
4.1	What Is Stationarity?	81
4.2	The Importance of Stationarity	82
4.3	Restrictions on AR coefficients Which Ensure Stationarity	83
4.3.1	Restrictions on AR(1) Coefficients	83
4.3.2	Restrictions on AR(2) Coefficients	84
4.3.3	Restrictions on AR(p) Coefficients	92
4.3.4	Characteristic and Inverse Characteristic Equations	93
4.3.5	Restrictions on ARIMA(p,q) Coefficients	94
4.4	The Connection Between AR and MA Processes	95
4.4.1	AR(1) to MA(∞)	95
4.4.2	AR(p) to MA(∞)	97
4.4.3	Invertibility: MA(1) to AR(∞)	97
4.5	What Are Unit Roots, and Why Are They Bad?	99
5	Non-stationarity and ARIMA(p,d,q) Processes	101
5.1	Differencing	101
5.1.1	Example of Differencing	102
5.2	The Random Walk	104
5.2.1	The Mean and Variance of the Random Walk	104
5.2.2	Taking the First Difference Makes it Stationary	105
5.3	The Random Walk with Drift	106
5.3.1	The Mean and Variance of the Random Walk with Drift	106
5.3.2	Taking the First Difference Makes it Stationary	107
5.4	Deterministic Trend	107
5.4.1	Mean and Variance	107
5.4.2	First Differencing Introduces an MA Unit Root	107
5.5	Random Walk with Drift vs Deterministic Trend	108
5.6	Differencing and Detrending Appropriately	109
5.6.1	Mistakenly Differencing (Overdifferencing)	113
5.6.2	Mistakenly Detrending	116

5.7	Replicating Granger and Newbold (1974)	117
5.8	Conclusion	121
6	Seasonal ARMA(p,q) Processes	123
6.1	Different Types of Seasonality	123
6.1.1	Deterministic Seasonality	125
6.1.2	Seasonal Differencing	126
6.1.3	Additive Seasonality	127
6.1.4	Multiplicative Seasonality	128
6.1.5	MA Seasonality	131
6.2	Identification	133
6.3	Invertibility and Stability	135
6.4	How Common are Seasonal Unit Roots?	135
6.5	Using De-seasonalized Data.....	136
6.6	Conclusion	137
7	Unit Root Tests	139
7.1	Introduction	139
7.2	Unit Root Tests.....	140
7.3	Dickey-Fuller Tests	141
7.3.1	A Random Walk vs a Zero-Mean AR(1) Process	142
7.3.2	A Random Walk vs an AR(1) Model with a Constant ..	146
7.3.3	A Random Walk with Drift vs a Deterministic Trend..	148
7.3.4	Augmented Dickey-Fuller Tests	150
7.3.5	DF-GLS Tests	152
7.3.6	Choosing the Lag Length in DF-Type Tests	153
7.4	Phillips-Perron Tests	156
7.5	KPSS Tests	158
7.6	Nelson and Plosser	160
7.7	Testing for Seasonal Unit Roots.....	168
7.8	Conclusion and Further Readings	169
8	Structural Breaks	171
8.1	Structural Breaks and Unit Roots	171
8.2	Perron (1989): Tests for a Unit Root with a Known Structural Break	173
8.3	Zivot and Andrews' Test of a Break at an Unknown Date	184
8.3.1	Replicating Zivot and Andrews (1992) in Stata	185
8.3.2	The <code>zandrews</code> Command	191
8.4	Further Readings	194
9	ARCH, GARCH and Time-Varying Variance	197
9.1	Introduction	197
9.2	Conditional vs Unconditional Moments	200
9.3	ARCH Models	201
9.3.1	ARCH(1)	201
9.3.2	AR(1)-ARCH(1)	208

9.3.3	ARCH(2)	212
9.3.4	ARCH(q)	216
9.3.5	Example 1: Toyota Motor Company	221
9.3.6	Example 2: Ford Motor Company	226
9.4	GARCH Models	229
9.4.1	GARCH(1,1)	229
9.4.2	GARCH(p,q)	233
9.5	Variations on GARCH	239
9.5.1	GARCH- t	240
9.5.2	GARCH-M or GARCH-IN-MEAN	246
9.5.3	Asymmetric Responses in GARCH	250
9.5.4	I-GARCH or Integrated GARCH	257
9.6	Exercises	259
10	Vector Autoregressions I: Basics	263
10.1	Introduction	263
10.1.1	A History Lesson	263
10.2	A Simple VAR(1) and How to Estimate it	266
10.3	How Many Lags to Include?	270
10.4	Expressing VARs in Matrix Form	272
10.4.1	Any VAR(p) Can be Rewritten as a VAR(1)	274
10.5	Stability	276
10.5.1	Method 1	276
10.5.2	Method 2	278
10.5.3	Stata Command Varstable	282
10.6	Long-Run Levels: Including a Constant	282
10.7	Expressing a VAR as a VMA Process	283
10.8	Impulse Response Functions	284
10.8.1	IRFs as the Components of the MA Coefficients	285
10.9	Forecasting	291
10.10	Granger Causality	295
10.10.1	Replicating Sims (1972)	297
10.10.2	Indirect Causality	300
10.11	VAR Example: GNP and Unemployment	302
10.12	Exercises	309
11	Vector Autoregressions II: Extensions	311
11.1	Orthogonalized IRFs	311
11.1.1	Order Matters in OIRFs	313
11.1.2	Cholesky Decompositions and OIRFs	315
11.1.3	Why Order Matters for OIRFs	324
11.2	Forecast Error Variance Decompositions	326
11.3	Structural VARs	329
11.3.1	Reduced Form vs Structural Form	329
11.3.2	SVARs are Unidentified	330
11.3.3	The General Form of SVARs	332

11.3.4	Cholesky is an SVAR	333
11.3.5	Long-Run Restrictions: Blanchard and Quah (1989) ..	336
11.4	VARs with Integrated Variables	339
11.5	Conclusion	340
12	Cointegration and VECMs	343
12.1	Introduction	343
12.2	Cointegration	343
12.3	Error Correction Mechanism	348
12.3.1	The Effect of the Adjustment Parameter	350
12.4	Deriving the ECM	350
12.5	Engle and Granger's Residual-Based Tests of Cointegration	351
12.5.1	MacKinnon Critical Values for Engle-Granger Tests ..	352
12.5.2	Engle-Granger Approach	354
12.6	Multi-Equation Models and VECMs	360
12.6.1	Deriving the VECM from a Simple VAR(2)	361
12.6.2	Deriving the VECM(k-1) from a Reduced-form VAR(k)	362
12.6.3	$\Pi = \alpha\beta'$ is Not Uniquely Identified	363
12.6.4	Johansen's Tests and the Rank of Π	364
12.7	IRFs, OIRFs and Forecasting from VECMs	376
12.8	Lag-Length Selection	376
12.9	Cointegration Implies Granger Causality	377
12.9.1	Testing for Granger Causality	378
12.10	Conclusion	379
12.11	Exercises	379
13	Conclusion	383
A	Tables of Critical Values	389
Bibliography	395
Index	405